215(2): Linear Velocity in a Precessing Planetary Orbit

In this case:

$$\left(\frac{d\underline{r}}{dt}\right)_{fixed} = \left(\frac{d\underline{r}}{dt}\right)_{rotating} + \underline{\omega} \times \underline{r} \quad -(1)$$

where

$$\underline{v}_{fixed} = \left(\frac{d\underline{r}}{dt}\right)_{fixed} = \dot{r}\left(\underline{e}_r\right)_f + r\dot{\theta}\,\underline{e}_\theta \quad -(2)$$

$$\underline{v}_{rotating} = \left(\frac{d\underline{r}}{dt}\right)_{rotating} = \dot{r}\left(\underline{e}_r\right)_r + r\dot{\beta}\,\underline{e}_\beta \quad -(3)$$

where:

$$\left(\underline{e}_r\right)_f = \underline{i}\cos\theta + \underline{j}\sin\theta \quad -(4)$$

$$\left(\underline{e}_r\right)_r = \underline{i}\cos\beta + \underline{j}\sin\beta \quad -(5)$$

$$\underline{e}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta \quad -(6)$$

$$\underline{e}_\beta = -\underline{i}\sin\beta + \underline{j}\cos\beta \quad -(7)$$

So:

$$\underline{v}_{fixed} - \underline{v}_{rotating}$$

$$= \dot{r}\left(\underline{i}\left(\cos\theta - \cos(x\theta)\right) + \underline{j}\left(\sin\theta - \sin(x\theta)\right)\right)$$

$$+ r\dot{\theta}\left(-\underline{i}\sin\theta + \underline{j}\cos\theta\right)$$

$$- r\dot{\beta}\left(-\underline{i}\sin(x\theta) + \underline{j}\cos(x\theta)\right)$$

2)

$$= \underline{i}\left[\dot{r}\left(\cos\theta - \cos(x\theta)\right) - r\dot{\theta}\sin\theta + r\dot{\beta}\sin(x\theta)\right]$$

$$+ \underline{j}\left[\dot{r}\left(\sin\theta - \sin(x\theta)\right) + r\dot{\theta}\cos\theta - r\dot{\beta}\cos(x\theta)\right]$$

$$= \underline{i}\left[\dot{r}\left(\cos\theta - \cos(x\theta)\right) - r\dot{\theta}\sin\theta + xr\dot{\theta}\sin(x\theta)\right]$$

$$+ \underline{j}\left[\dot{r}\left(\sin\theta - \sin(x\theta)\right) + r\dot{\theta}\cos\theta - xr\dot{\theta}\cos(x\theta)\right]$$

$$\boxed{\begin{aligned} &= \underline{i}\left[\dot{r}\left(\cos\theta - \cos(x\theta)\right) - r\dot{\theta}\left(\sin\theta - x\sin(x\theta)\right)\right] \\ &+ \underline{j}\left[\dot{r}\left(\sin\theta - \sin(x\theta)\right) + r\dot{\theta}\left(\cos\theta - x\cos(x\theta)\right)\right] \\ &= \underline{\omega} \times \underline{r} \end{aligned}}$$

— (8)

If $x = 1$ — (9)

then $\underline{\omega} \times \underline{r} = \underline{0}$ — (10)