217(2): Comparison of Precessing Ellipse and Einstein Theory

The precessing ellipse is given by:

$$\theta = \frac{1}{x} \sin^{-1}\left[\left(1 - \frac{1}{\epsilon^2}\right) + \frac{2\alpha}{\epsilon^2}\frac{1}{r} - \left(\frac{\alpha}{\epsilon}\right)^2 \frac{1}{r^2}\right]^{1/2} \quad -(1)$$

so the suggested plot is θ against r for values of ϵ and α with the constraint:

$$\sin^2(x\theta) \leq 1. \quad -(2)$$

For the Einstein theory:

$$\theta = \frac{1}{x} \sin^{-1}\left[\left(\frac{\alpha}{x\epsilon}\right)^2\left(\frac{1}{b^2} - \frac{1}{a^2}\right) + \frac{r_0}{ar} - \frac{1}{r^2} + \frac{r_0}{r^3}\right] \quad -(3)$$

with

$$\sin^2(x\theta) \leq 1. \quad -(4)$$

From eq (1):

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(5)$$

From eq. (3):

$$\cos(x\theta) = \left[1 - \left(\frac{\alpha}{x\epsilon}\right)^2\left[\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right]\right]^{1/2} \quad -(6)$$