228(8): Transmission Coefficient in the Presence of Absorption

The transmission coefficient for quantum tunnelling through a barrier of height V_0 and thickness $2a$ is:

$$T = 16\kappa^2 k^2 / \left(\left(e^{4\kappa a} + e^{-4\kappa a} \right) \left(k^2 + \kappa^2 \right) - 2 \left(\kappa^4 + k^4 - 6\kappa^2 k^2 \right) \right) \quad -(1)$$

If

$$x = \kappa a \quad -(2)$$

then T is sketched in Fig (1):

$x = \kappa a$

Fig (1)

It is maximized at

$$x = 0 \quad -(3)$$

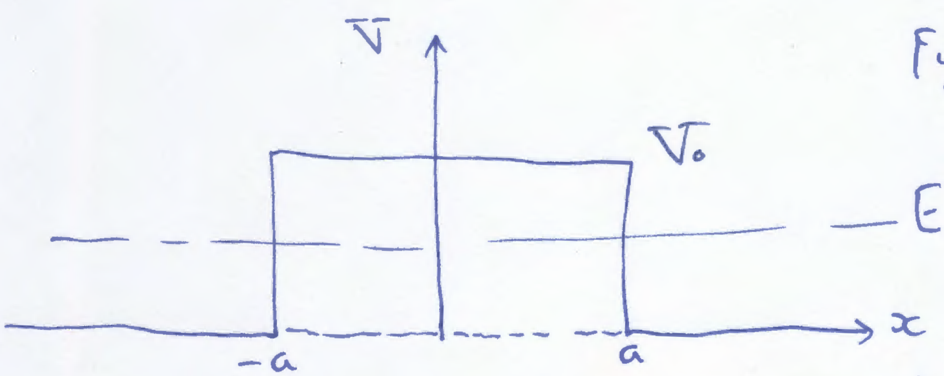

Fig (2)

The function dT/dx has a minimum at:

$$x = 0.25 \quad -(4)$$

The wavenumbers k and κ are defined when the particle approaches from the left into the barrier:

2)

$$p^2 = (\hbar k)^2 = 2mE \quad -(5)$$

$$p_1^2 = (\hbar \kappa)^2 = 2m(V_0 - E) \quad -(6)$$

therefore as E approaches V_0 the transmission coefficient is maximized for any thickness a.

From eqns (1), (5) and (6) it would be useful to plot T as a function of E and find dT/dE and d^2T/dE^2 by computer algebra, and plot these for different V_0 and a.

The transmission coefficient is maximized at 100% by tuning:

$$\boxed{E \rightarrow V_0} \quad -(7)$$

and this process can occur by absorption of energy E_1 from spacetime, so:

$$E + E_1 \rightarrow V_0 \quad -(8)$$

In order to obtain 100% transmission, the amount of spacetime energy needed is:

$$\boxed{E_1 = V_0 - E} \quad -(9)$$

3) It would also be useful to plot T versus K and T versus k, and to evaluate dT/dK, d^2T/dK^2, dT/dk and d^2T/dk^2.

Coulomb Barrier

For a given distance r_0 between the nucleus of atom 1 and the nucleus of atom 2, the Coulomb barrier is order of magnitude:

$$V_0 = \frac{e^2}{4\pi\epsilon_0 r_0} \quad -(10)$$

where

$$e = 1.602 \times 10^{-19}\ C$$

$$4\pi\epsilon_0 = 1.113 \times 10^{-10}\ J^{-1} C^2 m^{-1}$$

$$r_0 \sim 10^{-10}\ m$$

So if r is ~~one or~~ $10^{-10}\ m$:

$$V_0 = 2.306 \times 10^{-18}\ J \quad -(11)$$

So if the energy E of the incoming particle is this amount, there will be complete transmission through the Coulomb barrier. From eq. (5), the momentum of the incoming particle must be:

$$p^2 = 2mE \quad -(12)$$

$$= m^2 v^2$$

where v is its velocity. So:

4)

$$E = \frac{1}{2} m v^2 \quad -(13)$$

$$v = \left(\frac{2E}{m}\right)^{1/2} \quad -(14)$$

If the incoming particle is an electron then:

$$m = 9.10953 \times 10^{-31}\ kg \quad -(15)$$

and

$$v = 2.25 \times 10^{6}\ m s^{-1} \quad -(16)$$

The electron would have to be accelerated close to the speed of light to achieve:

$$E = V_0 \quad -(17)$$

i.e.

$$\frac{1}{2} m v^2 = \frac{e^2}{4\pi \epsilon_0 r_0} \quad -(18)$$

At the point (17) then:

$$T = 1.0 \quad -(19)$$

and the electron tunnels through the barrier for any a. This is a very rough and simple theory, but shows that low energy nuclear reaction can occur.