306(1): Angular Velocity and Areal Velocity in Terms of Torsion.

In general, the two torsion elements of a planar orbit are:

$$T^1{}_{01} = \frac{1}{c}\frac{df/dt}{(1+f)} \quad -(1)$$

and

$$T^1{}_{21} = \frac{1}{r}\frac{df/d\theta}{(1+f)} \quad -(2)$$

where:

$$f = r^2\left(\frac{d\theta}{dr}\right)^2 \quad -(3)$$

Dividing eq. (1) by eq. (2) the angular velocity is given by:

$$\omega = \frac{d\theta}{dt} = \frac{df}{dt}\frac{d\theta}{df} = \frac{c}{r}\left(\frac{T^1{}_{01}}{T^1{}_{21}}\right) \quad -(4)$$

and the areal velocity by:

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}cr\left(\frac{T^1{}_{01}}{T^1{}_{21}}\right) \quad -(5)$$

Both the angular velocity and the areal velocity are proportional to the ratio of two torsion elements of spacetime.

For the hyperbolic spiral:

$$T^1{}_{01} = \frac{2\theta}{c(1+\theta^2)}\frac{d\theta}{dt}, \quad T^1{}_{21} = \frac{2\theta}{r(1+\theta^2)} \quad -(6)$$

which is consistent with eq. (4).

In eq. (6):

$$\theta = r_0 / r \quad — (7)$$

Therefore:

$$T^1{}_{01} = -\frac{2 r_0^2}{c r (r^2 + r_0^2)} \frac{dr}{dt} \quad — (8)$$

$$T^1{}_{21} = \frac{2 r_0}{r^2 + r_0^2} \quad — (9)$$

From eq. (7):

$$\frac{d\theta}{dt} = r_0 \frac{d}{dt}\left(\frac{1}{r}\right) = r_0 \frac{df_1}{dr} \frac{dr}{dt} \quad — (10)$$

where

$$f_1 = 1/r \quad — (11)$$

so

$$\frac{d\theta}{dt} = -\frac{r_0}{r^2} \frac{dr}{dt} \quad — (12)$$

$$= \frac{c}{r}\left(\frac{T^1{}_{01}}{T^1{}_{21}}\right)$$

Therefore

$$\boxed{\frac{dr}{dt} = -c \frac{r}{r_0} \frac{T^1{}_{01}}{T^1{}_{21}}} \quad — (13)$$

For the hyperbolic spiral:

$$\boxed{\frac{dr}{dt} = -\frac{2}{r_0} \frac{dA}{dt}} \quad — (14)$$

As r becomes infinite the spiral becomes a straight line, in which case dA/dt is constant. So dr/dt is constant, giving the observed velocity curve.