206(2): Torsion Elements for the Elliptical Orbit

Consider the general result:

$$\frac{dA}{dt} = \frac{1}{2} cr \left(\frac{T'_{01}}{T'_{21}} \right) \quad -(1)$$

then it is always true that:

$$\boxed{\frac{T'_{01}}{T'_{21}} = \frac{2}{c} \left(\frac{dA}{dt} \right) \frac{1}{r}} \quad -(2)$$

For the ellipse, dA/dt is observed to be constant,

and

$$\frac{1}{r} = \frac{1}{\alpha} \left(1 + \epsilon \cos\theta \right) \quad -(3)$$

so

$$\frac{T'_{01}}{T'_{21}} = \frac{2}{\alpha c} \left(\frac{dA}{dt} \right) \left(1 + \epsilon \cos\theta \right) \quad -(4)$$

i.e.

$$\boxed{\frac{T'_{01}}{T'_{21}} = B \left(1 + \epsilon \cos\theta \right)} \quad -(5)$$

where B is a constant.

For the circle:

$$\epsilon = 0 \quad -(6)$$

so

$$\frac{T'_{01}}{T'_{21}} = \frac{2}{rc} \frac{dA}{dt} \quad -(7)$$

where

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega. \quad -(8)$$

So for a circle:

2)

$$\frac{T^{1}{}_{01}}{T^{1}{}_{21}} = \frac{\omega r}{c} = \frac{v}{c} \quad -(9)$$

Therefore : $T^{1}{}_{21} \gg T^{1}{}_{01}$. $-(10)$

The dominant torsion element is :

$$\boxed{T^{1}{}_{21} = \frac{1}{r} \frac{\partial f / \partial\theta}{1+f} \quad -(11)}$$

For a precessing ellipse:

$$\frac{T^{1}{}_{01}}{T^{1}{}_{21}} = \frac{2}{\alpha c}\left(\frac{dA}{dt}\right)\left(1+\epsilon\cos(x\,\theta)\right) \quad -(12)$$

but the areal velocity is no longer constant.

For a hyperbolic spiral :

$$\frac{T^{1}{}_{01}}{T^{1}{}_{21}} = \frac{2}{c}\left(\frac{dA}{dt}\right)\frac{\theta}{r_0}, \quad -(13)$$

and for a logarithmic spiral

$$\frac{T^{1}{}_{01}}{T^{1}{}_{21}} = \frac{2}{c}\left(\frac{dA}{dt}\right)\frac{1}{r_0}e^{-\alpha\theta} \quad -(14)$$

but : $T^{1}{}_{01} = T^{1}{}_{21} = 0$. $-(15)$