217(8): Asymptotes of the Fractal Hyperbola.

For the ordinary hyperbola:

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1 \quad -(1)$$

the asymptotes are found as follows. Let:

$$Y = mX + c \quad -(2)$$

be the asymptote. Then:

$$(a^2m^2 - b^2)X^2 + 2a^2mcX + a^2(b^2+c^2) = 0 \quad -(3)$$

The asymptote must get nearer and nearer to the hyperbola at infinity. So both roots of eq. (3) are infinite, i.e.

$$a^2m^2 - b^2 = 0 \quad -(4)$$

$$-2a^2mc = 0 \quad -(5)$$

so

$$\boxed{Y = \pm\frac{b}{a}X} \quad -(6)$$

are the asymptotes. From eq. (1):

$$Y = b^2\left(\frac{X^2}{a^2} - 1\right)^{1/2} \quad -(7)$$

so

$$\frac{dY}{dX} = 2\frac{X}{Y}\left(\frac{b^2}{a}\right)^2 = 2\alpha^2\frac{X}{Y} \quad -(8)$$

$$\boxed{\frac{dY}{dX} = 2\alpha^2\frac{X}{Y}} \quad -(9)$$

For the hyperbola with orbital centre at a focal point:

2)
$$\frac{(X - a\epsilon)^2}{a^2} - \frac{Y^2}{b^2} = 1 \quad - (10)$$

and the asymptotes are:
$$\boxed{Y = \pm \frac{b}{a}\left(X - a\epsilon\right)} \quad - (11)$$

where
$$X = -a\epsilon + r\cos\theta \quad - (12)$$
$$Y = r\sin\theta \quad - (13)$$

These are transformed into fractal hyperbolae by
$$\boxed{\theta \rightarrow x\theta} \quad - (14)$$

In the centred hyperbola of eq. (6) the asymptotes are straight lines:
$$\frac{Y}{X} = \tan\theta = \pm\frac{b}{a} \quad - (15)$$

but in the fractal hyperbola:
$$\boxed{\frac{Y}{X} = \tan(x\theta) = \pm\frac{b}{a}} \quad - (16)$$

with completely different properties.

Eq (11) is:
$$Y = \pm\frac{b}{a}X \mp b\epsilon \quad - (17)$$

and this becomes the fractal asymptote:

$$\frac{Y}{X} = \tan(x\theta) = \pm\frac{b}{a} \mp \frac{b\epsilon}{X} \quad -(18)$$

where

$$X = -a\epsilon + \frac{\alpha\cos(x\theta)}{1+\epsilon\cos(x\theta)} \quad -(19)$$

Similarly, eq. (9) for the fractal hyperbola becomes:

$$\boxed{\frac{dY}{dX} = 2\alpha^2\operatorname{cotan}(x\theta)} \quad -(20)$$