Note 216(1): Calculation of Photon Mass from Distance of Closest Approach.

Use the Newtonian equation:

$$v^2 = mG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(1)$$

where v is the velocity of the photon, m is the mass of the sun, G is the Newton constant, r the sun to photon distance and where

$$a = \frac{R_0}{1-\epsilon} \quad -(2)$$

where R_0 is the distance of closest approach and ϵ the eccentricity of the photon's orbit.

At closest approach:

$$v^2 = mG\left(\frac{2}{R_0} - \frac{1-\epsilon}{R_0}\right) \quad -(3)$$

$$= \frac{mG}{R_0}(1+\epsilon)$$

so

$$\boxed{1+\epsilon = \frac{R_0 v^2}{mG}} \quad -(4)$$

For the photon, assume that:

$$v \rightarrow c \quad -(5)$$

then

$$1+\epsilon = \frac{R_0 c^2}{mG}. \quad -(6)$$

Here:

$$R_\odot = 6.955 \times 10^{8} \, m$$
$$c = 2.998 \times 10^{8} \, m s^{-1}$$
$$M = 1.9891 \times 10^{30} \, kgm$$
$$G = 6.67428 \times 10^{-11} \, m^3 \, kg^{-1} \, s^{-2}$$

So
$$\boxed{\epsilon = 3.709 \times 10^{5}} \quad -(7)$$

<u>which</u> is <u>a hyperbola</u>:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(8)$$

In Newtonian theory:

$$\epsilon = \left(1 + \frac{2EL^2}{m^3 M^2 G^2} \right)^{1/2} \quad -(9)$$

where m is the photon mass, E the total energy and L the total angular momentum.

The photon is considered to be a particle of mass m attracted by the sun of mass M. From eqs. (7) and (9):

$$1 + \frac{2EL^2}{m^3 M^2 G^2} = 1.376 \times 10^{10} \quad -(10)$$

and to an excellent approximation:

3)

$$\boxed{m^3 = \frac{2EL^2}{1.376\, m^2 G^2} \times 10^{-10}} \quad -(11)$$

In a pure classical Newtonian theory:

$$E = \frac{1}{2}mv^2 - \frac{mMG}{r} \quad -(12)$$

and

$$L = mr^2\omega \quad -(13)$$

where ω is the angular velocity. So m cancels out on both sides of eq. (11).

In order to obtain an estimate of the photon mass the de Broglie equation is introduced:

$$E = \hbar\omega = \gamma mc^2 \quad -(14)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(15)$$

Here E is the total kinetic ~~energy of the~~ photon:

$$E = T + E_0 \quad -(16)$$

where the rest energy is:

$$E_0 = mc^2, \quad -(17)$$

and the relativistic kinetic energy is:

$$T = (\gamma - 1)mc^2 \quad -(18)$$

4) The Newtonian kinetic energy is the limit:

$$T \xrightarrow[v \ll c]{} mc^2\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right)$$

$$= mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right) \quad -(19)$$

$$= \frac{1}{2}mv^2$$

using:

$$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots \quad -(20)$$

For:

$$\frac{v}{c} = 0.32, \quad -(21)$$

$$\frac{1}{2}x = 0.05, \quad \frac{3}{8}x^2 = 0.00375 \quad -(22)$$

so the result (19) is true to a very good approximation up to v about half c.

The Newtonian total energy is obtained from the limit (19) of the Sommerfeld Hamiltonian:

$$H = E = (\gamma - 1)mc^2 + U(r) \quad -(23)$$

where $U(r)$ is the potential energy. So

$$E = \hbar\omega - mc^2 + U(r) \quad -(24)$$

From elementary particle table:

$$m \lesssim 10^{-52}\ \text{kg} \quad -(25)$$

At a visible frequency:

$$\omega \sim 10^{16}\ \text{rad s}^{-1} \quad -(26)$$

then

$$\hbar\omega \sim 10^{-18}\ \text{J}$$

$$mc^2 \sim 10^{-36}\ \text{J} \quad -(27)$$

$$U \sim 10^{-46}\ \text{J}$$

for

$$R_0 = 6.955 \times 10^{8}\ \text{m} \quad -(28)$$

so

$$\boxed{E = \hbar\omega} \quad -(29)$$

From eqs. (11) and (29):

$$\boxed{m^3 = 8.697 \times 10^{-69}\ L^2} \quad -(30)$$

at the visible frequency defined by eq. (26)

the angular momentum L is:

$$\underline{L} = \underline{\hbar} + \underline{L_{orb}} \quad -(31)$$

so:

$$L^2 = (\hbar + L_{orb})^2 \quad -(32)$$

If there were no orbital angular momentum then

$$L = \hbar = 1.05459 \times 10^{-34} \text{ Js} \quad -(33)$$

and

$$\boxed{m = 2.13 \times 10^{-46} \text{ kgm}} \quad -(34)$$

In general the orbital angular momentum of the photon may be non-zero, and a method must be found for estimating L_{orb}. This will be the subject of the next note.