228(6): Transmission Coefficient.

This is defined as:

$$T = \left|\frac{F}{A}\right|^2 = \frac{F}{A}\left(\frac{F}{A}\right)^* \quad -(1)$$

where

$$\frac{F}{A} = \frac{e^{-2ika}}{\cosh(2\kappa a) + i\frac{\epsilon}{2}\sinh(2\kappa a)} \quad -(2)$$

$$\left(\frac{F}{A}\right)^* = \frac{e^{2ika}}{\cosh(2\kappa a) - i\frac{\epsilon}{2}\sinh(2\kappa a)} \quad -(3)$$

So:

$$\boxed{T = \left(\cosh^2(2x) + \frac{\epsilon^2}{4}\sinh^2(2x)\right)^{-1}} \quad -(4)$$

where

$$\epsilon = \frac{\kappa}{k} - \frac{k}{\kappa} = \frac{x}{ka} - \frac{ka}{x} \quad -(5)$$

The maximum value of T is given by:

$$\frac{dT}{dx} = 0 \quad -(6)$$

Computer algebra can be used to determine whether this maximum value is enough to cause a nuclear fusion.

Here:

$$x = \kappa a. \quad -(7)$$

In the limit...

2)

$$\kappa a \gg 1 \quad \text{— (8)}$$

$$T \rightarrow \left(\frac{1}{4} e^{4x}\left(1 + \frac{\epsilon^2}{4}\right)\right)^{-1}$$

$$= \frac{4e^{-4\kappa a}}{1 + \frac{1}{4}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)^2}$$

$$= \frac{16e^{-4\kappa a}}{4 + \left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)^2}$$

$$= \frac{16e^{-4\kappa a}k^2\kappa^2}{4k^2\kappa^2 + (\kappa^2 - k^2)^2} \quad \text{— (9)}$$

$$T = 16e^{-4\kappa a}\left(\frac{k\kappa}{k^2+\kappa^2}\right)^2 \quad \checkmark\checkmark$$

The result by Merzbacher, his eq. (6.45), is correct. The accurate expression is:

$$T = 1\Big/\left[\cosh^2(2x) + \frac{1}{4}\left(\frac{x}{ka} - \frac{ka}{x}\right)^2 \sinh^2(2x)\right]$$

$$\text{— (10)}$$

Where:

$$\cosh(2x) = \frac{1}{2}\left(e^{2x} + e^{-2x}\right) \quad -(11)$$

$$\sinh(2x) = \frac{1}{2}\left(e^{2x} - e^{-2x}\right), \quad -(12)$$

So

$$\cosh^2(2x) = \frac{1}{4}\left(e^{4x} + e^{-4x} + 2\right) \quad -(13)$$

$$\sinh^2(2x) = \frac{1}{4}\left(e^{4x} + e^{-4x} - 2\right) \quad -(14)$$

Therefore

$$T = \frac{4}{A} \quad -(15)$$

where:

$$A = e^{4x} + e^{-4x} + 2 + \frac{1}{4}\left(\frac{x^2 - k^2a^2}{xka}\right)^2\left(e^{4x} + e^{-4x} - 2\right)$$

$$= \left(e^{4x} + e^{-4x}\right)\left(1 + \frac{1}{4}\left(\frac{x^2 - k^2a^2}{xka}\right)^2\right) - 2\left(\frac{1}{4}\left(\frac{x^2 - k^2a^2}{xka}\right)^2 - 1\right)$$

$$= \left(e^{4x} + e^{-4x}\right)\left(\frac{4(xka)^2 + (x^2 - k^2a^2)^2}{4(xka)^2}\right) - 2\left(\frac{(x^2 - k^2a^2)^2 - 4(xka)^2}{4(xka)^2}\right) \quad -(16)$$

4) So:

$$T = \frac{16x^2k^2a^2}{B} \quad -(17)$$

where

$$B = \left(e^{4x} + e^{-4x}\right)\left(x^2 + k^2a^2\right)^2$$
$$\mp 2\left(x^4 + k^4a^4 - 6x^2k^2a^2\right) \quad -(18)$$

where

$$x = \kappa a \quad -(19)$$

Therefore:

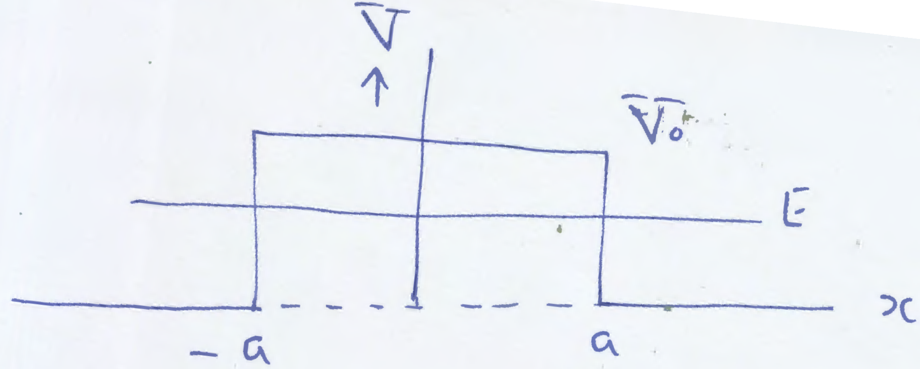
$$T = \frac{16\kappa^2k^2a^4}{\left(e^{4\kappa a} + e^{-4\kappa a}\right)\left(\kappa^2 + k^2\right)^2a^4 \mp 2\left(\kappa^4a^4 + k^4a^4 - 6\kappa^2k^2a^4\right)}$$

$$\boxed{T = \frac{16\kappa^2k^2}{\left(e^{4\kappa a} + e^{-4\kappa a}\right)\left(k^2 + \kappa^2\right)^2 \mp 2\left(\kappa^4 + k^4 - 6\kappa^2k^2\right)}} \quad -(20)$$

In this notation

$$k^2 = \frac{2mE}{\hbar^2}, \quad \kappa^2 = \frac{2m\left(V_0 - E\right)}{\hbar^2} \quad -(21)$$

5)



The transmission coefficient is maximized at:

$$\frac{dT}{dK} = 0. \quad -(22)$$

and this can be worked out with computer algebra to find the value of K at which T is a maximum.

If E is constant, V_0 can be varied to find the maximum T.

The momentum outside the barrier is

$$p = \hbar k \quad -(23)$$

and the momentum inside the barrier is

$$p = \hbar K. \quad -(24)$$

If the maximum momentum inside the barrier is:

$$p_{max} = \hbar K_{max} \quad -(25)$$

then a spacetime momentum p_{max} can be tuned to resonance to maximise the probability of fusion.