228(11): Properties of the Transmission Coefficient.

This is:

$$T = \frac{8\kappa^2 k^2}{(k^2+\kappa^2)^2\cosh(4\kappa a) - (\kappa^4 + k^4 - 6\kappa^2 k^2)} \quad -(1)$$

If:

$$\kappa = 0 \quad -(2)$$

then:

$$\cosh(4\kappa a) = 1 \quad -(3)$$

and

$$T = 0. \quad -(4)$$

Here:

$$k^2 = \frac{2mE}{\hbar^2}, \quad \kappa^2 = \frac{2m}{\hbar^2}(V_0 - E). \quad -(5)$$

In the limit:

$$x \gg 1 \quad -(6)$$

where

$$x = \kappa a \quad -(7)$$

then

$$T \rightarrow 16 e^{-4x}\left(\frac{k\kappa}{k^2+\kappa^2}\right)^2 \quad -(8)$$

$$= 16 e^{-4x}\left(\frac{akx}{a^2k^2+x^2}\right)^2$$

So in this limit the maximum of T may be found by:

2)

$$\frac{dT}{dx} = 0 \quad -(9)$$

From eq. (8), the minimum value of T occurs at

$$x = 0, \; T = 0. \quad -(10)$$

We have: $\quad -(11)$

$$\frac{dT}{dx} = -64 e^{-4x}\left(\frac{akx}{a^2k^2+x^2}\right)^2 + 16 e^{-4x}\frac{d}{dx}\left(\frac{akx}{a^2k^2+x^2}\right)^2$$

where:

$$\frac{d}{dx}\left(\frac{akx}{a^2k^2+x^2}\right)^2 = 2\left(\frac{akx}{a^2k^2+x^2}\right)\left(\frac{ak(a^2k^2+x^2)-2akx^2}{(a^2k^2+x^2)^2}\right)$$

$$= 2\left(\frac{akx}{a^2k^2+x^2}\right)\frac{ak(a^2k^2-x^2)}{(a^2k^2+x^2)^2} \quad -(12)$$

Therefore:

$$\frac{dT}{dx} = \frac{32e^{-4x}akx}{(a^2k^2+x^2)^2}\left[\frac{ak(a^2k^2-x^2)}{a^2k^2+x^2} - 2akx\right] = 0$$

$-(13)$

Either $\quad x = 0, \; T = 0 \quad -(14)$

which is a minimum, or:

$$a^2k^2 - x^2 - 2x(a^2k^2+x^2) = 0 \quad -(15)$$

i.e. $\quad a^2k^2 - x^2 - 2xa^2k^2 - 2x^3 = 0 \quad -(16)$

3) This is a cubic equation:

$$2x^3 + x^2 + 2xa^2k^2 - a^2k^2 = 0 \quad -(17)$$

The three roots x can be found by computer algebra.

In general, if:

$$ax^3 + bx^2 + cx + d = 0 \quad -(18)$$

then

$$y^3 + Ay = B \quad -(19)$$

where

$$y = x + \frac{b}{3a}, \quad -(20)$$

$$A = \frac{1}{a}\left(c - \frac{b^2}{3a}\right), \quad -(21)$$

$$B = \frac{1}{a}\left(\frac{bc}{3a} - \frac{2b^3}{27a^2} - d\right). \quad -(22)$$

$-(23)$

If

$$y = s - t$$

$$A = 3st, \quad B = s^3 - t^3 \quad -(24)$$

then

$$u^2 + Bu - \frac{A^3}{27} = 0 \quad -(25)$$

where

$$u = t^3. \quad -(26)$$
