216(7): Calculation of Orbital Deflection

The calculation starts with:

$$\epsilon = \frac{1}{\sin\phi} \quad -(1)$$

the angle ϕ of the conjugate hyperbola is $\pi/2$ minus the angle of the original. Here:

$$\frac{1}{\sin\phi} = \left(1 + \frac{b^2}{a^2}\right)^{1/2} \quad -(2)$$

where

$$\alpha = b^2 / a. \quad -(3)$$

These are properties of the hyperbola. So:

$$\sin\phi = \frac{1}{\epsilon} \quad -(4)$$

and for small angles:

$$\sin\phi \sim \phi = \frac{1}{\epsilon} \quad -(5)$$

The angle of deflection is:

$$\boxed{\Delta\phi = 2\phi = \frac{2}{\epsilon}} \quad -(6)$$

For the precessing hyperbola:

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad -(7)$$

the orbital velocity was shown in UFT215 to be:

$$v^2 = \left(\frac{L}{m\alpha}\right)^2 \left(\frac{2x^2\alpha}{r} + x^2(\epsilon^2 - 1) - \left(\frac{\alpha}{r}\right)^2 (x^2 - 1)\right) \quad -(8)$$

2) At the distance of closest approach:

$$r = R_0 \quad -(9)$$

and

$$\cos(x\theta) = 1 \quad -(10)$$

so

$$d = R_0(1+\epsilon) \quad -(11)$$

and

$$v^2 = \frac{x^2 L^2}{m^2 d R_0}(1+\epsilon) - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0^2}\right) \quad -(12)$$

i.e.

$$\frac{x^2 L^2}{m^2 d R_0}(1+\epsilon) = v^2 - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0^2}\right) \quad -(13)$$

and

$$\epsilon = \frac{1}{\sin\phi} \quad -(14)$$

so

$$1+\epsilon = \frac{m^2 d R_0}{x^2 L^2}\left(v^2 - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0^2}\right)\right)$$

and

$$\sin\phi = \frac{1}{\epsilon} = \left[\frac{m^2 d R_0}{x^2 L^2}\left(v^2 - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0^2}\right)\right) - 1\right]^{-1} \quad -(15)$$

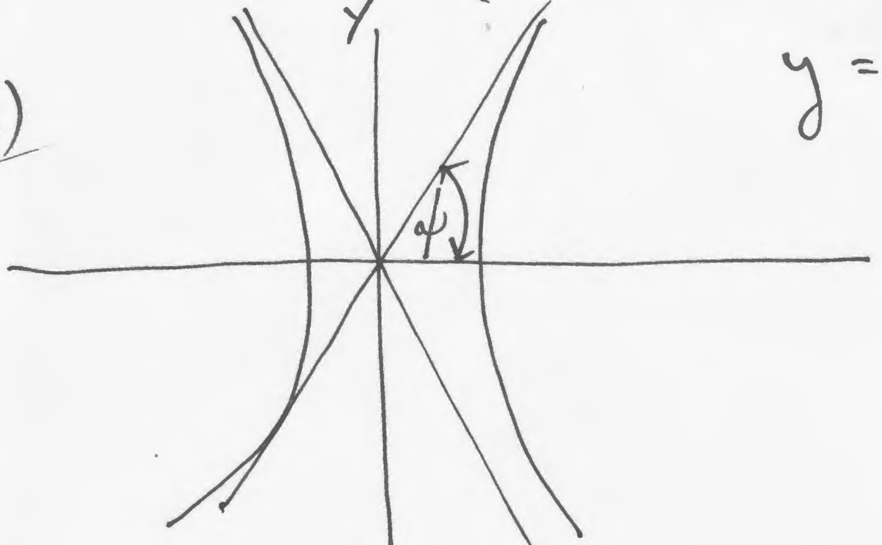
The asymptotes of the hyperbola are:

$$y = \pm \frac{a}{b} x \quad \text{— (16)}$$

so
$$\tan \phi = \frac{a}{b} \quad \text{— (17)}$$

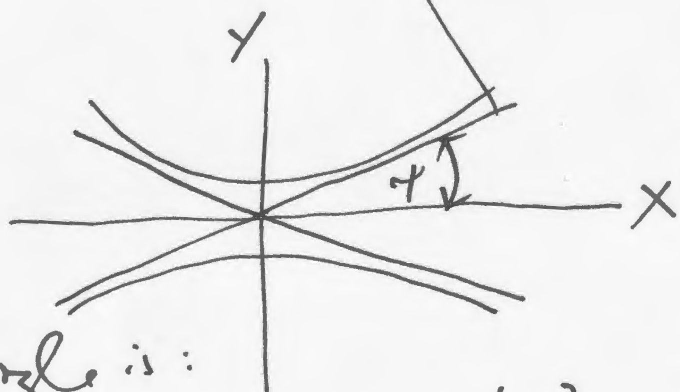
$$\sin \phi = \frac{a}{(a^2+b^2)^{1/2}} \quad \text{— (18)}$$

Fig (1)



$$y = \pm \frac{b}{a} x$$

Fig (2)



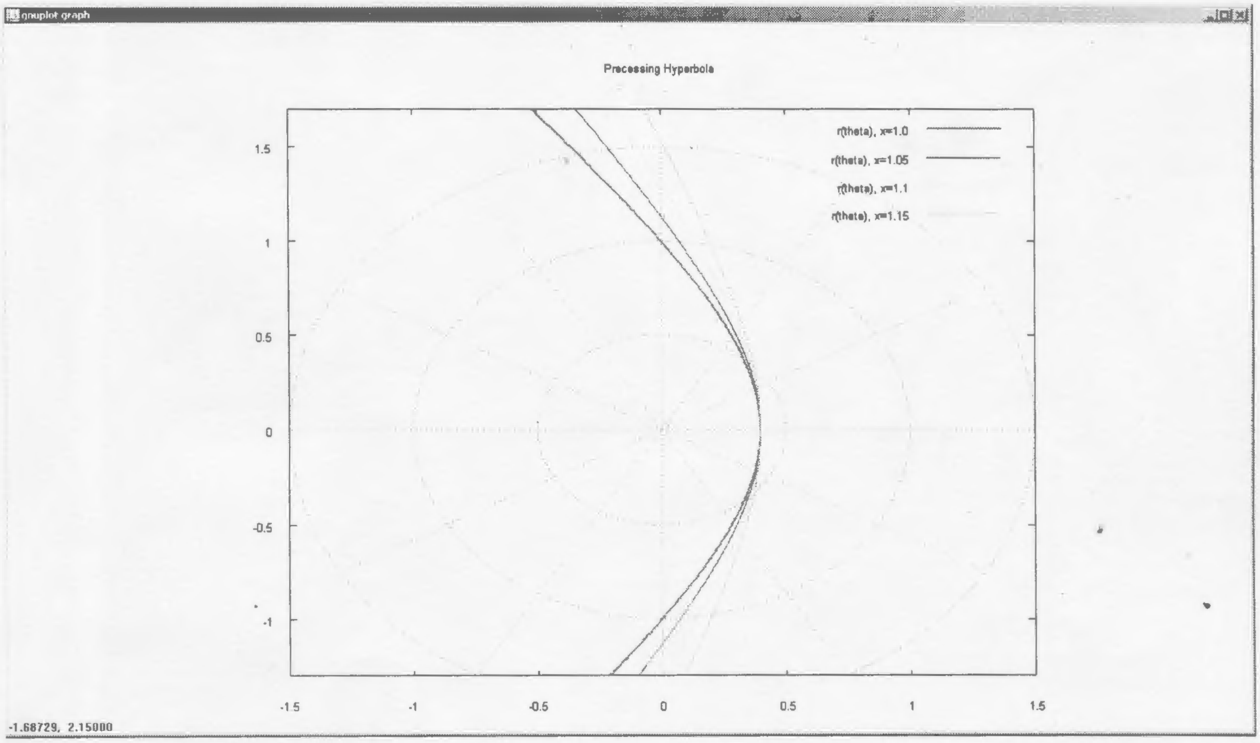
$$y = \pm \frac{a}{b} x$$

The deflection angle is:

$$\sin \phi = \frac{1}{\epsilon} = \left(\frac{a^2}{a^2+b^2} \right)^{1/2} \quad \text{— (19)}$$

and is the one in Figure 2. The total deflection is:

$$2\phi = 2 \sin^{-1} \frac{1}{\epsilon} \quad \text{— (20)}$$



Effect of increasing x on the hyperbola:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)}$$