209(1): Time Evolution and Animation of Orbital Trajectories in a Galaxy

If the orbit is assumed to be the hyperbolic spiral:

$$r(t) = \frac{r_0}{\theta(t)} \quad -(1)$$

then

$$\omega(t) = \frac{d\theta}{dt} = \frac{\omega_0}{\theta(t)} \quad -(2)$$

so

$$\omega_0 \, dt = \theta(t) \, d\theta \quad -(3)$$

and

$$\omega_0 \int dt = \int \theta(t) \, d\theta \quad -(4)$$

therefore:

$$\omega_0 t + C_1 = \frac{\theta^2}{2} + C_2 \quad -(5)$$

or

$$\boxed{\theta^2 = 2(\omega_0 t + C)} \quad -(6)$$

where

$$C = C_1 - C_2 \quad -(7)$$

is the constant of integration. It follows that

$$\theta(t) = \sqrt{2}\,(\omega_0 t + C)^{1/2}. \quad -(8)$$

The dynamics of a whirlpool galaxy may now be animated.

From UFT 208:

$$r(t) = \frac{r_0}{\theta(t)} = \frac{r_0}{\sqrt{2}}\left(\omega_0 t + C\right)^{-1/2} \quad -(9)$$

and this gives the dynamics of the position of the star as a function of time. The way the star moves can be animated.

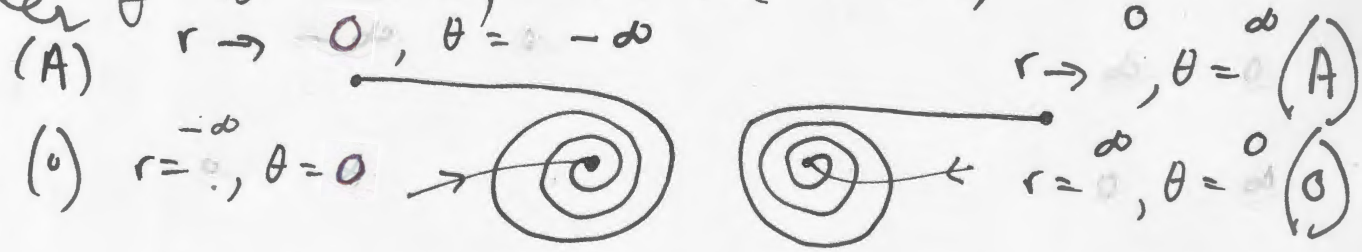
The torsional dynamics are given by:

$$T^1_{01}(t) = \frac{\omega_0}{c\left(1 + 2(\omega_0 t + C)\right)} \quad -(10)$$

and the orbital linear velocity is given by:

$$v(t) = \frac{\omega_0 r_0}{2(\omega_0 t + C)}\left(1 + \frac{1}{2(\omega_0 t + C)}\right) \quad -(11)$$

In the hyperbolic spiral the distance r is infinite when θ is zero, and vice versa,



so eq. (9) has to be interpreted in this way. Note carefully that this does not mean that the star

3)

From eq. (6) it is seen that the angle $\theta(t)$ increases with time, so the star travels from the centre outwards. The sense of the spiral is:

$$r = \frac{r_0}{\theta} \quad (\text{counterclockwise}) \qquad -(12)$$

and

$$r = -\frac{r_0}{\theta} \quad (\text{clockwise}) \qquad -(13)$$

As the star travels from the centre O to a point A its torsion and orbital linear velocity decrease with time. During this interval of time the angle θ increases to infinity for counter-clockwise rotation and to $-\infty$ for clockwise rotation. Finally the angular velocity of the star decreases with time as follows:

$$\omega(t) = \frac{d\theta}{dt} = \frac{\omega_0}{\sqrt{2}\,(\omega_0 t + C)} \qquad -(14)$$

So the dynamics of the star in a galaxy can be worked out completely by assuming a given orbital function. Any orbital function can be used provided that it is based on experimental data.