215(4): Self Consistent Calculation of Force in the Static Frame.

In the static frame the acceleration is:

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta \quad -(1)$$

and the orbit is:

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)}. \quad -(2)$$

The conserved angular momentum is:

$$L = mr^2\frac{d\theta}{dt} \quad -(3)$$

in the static frame, as shown in note 215(3).

From eq. (2):

$$\frac{dr}{d\theta} = \frac{x\epsilon}{\alpha}r^2\sin(x\theta) \quad -(4)$$

and

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{xL\epsilon}{m\alpha}\sin(x\theta). \quad -(5)$$

From eq. (3)

$$\dot{\theta} = \frac{L}{mr^2} \quad -(6)$$

Therefore:

$$\ddot{r} = \frac{xL\epsilon}{m\alpha}\frac{d}{dt}\left(\sin(x\theta)\right) = \frac{x^2L\epsilon}{m\alpha r^2}\cos(x\theta) \quad -(7)$$

) and

$$\ddot{\theta} = \frac{L}{m}\frac{d}{dt}\left(\frac{1}{r^2}\right) \quad -(8)$$

$$= -\frac{2L^2 \epsilon x}{m^2 \alpha r^3} \sin(x\theta)$$

Now use:

$$\alpha = \frac{L^2}{m^2 M G} \quad -(9)$$

then:

$$\ddot{r} = \frac{x^2 M \epsilon G}{r^2}\cos(x\theta). \quad -(10)$$

Note that:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\frac{2xL^2\epsilon}{m^2\alpha r^2}\sin(x\theta) + \frac{2xL^2\epsilon}{m^2\alpha r^2}\sin(x\theta) = 0 \quad -(11)$$

So:

$$\underline{a} = \left(\frac{x^2 M \epsilon G}{r^2}\cos(x\theta) - \frac{L^2}{m^2 r^3}\right)\underline{e}_r \quad -(12)$$

where:

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{L^2}{m^2 M G r} - 1\right) \quad -(13)$$

i.e.

$$\boxed{\underline{a} = \left(-\frac{x^2 M G}{r^2} + (x^2 - 1)\frac{L^2}{m^2 r^3}\right)\underline{e}_r}$$

$-(14)$

The force is:

$$\underline{F} = m\,\underline{a} \quad -(15)$$

Eq. (15) is the same result as obtained in UFT 214 QED.

The method is self consistent.

Summary

1) Static frame force $\quad -(16)$

$$\underline{F}_s = \left(-x^2 \frac{mMG}{r^2} + (x^2 - 1)\frac{L^2}{mr^3} \right) \underline{e}_{rs}$$

$$\underline{e}_{rs} = \underline{i} \cos\theta + \underline{j} \sin\theta \quad -(17)$$

2) Moving frame force

$$\underline{F}_m = -\frac{mMG}{r^2}\,\underline{e}_{rm} \quad -(18)$$

$$\underline{e}_{rm} = \underline{i} \cos(x\theta) + \underline{j} \sin(x\theta) \quad -(19)$$