222(5): Circumference of a Precessing Orbit

In this case use the general formula:

$$dA = \frac{1}{2} r^2 \, d\theta \quad — (1)$$

and

$$\frac{dA}{dr} = \frac{dA}{dt} \frac{dt}{dr} \quad — (2)$$

with:

$$\frac{dA}{dt} = \frac{L}{2\mu}, \quad \frac{dr}{dt} = -\frac{L}{\mu} \frac{d}{d\theta}\left(\frac{1}{r}\right) \quad — (3)$$

The precessing conical section is defined by:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon \cos(x\theta)\right) \quad — (4)$$

So:

$$\boxed{\frac{dr}{dt} = \left(\frac{\epsilon x L}{\alpha \mu}\right) \sin(x\theta)} \quad — (5)$$

This function can be observed in astronomy, and can be graphed.

Therefore:

$$\boxed{\frac{dA}{dr} = \frac{\alpha}{2 \epsilon x \sin(x\theta)}} \quad — (6)$$

It would also be interesting to graph this function.

The effect of increasing x on eqs. (5) and (6) would be considerable, producing a lot of new information. Polar graphs should be used.

2)

In these equations:

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{d}{r} - 1\right) \quad -(7)$$

so:

$$\sin^2(x\theta) + \cos^2(x\theta) = 1 \quad -(8)$$

from which:

$$\frac{1}{\sin(x\theta)} = \frac{\epsilon r}{(\epsilon^2 r^2 - (d-r)^2)^{1/2}} \quad -(9)$$

Therefore:

$$\boxed{\frac{dA}{dr} = \frac{d}{2x} \cdot \frac{r}{(\epsilon^2 r^2 - (d-r)^2)^{1/2}}} \quad -(10)$$

It would be interesting to graph dA/dr as a function of r for given x, and as a function of x for given r, and as a three dimensional plot.

The area of the orbit is given by:

$$A = \frac{d}{2x}\int_0^R \frac{r\,dr}{(\epsilon^2 r^2 - (d-r)^2)^{1/2}} \quad -(11)$$

3) where R is the circumference of the orbit.

The area of the precessing orbit is also given by eq. (7) of UFT 220:

$$A = \frac{1}{2} \int \left(\frac{\alpha}{1 + \epsilon \cos(x\theta)} \right)^2 d\theta \quad -(12)$$

If θ is changed from 0 to 2π the area is:

$$A = \frac{1}{2} \int_0^{2\pi} \left(\frac{\alpha}{1 + \epsilon \cos(x\theta)} \right)^2 d\theta \quad -(13)$$

If R is the circumference drawn out by changing θ from 0 to 2π then:

$$\int_0^R \frac{r\,dr}{\left(\epsilon^2 r^2 - (\alpha - r)^2\right)^{1/2}} = \frac{x}{\alpha} \int_0^{2\pi} \left(\frac{\alpha}{1 + \epsilon \cos(x\theta)} \right)^2 d\theta \quad -(14)$$

Therefore R can be found from this equation, and R can be plotted as a function of x.