228(2): Enhanced Quantum Tunnelling, Basic Definitions.

In standard notation, quantum tunnelling depends on increasing the value of the wave number:

$$\kappa = \left| \left(\frac{2m}{\hbar^2} (E - V) \right)^{1/2} \right| \quad -(1)$$

where the absolute value of κ is implied, because:

$$V > E \quad -(2)$$

The greater the value of κ, the greater the probability of quantum tunnelling. In the context of low energy nuclear fusion, the fusion of two atoms may take place by quantum tunnelling if V is large enough. The energy for ~~low~~ fusion therefore originates in V. More accurately, E is the hamiltonian H, so

$$\kappa = \left| \left(\frac{2m}{\hbar^2} (H - V) \right)^{1/2} \right| \quad -(2)$$

where

$$H = T + V \quad -(3)$$

where T is the kinetic energy and V the potential energy. The momentum in eq. (1) is:

$$p = \hbar \kappa \quad -(4)$$

so

2)

$$p^2 = 2m(E - V) \quad -(5)$$

i.e $$T = \frac{p^2}{2m} \quad -(6)$$

The probability of quantum tunnelling is maximized by making T as large as possible.

The accurate way of writing the Schroedinger equation is:

$$\hat{H}\phi = H\phi \quad -(7)$$

where $$\hat{H} = \frac{\hat{p}^2}{2m} + V, \quad -(8)$$

$$\hat{p} = -i\hbar\underline{\nabla}. \quad -(9)$$

The H on the right hand side of eq. (7) is the total energy:

$$H = T + V. \quad -(10)$$

In relativistic classical physics:

$$H = mc^2 + T + V \quad -(11)$$

$$= E + V$$

where $$E = \gamma mc^2 \quad -(12)$$

$$T = (\gamma - 1)mc^2. \quad -(13)$$

So it is important to distinguish between E and H.

3) The fermion equation is a quantization of eq. (11), and is:

$$(H - V + c\underline{\sigma}\cdot\underline{p})\phi^L = mc^2\phi^R \quad -(14)$$

$$(H - V - c\underline{\sigma}\cdot\underline{p})\phi^R = mc^2\phi^L. \quad -(15)$$

These two equations can be combined to give:

$$((H-V)^2 - c^2\underline{\sigma}\cdot\underline{p}\,\underline{\sigma}\cdot\underline{p})\phi^L = m^2c^4\phi^L \quad -(16)$$

where

$$H - V = E = i\hbar\frac{\partial}{\partial t} \quad -(17)$$

$$p = -i\hbar\underline{\nabla} \quad -(18)$$

so

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\phi^L = 0, \quad -(19)$$

where

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2. \quad -(20)$$

Similarly:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\phi^R = 0. \quad -(21)$$

Here:

$$\phi^L = \begin{bmatrix}\phi_1^L \\ \phi_2^L\end{bmatrix}, \quad \phi^R = \begin{bmatrix}\phi_1^R \\ \phi_2^R\end{bmatrix} \quad -(22)$$

Therefore:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)q^a_\mu = 0 \quad -(23)$$

4)

where:

$$q^a_\mu = \begin{bmatrix} \phi^L_1 & \phi^L_2 \\ \phi^R_1 & \phi^R_2 \end{bmatrix} \quad - (24)$$

is a Cartan tetrad, defined by:

$$\begin{bmatrix} \phi^L \\ \phi^R \end{bmatrix} = \begin{bmatrix} \phi^L_1 & \phi^L_2 \\ \phi^R_1 & \phi^R_2 \end{bmatrix} \begin{bmatrix} \phi^1 \\ \phi^2 \end{bmatrix} \quad - (25)$$

Therefore eq. (23) is an example of the ECE wave equation:

$$(\square + R)\, q^a_\mu = 0 \quad - (26)$$

of tetrad which is a way of writing the Cartan geometry postulate of

$$D_\nu q^a_\mu = 0. \quad - (27)$$

So the origin of the energy observed in low energy nuclear reaction is the geometry of spacetime.

Eq. (16) can be written as:

$$((H - V)^2 - m^2 c^4)\,\phi^L = c^2\, \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p}\, \phi^L \quad - (28)$$

i.e.

$$(H - V - mc^2)\,\phi^L = \left(\frac{c^2\, \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p}}{H - V + mc^2} \right) \phi^L \quad - (29)$$

5) So:

$$H\phi^L = \left(mc^2 + V + \frac{c^2 \underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{p}}{H - V + mc^2} \right) \phi^L \quad -(30)$$

$$= \left(mc^2 + V + \frac{c^2 \underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{p}}{E + mc^2} \right) \phi^L ,$$

$$\boxed{H\phi^L = \left(mc^2 + V + \frac{\underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{p}}{m(\gamma + 1)} \right) \phi^L} \quad -(31)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} , \quad -(32)$$

$$\underline{p} = \gamma m \underline{v} . \quad -(33)$$

In the non-relativistic approximation:

$$v << c \quad -(34)$$

eq. (31) becomes:

$$H\phi^L = \left(mc^2 + V + \frac{p^2}{2m} \right) \phi^L . \quad -(35)$$

The Schroedinger equation is recovered in the usual notation if:

6)

$$E := H - mc^2 \quad -(36)$$

so

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\phi^L = E\phi^L, \quad -(37)$$

or

$$-\frac{\hbar^2}{2m}\nabla^2\phi^L = (E - V)\phi^L, \quad -(38)$$

$$\nabla^2\phi^L = -\frac{2m}{\hbar^2}(E - V)\phi^L \quad -(39)$$

The solutions of this equation lead to eq. (1).

From eq. (31) the relativistic version of eq. (39) is:

$$(H - mc^2 - V)\phi^L = \frac{p^2}{m(\gamma+1)}\phi^L \quad -(40)$$

so:

$$-\nabla^2\phi^L = \frac{m}{\hbar^2}(\gamma+1)(H - mc^2 - V)\phi^L \quad -(41)$$

so eq. (1) becomes:

$$R = \left|\left(\frac{m}{\hbar^2}(\gamma+1)(H - mc^2 - V)\right)^{1/2}\right| \quad -(42)$$

For an ultra-relativistic particle, quantum

7) tunnelling is maximised by:

$$v \to c \quad -(43)$$

for a given V, because:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \to \infty . \quad -(44)$$

Therefore low energy nuclear reaction is most probable for a large V and ultra-relativistic impact velocity.
