228(3): Loss of Information in the Dirac Approximation.

The fermion eq. leads to eq. (30) of note 228(2):

$$H\phi^L = \left(mc^2 + V + c^2 \frac{\underline{\sigma} \cdot \underline{p}\ \underline{\sigma} \cdot \underline{p}}{H - V + mc^2} \right) \phi^L \quad -(1)$$

$$= \left(mc^2 + V + \frac{\underline{\sigma} \cdot \underline{p}\ \underline{\sigma} \cdot \underline{p}}{(\gamma + 1) m} \right) \phi^L$$

without any approximation.

The Dirac equation is greatly improved by use of the fermion equation, which leads to eq. (1), without use of the 4 x 4 Dirac matrices. The usual results of the Dirac approximation are obtained from eq. (1) using

$$H = T + V + mc^2$$

$$\sim \gamma mc^2 + V \quad -(2)$$

$$\sim 2mc^2$$

So:

$$V << E, \quad -(3)$$

$$E \simeq (\gamma - 1) mc^2 \sim mc^2,$$

$$\gamma \sim 1$$

So eq. (1) becomes:

2) $$H\phi^L \doteq \left(mc^2 + V + c^2 \underline{\sigma}\cdot\underline{p}\left(\frac{1}{2mc^2 - V}\right)\underline{\sigma}\cdot\underline{p}\right)\phi^L \quad -(4)$$

It is now assumed that:

$$V \ll 2mc^2 \quad -(5)$$

so

$$H\phi^L \doteq \left(mc^2 + V + \frac{c^2}{2mc^2}\underline{\sigma}\cdot\underline{p}\left(1 + \frac{V}{2mc^2}\right)\underline{\sigma}\cdot\underline{p}\right)\phi^L$$

$$= \left(mc^2 + V + \frac{1}{2m}\underline{\sigma}\cdot\underline{p}\left(1 + \frac{V}{2mc^2}\right)\underline{\sigma}\cdot\underline{p}\right)\phi^L \quad (6)$$

which is the well known relativistic quantum hamiltonian of Dirac, with:

$$\underline{p} = -i\hbar\underline{\nabla} \quad -(7)$$

on the right hand side. Eq. (7) is the Schroedinger postulate. Therefore:

$$H\phi^L = \hat{H}\phi^L \quad -(8)$$

where the hamiltonian operator is:

$$\hat{H} = mc^2 + V - \frac{\hbar^2}{2m}\underline{\sigma}\cdot\underline{\nabla}\left(1 + \frac{V}{2mc^2}\right)\underline{\sigma}\cdot\underline{\nabla} \quad -(9)$$

3) The Dirac theory is used for the interaction of a charge e with the electromagnetic potential ϕ on the U(1) level:

$$V = e\phi \quad -(10)$$

On the ECE level there are extra terms giving more information. Therefore eq. (9) needs to be developed on the ECE level. When the vector potential $\underline{A}$ is included on the U(1) level (now obsolete), eq. (9) predicts electron spin resonance and other effects.

Much of the acceptance of the Dirac equation rested on these crude approximations, which eliminate a lot of valuable information.

The unapproximated equation (1) leads to relativistically enhanced quantum tunnelling, as in note 228(2).

When considering the interaction of a particle of mass m_1 with a particle of mass m_2, eq. (1) becomes a quantization of the relativistic

4) Hamilton Jacobi equation:

$$\left(p_1^\mu + p_2^\mu\right)\left(p_{\mu 1} + p_{\mu 2}\right) = \underline{M}^2 c^2 \quad -(11)$$

which is obtained from eqns. (28) and (32) of note 227(1). Here:

$$M^2 = m_1^2 + m_2^2 + 2 m_1 m_2 \left(\gamma_1 \gamma_2 - \left(\gamma_1^2 - 1\right)^{1/2}\left(\gamma_2^2 - 1\right)^{1/2} \cos\theta\right) \quad -(12)$$

Eq. (11) may be described as ~~a~~ result of ~~the~~ minimal prescription

$$p_1^\mu \rightarrow p_1^\mu + p_2^\mu . \quad -(13)$$

In a purely kinetic process:

$$p_1^\mu = \left(\frac{E_1}{c}, \underline{p_1}\right) \quad -(14)$$

$$p_2^\mu = \left(\frac{E_2}{c}, \underline{p_2}\right) \quad -(15)$$

and there is no consideration of potential energy. In this case:

$$H_1 = E_1 = T_1 + m_1 c^2 \quad -(16)$$

$$H_2 = E_2 = T_2 + m_2 c^2 \quad -(17)$$

and eq. (32) of note 227(1) gives:

5)
$$(H_1 + H_2)^2 = c^2(\underline{p}_1 + \underline{p}_2)\cdot(\underline{p}_1 + \underline{p}_2) + \underline{M}^2 c^4 \quad -(18)$$

which quantizes to the fermion equation:

$$\left((H_1 + H_2) + c\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)\right)\phi^L = \underline{M}c^2\phi^R \quad -(19)$$

$$\left((H_1 + H_2) - c\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)\right)\phi^R = \underline{M}c^2\phi^L \quad -(20)$$

so

$$\left((H_1 + H_2)^2 - \underline{M}^2 c^4\right)\phi^L = c^2\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)\phi^L \quad -(21)$$

i.e.

$$(H_1 + H_2)\phi^L = \left(\underline{M}c^2 + \frac{c^2\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)}{H_1 + H_2 + \underline{M}c^2}\right)\phi^L$$

$$= \left(\underline{M}c^2 + \frac{c^2\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)\underline{\sigma}\cdot(\underline{p}_1 + \underline{p}_2)}{(\gamma_1 m_1 + \gamma_2 m_2 + \underline{M})}\right)\phi^L \quad -(22)$$

This is the required generalization of eq. (1). More generally, potential energy is needed, so this will be the subject of the next note.