217(5): Representation of Precessing Hyperbola.

In polar representation:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)}, \quad -(1)$$

$$\alpha = a(\epsilon^2 - 1), \quad -(2)$$

$$\epsilon > 1. \quad -(3)$$

In Cartesian representation:

$$\frac{(X - a\epsilon)^2}{a^2} - \frac{Y^2}{b^2} = 1 \quad -(4)$$

where

$$X = -a\epsilon + r\cos\theta \quad -(5)$$

$$Y = r\sin\theta \quad -(6)$$

$$r = -a - \epsilon X, \quad -(7)$$

$$\epsilon = \left(1 + \frac{b^2}{a^2}\right)^{1/2}. \quad -(8)$$

So:

$$X = -a\epsilon + \frac{\alpha\cos(x\theta)}{1 + \epsilon\cos(x\theta)} \quad -(9)$$

$$Y = \frac{\alpha\sin(x\theta)}{1 + \epsilon\cos(x\theta)} \quad -(10)$$

i.e

$$X = \frac{\alpha\epsilon}{1 - \epsilon^2} + \frac{\alpha\cos(x\theta)}{1 + \epsilon\cos(x\theta)} \quad -(11)$$

$$Y = \frac{\alpha\sin(x\theta)}{1 + \epsilon\cos(x\theta)}$$

As x is varied, X and Y will produce many different patterns in mathematics