216(5): The Condition at Which the Precessing Conical Section Becomes a Hyperbolic Spiral.

It is now known that the precessing conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(1)$$

produces a vast array of hitherto unknown planar orbits from a new universal force law. In the small angle limit the deflection of m due to M is:

$$\Delta\theta = \frac{2}{\epsilon x}. \quad -(2)$$

Consider the hyperbolic spiral:

$$r = \frac{r_0}{x\theta} \quad -(3)$$

For the sake of simplicity assume that

$$r_0 = \alpha \quad -(4)$$

then

$$r = \frac{\alpha}{x\theta}. \quad -(5)$$

Eq (1) becomes eq (5) when:

$$\boxed{1 + \epsilon \cos(x\theta) = x\theta} \quad -(6)$$

The solution to eq. (6) can be found

graphically by plotting the left hand side and right hand side as a function of x. If the solution of eq. (6) is denote θ_0 then:

$$\boxed{r = \frac{\alpha}{1+\epsilon\cos(x\theta_0)} = \frac{\alpha}{x\theta_0}} \quad -(7)$$

Under this condition the function is both a precessing conical section and a hyperbolic spiral.

Thus far the precession factor x has been considered to be a constant, but if x is allowed to vary, and if

$$\phi := x\theta_0 \quad -(8)$$

then:

$$\boxed{r = \frac{\alpha}{1+\epsilon\cos\phi} = \frac{\alpha}{\phi}} \quad -(9)$$

where ϕ is an angle that varies.

Under these conditions the conical section orbit becomes a hyperbolic spiral orbit.