205(1): Calculation of Torsion from Orbit.

For a given orbit represented by:

$$f(r, \theta) = \frac{d\theta}{dr} \quad -(1)$$

The constrained Minkowski metric is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - \left(1 + r^2 \left(\frac{d\theta}{dr}\right)^2\right) dr^2 \quad -(2)$$

So

$$g_{00} = 1, \quad g_{11} = -\left(1 + r^2 \left(\frac{d\theta}{dr}\right)^2\right) \quad -(3)$$

The antisymmetric connection is calculated using eq. (19) of UFT188:

$$\Gamma^{\alpha}_{\rho\alpha} = \frac{1}{2} g^{\alpha\alpha} \partial_\rho g_{\alpha\alpha} \quad -(4)$$

The two connections of relevance are:

$$\Gamma^{1}_{01} = \frac{1}{2} g^{11} \partial_0 g_{11} \quad -(5)$$

$$\Gamma^{0}_{10} = \frac{1}{2} g^{00} \partial_1 g_{00} \quad -(6)$$

The Riemann torsion from eq. (5) is:

$$T^{1}_{01} = 2\Gamma^{1}_{01}. \quad -(7)$$

The inverse metric element is calculated from:

$$g^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} g_{\alpha\beta} \quad -(8)$$

2)

So
$$g^{11} = g^{11} g^{11} g_{11} \quad -(9)$$

and
$$g^{11} = \frac{1}{g_{11}} : \quad -(10)$$

So
$$\Gamma^0_{10} = \frac{1}{2g_{00}} \partial_1 g_{00} \quad -(11)$$
$$\Gamma^1_{01} = \frac{1}{2g_{11}} \partial_0 g_{11} \quad -(12)$$

From eq. (3):
$$\Gamma^0_{10} = 0, \quad -(13)$$

and:
$$\partial_0 g_{11} = -\frac{1}{c} \frac{\partial}{\partial t} \left(1 + r^2 \left(\frac{d\theta}{dr} \right)^2 \right) \quad -(14)$$

Elliptical Orbit

In this case:
$$r = \frac{\alpha}{1 + \epsilon \cos\theta}, \quad -(15)$$
$$\frac{dr}{d\theta} = \frac{\epsilon}{\alpha} r^2 \sin\theta, \quad -(16)$$
$$r^2 \left(\frac{d\theta}{dr} \right)^2 = \left(\frac{\alpha}{\epsilon} \right)^2 \cdot \frac{1}{r^2 \sin^2\theta} \quad -(17)$$
$$= \frac{1}{\epsilon^2} \left(\frac{1 + \epsilon \cos\theta}{\sin\theta} \right)^2 \quad -(18)$$

Therefore:

$$\partial_0 g_{11} = -\frac{1}{c}\frac{\partial}{\partial t}\left(1 + \frac{1}{\epsilon^2}\left(\frac{1+\epsilon\cos\theta}{\sin\theta}\right)^2\right) \quad -(19)$$

Now use:

$$\frac{df}{dt} = \frac{df}{d\theta}\frac{d\theta}{dt}. \quad -(20)$$

So:

$$\frac{d\sin\theta(t)}{dt} = \frac{d\theta}{dt}\cos\theta(t), \quad -(21)$$

$$\frac{d\cos\theta(t)}{dt} = -\frac{d\theta}{dt}\sin\theta(t). \quad -(22)$$

So:

$$\frac{d}{dt}\left(\frac{1+\epsilon\cos\theta(t)}{\sin\theta(t)}\right) = -\epsilon\frac{d\theta}{dt}\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}\right)$$

$$= -\frac{\epsilon}{\sin^2\theta}\frac{d\theta}{dt} \quad -(23)$$

Now use:

$$\frac{d}{dt}f^2(\theta) = \frac{d}{dt}\left(f(\theta)f(\theta)\right) \quad -(24)$$
$$= 2f(\theta)\frac{df(\theta)}{dt}$$

So:

$$\frac{d}{dt}\left(\frac{1+\epsilon\cos\theta}{\sin\theta}\right)^2$$
$$= -2\epsilon\left(\frac{1+\epsilon\cos\theta}{\sin^3\theta}\right)\frac{d\theta}{dt} \quad -(25)$$

From eqs. (14), (19) and (25):

$$\partial_0 g_{11} = \frac{2\epsilon}{c}\left(\frac{1+\epsilon\cos\theta}{\sin^3\theta}\right)\omega \quad -(26)$$

where

$$\omega = \frac{d\theta}{dt} \quad -(27)$$

$$-(28)$$

Therefore:

$$\boxed{\partial_0 g_{11} = \frac{2\epsilon d}{cr}\left(1-\frac{1}{\epsilon^2}\left(\frac{d}{r}-1\right)\right)^{-3/2}\frac{d\theta}{dt}}$$

The required connection is:

$$\Gamma^1_{01} = \frac{1}{2}g^{11}\partial_0 g_{11} \quad -(29)$$

where

$$g^{11} = \frac{1}{g_{11}} = -\left(1+r^2\left(\frac{d\theta}{dr}\right)^2\right)^{-1} \quad -(30)$$

$$= -\left(1+\left(\frac{d}{\epsilon}\right)^2\frac{1}{r^2}\left(1-\frac{1}{\epsilon^2}\left(\frac{d}{r}-1\right)\right)^{-1}\right)^{-1}$$

So:

$$\boxed{\Gamma^1_{01} = -\frac{2\epsilon d}{cr}\left[\frac{\left(1-\frac{1}{\epsilon^2}\left(\frac{d}{r}-1\right)\right)^{-3/2}}{1+\left(\frac{d}{\epsilon}\right)^2\frac{1}{r^2}\left(1-\frac{1}{\epsilon^2}\left(\frac{d}{r}-1\right)\right)^{-1}}\right]\frac{d\theta}{dt}}$$

$$-(31)$$

and

$$T^1_{01} = 2\Gamma^1_{01}. \quad -(32)$$

Therefore the torsion is characterized by ϵ and α, the parameters of the orbit. These are determined by observation. Therefore the torsion may be observed experimentally. The angular velocity may be observed experimentally through the areal velocity:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \quad -(33)$$

So

$$\boxed{\frac{d\theta}{dt} = \frac{2}{r^2} \frac{dA}{dt}} \quad -(34)$$

Eq. (34) is the second law of Johannes Kepler (1571 - 1630), published in 1609 after a study of the data of Tycho Brahe (1546 - 1601) on Mars. Eq. (34) is true for all orbits.

Conclusion

The Riemann torsion (32) may be observed for any orbit by observing its parameters. For the ellipse they are α, ϵ and dA/dt.

The origin of all orbits is torsion, and the orbit constrains the Minkowski metric.