217(10): Comparison of Orbital Linear Velocities

1) Fractal Orbits

The orbital linear velocity is given by:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 . \quad -(1)$$

Use:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad -(2)$$

so

$$v^2 = \left(\frac{d\theta}{dt}\right)^2\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right) -(3)$$

The fractal orbits are:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(4)$$

so

$$\frac{dr}{d\theta} = \left(\frac{x\epsilon}{\alpha}\right) r^2 \sin(x\theta), -(5)$$

and:

$$v^2 = \left(\frac{d\theta}{dt}\right)^2 r^2 \left(1 + r^2\left(\frac{x\epsilon}{\alpha}\right)^2 \sin^2(x\theta)\right). -(6)$$

Now use:

$$\frac{d\theta}{dt} = \frac{L}{mr^2} \quad -(7)$$

so

$$v^2 = \left(\frac{L}{mr}\right)^2\left(1 + \left(\frac{x\epsilon r}{\alpha}\sin(x\theta)\right)^2\right) -(8)$$

so the orbital linear velocity is:

$$v = \frac{L}{mr}\left(1 + \left(\frac{x\epsilon \sin(x\theta)}{\alpha}\right)^2\right)^{1/2} \quad -(9)$$

where
$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(10)$$

$$\alpha = \frac{L^2}{mk}, \quad \epsilon = \left(1 + \frac{2EL^2}{mk^2}\right)^{1/2}, \quad -(11)$$

$$k = mMG.$$

Eq. (9) will give an essentially infinite new subject area in orbital dynamics. For example, if $x \sim 1$ the precessing orbits of the solar system are obtained.

<u>Suggested Plots</u>

1) v against θ for $x = 1$ (Newtonian)
2) v against θ for $x \sim 1$ (precessing orbits).
3) v against θ for x increased slightly from $x \sim 1$ to study the precise way in which the orbit starts to deviate from the precessing ellipse.
4) v against θ for x in general.

The results from EGR are as follows:

3)
$$\frac{dr}{d\theta} = r^2\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{1/2} \quad -(12)$$

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E}, \quad r_0 = \frac{2MG}{c^2}, \quad -(13)$$

$$L = mr^2\frac{d\theta}{d\tau}; \quad \omega = \frac{cb}{r^2}\left(1-\frac{r_0}{r}\right), \quad -(14)$$

$$L = mr^2\omega\frac{dt}{d\tau}. \quad -(15)$$

So
$$\omega = \frac{L}{mr^2}\frac{d\tau}{dt} = \frac{d\theta}{dt}, \quad -(16)$$

where
$$\frac{d\tau}{dt} = \frac{mc^2}{E}\left(1-\frac{r_0}{r}\right) \quad -(17)$$

so
$$\frac{d\theta}{dt} = \frac{Lc^2}{Er^2}\left(1-\frac{r_0}{r}\right). \quad -(18)$$

The orbital linear velocity in EGR is therefore:

$$v = \frac{cb}{r}\left(1-\frac{r_0}{r}\right)\left(r^2\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right) + 1\right)^{1/2}$$

$-(19)$

Compare eq. (19) graphically with eq (9).
The results will be completely different.