217(12): Orbits from the Schroedinger Orbitals for H

The orbits are given by:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(1)$$

where

$$x = 4.474467 \times 10^{19} \quad -(2)$$

The half right latitude is:

$$\alpha = \frac{\ell(\ell+1)\hbar^2}{(x^2-1)m^2 M G} \quad -(3)$$

where m is the electron mass and M the proton mass.

So:

$$\alpha = 5.33413 \times 10^{-11} \ell(\ell+1) \quad -(4)$$

The eccentricity is given by:

$$\epsilon = \left(1 + \frac{2|E|\alpha}{mMG}\right)^{1/2} \quad -(5)$$

$$= \left(1 + 1.049296 \times 10^{57} \ell(\ell+1)|E|\right)^{1/2}.$$

The magnitude of the energy levels of H is given by:

$$|E| = \left(\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\right)\frac{1}{n^2} \quad -(6)$$

$$= \frac{2.2 \times 10^{-18}}{n^2} \text{ J}$$

where n is the principal quantum number.

2) So:

$$\epsilon = \left(1 + 2.30845 \times 10^{39} \frac{\ell(\ell+1)}{n^2}\right)^{1/2}$$

$$\alpha = 5.33413 \times 10^{-11} \ell(\ell+1)$$

$$x = 4.474467 \times 10^{19}$$

Orbital	n	ℓ
1s	1	0
2s	2	0
2p	2	1
3s	3	0
3p	3	1
3d	3	2

So α and ϵ are different for each orbital, but x is the same.

Notes

1) The Bohr radius is:

$$r_B = 5.29177 \times 10^{-11} \, m$$

and is very close to α for the 2p and 3p orbitals.

2) For the s orbitals, $\alpha = 0$, there is no resultant angular momentum.