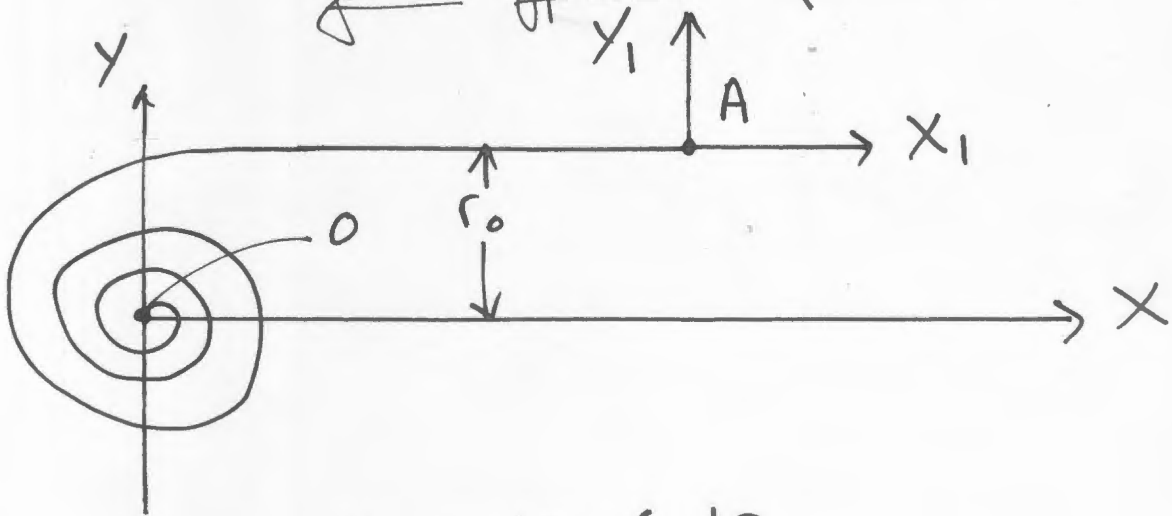
## 209(3): Definition of Coordinate System for the Hyperbolic Spiral




## X – Y Coordinate System

In this system:

$$r = \frac{r_0}{\theta} \quad -(1)$$

where

$$X = r\cos\theta,\ Y = r\sin\theta,\quad \tan\theta = \frac{Y}{X} \quad -(2)$$

$$r^2 = X^2 + Y^2$$

so

$$X = \frac{r_0}{\theta}\cos\theta,\quad Y = \frac{r_0}{\theta}\sin\theta. \quad -(3)$$

As:

$$\theta \rightarrow 0 \quad -(4)$$

$$X \rightarrow \infty,\quad Y \rightarrow r_0. \quad -(5)$$

The point A is defined by:

$$r \rightarrow \infty,\quad \theta \rightarrow 0 \quad -(6)$$

This system is not suitable for a whirlpool galaxy because it describes an orbit in which a particle of mass $m$ <u>starts</u> at $A$ at time $t = 0$, and spirals <u>into</u> $O$ a time $t$ later. In Newtonian dynamics this needs an inverse cubed law in $r$ as first shown by Cotes. We are developing a new relativity and new dynamics which is <u>non</u> - <u>Newtonian</u>. In the whirlpool galaxy a star of mass $m$ starts at $O$ and finishes at $A$. The motion is due to the torsion and curvature of spacetime. These are different basic concepts from those of Newton and Einstein. In the new theory the angle $\theta(t)$ is defined by:

$$\theta(t) = \sqrt{2}\left(\omega_0 t + C\right)^{1/2} \quad -(7)$$

<u>and increases with time</u>. At the point $A$ in the new theory:

$$\boxed{\theta \rightarrow \infty, \quad r \rightarrow 0.} \quad -(8)$$

## <u>$X_1 - Y_1$ Coordinate System</u>

By reference to the figure the point $A$ is at

$$X_1 = 0, \quad Y_1 = 0 \quad -(9)$$
$$X \rightarrow \infty, \quad Y = r_0$$

So:

3)

$$Y_1 = Y - r_0 \qquad (10)$$

$$X_1 = X - \infty$$

and $$r^2 = X_1^2 + Y_1^2 \qquad (11)$$

$$r = \frac{r_0}{\theta} \qquad (12)$$

Therefore in this coordinate system, point A is at

$$r^2 = X_1^2 + Y_1^2 = 0 \qquad (13)$$

so eq. (8) follows. The point O is at:

$$r^2 = X_1^2 + Y_1^2 = \infty \qquad (14)$$

because $$X = 0, \; Y = 0. \qquad (15)$$

The time at point A is $t$ and the time at point O is zero. The time is the measured in the observer frame because the shift:

$$(X, Y) \rightarrow (X_1, Y_1) \qquad (16)$$

is a shift in space. The proper time $\tau$ is the time measured in the <u>moving</u> frame $(X_1, Y_1)$.

In the figure, $(X_1, Y_1)$ is not moving with respect to $(X, Y)$, there is no time in the transformation (10). The linear orbital velocity at the

4) point A is, from note 209(1):

$$v(t) = \frac{\omega_0 r_0}{2(\omega_0 t + C)}\left(1 + \frac{1}{2(\omega_0 t + C)}\right) \quad -(17)$$

It is seen from the figure that the velocity is
in the $X_1$ axis. The particle or star m is
moving in a straight line. The time t is of
the order of millions of years. By observation,
this velocity becomes constant as $r \rightarrow \infty$. So
at point A the linear velocity of the star becomes
become constant. The time interval t is
effectively infinite, and so
eq. (17) is therefore constant. The time interval t is
$v(t)$ is constant.
very large. It was calculated from eq.
(4) of note 209(1):

$$\omega_0 \int dt = \int \theta(t)\, d\theta \quad -(17)$$

If it is assumed that the time elapsed from
point o to A is $\tau$, and that the angle at point
A is $\theta_1$, then:

$$\omega_0 \int_0^{\tau} dt = \int_0^{\theta_1} \theta(t)\, d\theta \quad -(18)$$

and:

5)

$$\omega_0 \tau = \theta_1^2 / 2 \quad -(19)$$

Therefore in eq. (17):

$$v(\tau) = \frac{\omega_0 r_0}{2\omega_0 \tau}\left(1 + \frac{1}{2\omega_0 \tau}\right) \quad -(20)$$

i.e.

$$\boxed{v(\tau) \xrightarrow[r \to \infty]{} \frac{r_0}{2\tau}\left(1 + \frac{1}{2\omega_0 \tau}\right)} \quad -(21)$$

To observers in the Hubble space telescope the velocity appears constant, because the star in the galaxy has evolved to its final state.