228(10): Double Check of Transmission Coefficient.

This is defined by Merzbacher as:

$$T = \left|\frac{F}{A}\right|^2 = \frac{F}{A}\left(\frac{F}{A}\right)^* \quad -(1)$$

where

$$\frac{F}{A} = \frac{e^{-2ika}}{\cosh(2\kappa a) + i\frac{\epsilon}{2}\sinh(2\kappa a)} \quad -(2)$$

$$\left(\frac{F}{A}\right)^* = \frac{e^{2ika}}{\cosh(2\kappa a) - i\frac{\epsilon}{2}\sinh(2\kappa a)} \quad -(3)$$

So

$$T = \frac{1}{\cosh^2(2x) + \frac{\epsilon^2}{4}\sinh^2(2x)} \quad -(4)$$

where

$$\epsilon = \frac{\kappa}{k} - \frac{k}{\kappa} = \frac{x}{ka} - \frac{ka}{x}, \quad -(5)$$

$$\cosh(2x) = \frac{1}{2}\left(e^{2x} + e^{-2x}\right) \quad -(6)$$

$$\sinh(2x) = \frac{1}{2}\left(e^{2x} - e^{-2x}\right). \quad -(7)$$

In the limit:

$$x \gg 1 \quad -(8)$$

$$T \to \left(\frac{1}{4}e^{4x}\left(1 + \frac{\epsilon^2}{4}\right)\right)^{-1} \quad -(8)$$

$$= \frac{4e^{-4x}}{1 + \frac{1}{4}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)^2} \quad -(9)$$

2)

$$= \frac{16 e^{-4\kappa a}}{4 + \left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)^2} \quad -(10)$$

$$= \frac{16 e^{-4x} k^2 \kappa^2}{4k^2\kappa^2 + (\kappa^2 - k^2)^2}$$

$$T = 16 e^{-4\kappa a} \left(\frac{k\kappa}{k^2+\kappa^2}\right)^2 \quad \checkmark\checkmark \quad -(10)$$

This is Merzbacher's eq. (6.45) QED.

The accurate equation is (4), in which:

$$\cosh^2(2x) = \frac{1}{4}\left(e^{4x} + e^{-4x} + 2\right) \quad -(11)$$

$$\sinh^2(2x) = \frac{1}{4}\left(e^{4x} + e^{-4x} - 2\right) \quad -(12)$$

Therefore:

$$T = \frac{4}{A} \quad -(13)$$

where:

$$A = e^{4x} + e^{-4x} + 2 + \frac{1}{4}\left(\frac{x^2 - k^2a^2}{xka}\right)^2 \left(e^{4x} + e^{-4x} - 2\right)$$

$$= \left(e^{4x} + e^{-4x}\right)\left(1 + \frac{1}{4}\left(\frac{x^2 - k^2a^2}{xka}\right)^2\right) - 2\left(\frac{1}{4}\left(\frac{x^2 - k^2a^2}{xka}\right)^2 - 1\right) \quad -(14)$$

So:

$$A = \left(e^{4x} + e^{-4x}\right)\left(\frac{4(xka)^2 + (x^2 - k^2a^2)^2}{4(xka)^2}\right) - 2\left(\frac{(x^2 - k^2a^2)^2 - 4(xka)^2}{4(xka)^2}\right) \quad -(15)$$

So:

$$T = 16x^2k^2a^2 / B \quad -(16)$$

where

$$B = \left(e^{4x} + e^{-4x}\right)\left(x^2 + k^2a^2\right)^2 - 2\left(x^4 + k^4a^4 - 6x^2k^2a^2\right) \quad -(17)$$

Here

$$x = \kappa a \quad -(18)$$

so

$$T = \frac{16\kappa^2 k^2}{\left(e^{4\kappa a} + e^{-4\kappa a}\right)\left(k^2 + \kappa^2\right)^2 - 2\left(\kappa^4 + k^4 - 6\kappa^2 k^2\right)} \quad -(18)$$

where:

$$k^2 = \frac{2mE}{\hbar^2}, \quad \kappa^2 = \frac{2m(V_0 - E)}{\hbar^2} \quad -(19)$$

Eq. (18) is:

$$\boxed{T = \frac{8\kappa^2 k^2}{\left(k^2 + \kappa^2\right)^2 \cosh(4\kappa a) - \left(\kappa^4 + k^4 - 6\kappa^2 k^2\right)}} \quad -(20)$$