205(2): Interpretation of the Evans Identity for Any Orbit.

The ~~four~~ two identities of relevance are:

$$D_0 T^1{}_{01} := R^1{}_{001} \quad -(1)$$

and

$$D_2 T^1{}_{21} := R^1{}_{221} \quad -(2)$$

Eq. (1) gives:

$$6\frac{d^2 f}{dt^2}(1+f) := 5\left(\frac{df}{dt}\right)^2 \quad -(3)$$

and eq. (2) gives:

$$6\frac{d^2 f}{d\theta^2}(1+f) := 5\left(\frac{df}{d\theta}\right)^2 \quad -(4)$$

where

$$f = r^2\left(\frac{d\theta}{dr}\right)^2 \quad -(5)$$

These equations are true for any orbit.

From eqs. (3) and (4):

$$\boxed{\frac{d^2 f}{dt^2} := \left(\frac{d\theta}{dt}\right)^2 \frac{d^2 f}{d\theta^2}} \quad -(6)$$

i.e.

$$\left(\frac{d\theta}{dt}\right)^2 := \left(\frac{d^2 f}{dt^2}\right)\left(\frac{d\theta^2}{d^2 f}\right) \quad -(7)$$

Eq. (7) is an exact identity, Q.E.D.

2) The derivative is defined by:

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left(\frac{f(x + \delta x) - f(x)}{\delta x} \right) \quad -(8)$$

where

$$y = f(x)$$

The second derivative is defined by:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \lim_{\delta x \to 0} \left(\frac{g(x + \delta x) - g(x)}{\delta x} \right) \quad -(9)$$

where:

$$g = \frac{dy}{dx}. \quad -(10)$$

We have:

$$\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right). \quad -(10)$$

Therefore:

$$\frac{d^2 y}{dx^2} = \lim_{\delta x \to 0} \left(\lim_{\delta x \to 0} \left(\frac{f(x + \delta x + \delta x) - f(x + \delta x) - (f(x + \delta x) - f(x))}{(\delta x)^2} \right) \right)$$

$$= \lim_{\delta x \to 0} \left(\lim_{\delta x \to 0} \left(\frac{f(x + 2\delta x) - 2 f(x + \delta x) - f(x)}{(\delta x)^2} \right) \right) \quad (11)$$

In eq. (7), note that:

$$f(\theta) = f(t) \quad -(12)$$

because f as a function of θ is the same as f as a function of t. In other words:

3) $$f(\theta, t) := f(\theta, t) \quad - (13)$$

It follows from eqs. (11) and (12) that:

$$\left(\frac{d^2 f}{dt^2}\right)\left(\frac{d\theta^2}{d^2 f}\right) = \lim_{\delta\theta \to 0}\left(\lim_{\delta t \to 0}\left(\frac{\delta\theta}{\delta t}\right)^2\right)$$

$$= \left(\frac{d\theta}{dt}\right)^2 \quad - (14)$$

Q.E.D.

So the Evans identity for any orbit is the exact identity:

$$\left(\frac{d\theta}{dt}\right)^2 := \left(\frac{d\theta}{dt}\right)^2 \quad - (15)$$

This check shows that the concepts and algebra are correct:

1) the connection is antisymmetric;
2) The Evans identity is true;
3) The metric compatibility theorem is true for the antisymmetric connection;
4) All orbits are due to Cartan's geometry;
5) the Evans identity is an example of Cartan's identity.