206(11): Reduction of the Schwarzschild Metric to the Minkowski Limit.

The Schwarzschild metric is:

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\frac{1}{a^2}\right) - \left(1-\frac{r_0}{r}\right)r^2 \quad -(1)$$

after reduction to the orbital equation.

The constrained Minkowski metric gives:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{c^2 L_c^2}{E^2 - m^2 c^4} - r^2 \quad -(2)$$

where L_c is the constrained angular momentum. In the Newtonian limit eq. (2) reduces to:

$$\left(\frac{dr}{d\theta}\right)^2 = 2mr^4\left(\frac{E_N - V(r)}{L^2}\right) - r^2 \quad -(3)$$

The result (1) can only reduce to the result (3) if:

$$r \rightarrow \infty \quad -(4)$$

in which case:

$$\frac{1}{b^2} - \frac{1}{a^2} + \frac{r_0}{r}\frac{1}{a^2} \rightarrow \frac{2m(E_N - V(r))}{L^2}$$

$$= \frac{2mT}{L^2} \quad -(5)$$

where the Newtonian kinetic energy is:

$$T = \frac{1}{2}mv^2 \quad -(6)$$

So:

a)

$$\frac{1}{b^2} - \frac{1}{a^2} + \frac{r_0}{r a^2} \rightarrow \frac{m^2 v^2}{L^2} \quad -(7)$$

i.e. $$\frac{E^2 - m^2 c^4}{c^2 L^2} + \frac{r_0}{r a^2} \rightarrow \frac{m^2 v^2}{L^2} \quad -(8)$$

This is true if and only if:

$$E^2 - m^2 c^4 \rightarrow c^2 p^2, \quad -(9)$$

$$p = \gamma m v \rightarrow m v \quad -(10)$$

i.e. $$v \ll c \quad -(11)$$

and $$r \rightarrow \infty. \quad -(12)$$

In this case however, there is no orbit.

Therefore the Einstein theory never reduces to the Newtonian theory of orbits, QED.