205(4) : Rigorous Proof of the Evans Identity for All Orbits

As in note 205(2), the identity is:

$$D_0 T^1{}_{01} := R^1{}_{001} \quad -(1)$$

$$D_2 T^1{}_{21} := R^1{}_{221} \quad -(2)$$

which reduces to:

$$6 \frac{d^2 f}{dt^2} (1+f) = 5 \left(\frac{df}{dt} \right)^2 \quad -(3)$$

$$6 \frac{d^2 f}{d\theta^2} (1+f) = 5 \left(\frac{df}{d\theta} \right)^2 \quad -(4)$$

i.e.

$$\frac{d^2 f}{dt^2} := \left(\frac{d\theta}{dt} \right)^2 \frac{d^2 f}{d\theta^2} \quad -(5)$$

To prove eq. (5) use:

$$g = \frac{df}{dt}, \quad \frac{dg}{dt} = \frac{d^2 f}{dt^2} = \frac{dg}{d\theta} \frac{d\theta}{dt} = \frac{d^2 f}{d\theta\, dt} \frac{d\theta}{dt} \quad -(6)$$

and

$$h = \frac{df}{d\theta}, \quad \frac{dh}{d\theta} = \frac{d^2 f}{d\theta^2} = \frac{dh}{dt} \frac{dt}{d\theta} = \frac{d^2 f}{dt\, d\theta} \frac{dt}{d\theta} \quad -(7)$$

By isotropy:

$$\frac{d^2 f}{d\theta\, dt} = \frac{d^2 f}{dt\, d\theta} . \quad -(8)$$

Finally divide eq. (6) by eq (7) to give eq. (5).

QED.