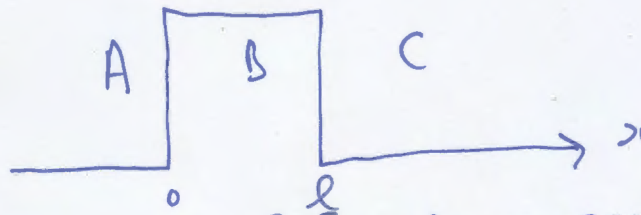
228(1): Spacetime Enhanced Quantum Tunnelling in Low Energy Nuclear Reactions

This mechanism is based on the enhancement of particle momentum in quantum tunnelling, which in this first note is developed in a non-relativistic limit.

Fig (1)



Consider the one dimensional Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V = E, \quad -(1)$$

where:

$$V = 0, \quad x < 0 \quad -(2)$$
$$V = V, \quad 0 \leq x \leq l \quad -(3)$$
$$V = 0, \quad x > l \quad -(4)$$

with reference to Fig. (1).

The solutions are as follows:

$$\psi = A e^{ikx} + B e^{-ikx}, \quad x < 0 \quad -(5)$$
$$k = \left(\frac{2mE}{\hbar^2}\right)^{1/2},$$

$$\psi = A' e^{ik'x} + B' e^{-ik'x}, \quad -(6)$$
$$k' = \left(\frac{2m}{\hbar^2}(E - V)\right)^{1/2},$$

2)
$$\psi = A'' e^{ikx} + B'' e^{-ikx}, \quad -(7)$$
$$k = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$$

Classically, if $E < V$ the particle cannot enter the barrier. In quantum mechanics however it can enter the barrier by quantum tunnelling.

When:
$$x = 0 \quad -(8)$$
then from eqs. (5) and (6):
$$\psi = A + B = A' + B' \quad -(9)$$

When:
$$x = l \quad -(10)$$
then eqs (6) and (7) give:
$$A' e^{-\kappa l} + B' e^{\kappa l} = A'' e^{ikl} + B'' e^{-ikl} \quad -(11)$$

where:
$$k = i\kappa \quad -(12)$$
$$\kappa = \left(\frac{2m}{\hbar^2}(E - V)\right)^{1/2} \quad -(13)$$

when
$$E < V \quad -(14)$$

k is imaginary, κ is real valued.

Now consider:

3)
$$\frac{\partial\phi}{\partial x} = ikAe^{ikx} - ikBe^{-ikx}, \quad x < 0$$
$$\frac{\partial\phi}{\partial x} = ik'A'e^{ik'x} - ik'B'e^{-ik'x}, \quad 0 \leq x \leq l,$$
$$\frac{\partial\phi}{\partial x} = ikA''e^{ikx} - ikB''e^{-ikx}, \quad x > l$$
$$\frac{\partial\phi}{\partial x} = ikA''e \quad -(15)$$

Then:
$$ikA - ikB = -\kappa A' + \kappa B' \quad -(16)$$
$$-\kappa A'e^{\kappa l} + \kappa B'e^{\kappa l} = ikA''e^{ikl} - ikB''e^{ikl} \quad -(17)$$

Consider particles prepared with momentum toward the right, then:
$$B'' = 0 \quad -(18)$$

Then:
$$ik(A-B) = \kappa(B'-A') \quad -(19)$$
$$\kappa e^{\kappa l}(B'-A') = ikA''e^{ikl} \quad -(20)$$
$$A + B = A' + B' \quad -(21)$$
$$A'e^{-\kappa l} + B'e^{\kappa l} = A''e^{ikl} \quad -(22)$$

The probability of finding the particle inside the barrier is:
$$\boxed{P = \frac{|A''|^2}{A^2} = \frac{A''A''^*}{A^2}} \quad -(23)$$

4) using eq. (21):

$$A + B = A' + B' \quad -(21)$$

and from eq. (19):

$$ik(A - B) = \kappa(B' - A') \quad -(22)$$

with:

$$k = i\kappa \quad -(23)$$

so

$$-\kappa(A - B) = \kappa(B' - A') \quad -(24)$$

$$A - B = A' - B' \quad -(25)$$

Adding eqs. (21) and (25):

$$A = A' \quad -(26)$$

Subtracting:

$$B = B'. \quad -(27)$$

Eq. (20) is:

$$\kappa e^{\kappa l}(B' - A') = -\kappa A'' e^{-\kappa l} \quad -(28)$$

so

$$A'' = e^{2\kappa l}(A' - B') = e^{2\kappa l}(A - B) \quad -(29)$$

Eq. (22) is:

$$A' e^{-\kappa l} + B' e^{\kappa l} = A'' e^{-\kappa l}$$

i.e.

$$A'' = A' + B' e^{2\kappa l} \quad -(30)$$

5)

Using eqs. (26) and (27):

$$A'' = A + Be^{2\kappa l} \quad -(31)$$

From eqs. (29) and (31):

$$e^{2\kappa l}(A-B) = A + Be^{2\kappa l} \quad -(32)$$

i.e $\quad A\left(1-e^{2\kappa l}\right) = -2Be^{2\kappa l},$

$$A\left(e^{2\kappa l}-1\right) = 2Be^{2\kappa l}$$

$$B = \frac{1}{2}Ae^{-2\kappa l}\left(e^{2\kappa l}-1\right)$$

$$\boxed{B = \frac{1}{2}A\left(1-e^{-2\kappa l}\right)} \quad -(32).$$

Therefore $\quad A-B = \frac{A}{2} + \frac{A}{2}e^{-2\kappa l}$

$$= \frac{1}{2}A\left(1+e^{-2\kappa l}\right) \quad -(33)$$

It follows that:

$$\frac{A''}{A-B} = 2\frac{A''}{A}\left(1+e^{-2\kappa l}\right)^{-1} \quad -(34)$$

$$= e^{2\kappa l}$$

so:

$$\frac{A''}{A} = \frac{e^{2\kappa\ell}}{2\left(1+e^{-2\kappa\ell}\right)} \quad -(35)$$

This is an obviously incorrect result because

$$\frac{A''}{A} \geq 1 \quad -(36)$$