205(5): Some Results for the Hyperbolic Spiral.

The two non-vanishing torsions are:

$$T^1{}_{01} = \frac{2\theta}{c(1+\theta^2)} \frac{d\theta}{dt} \quad -(1)$$

$$T^1{}_{12} = \frac{2\theta}{r(1+\theta^2)} \quad -(2)$$

Here

$$\theta r = r_0. \quad -(3)$$

So:

$$\boxed{T^1{}_{12} = \frac{2 r_0}{r^2 + r_0^2}} \quad -(4)$$

In eq. (1):

$$\frac{d\theta}{dt} = \frac{d\theta}{dr}\frac{dr}{dt} = -\frac{r_0}{r^2}\frac{dr}{dt} \quad -(5)$$

so

$$\boxed{T^1{}_{01} = -\frac{2 r_0^2}{c r (r^2 + r_0^2)} \frac{dr}{dt}} \quad -(6)$$

The total linear velocity squared is:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 \quad -(7)$$

$$= \left(1 - r_0^2\right)\left(\frac{dr}{dt}\right)^2$$

$$= r^2\left(\frac{r^2}{r_0^2} + 1\right)\left(\frac{d\theta}{dt}\right)^2$$

$$= \frac{r_0^2}{\theta^2}\left(1 + \frac{1}{\theta^2}\right)\left(\frac{d\theta}{dt}\right)^2$$

As:

$$r \rightarrow \infty \qquad (8)$$

then

$$\theta \rightarrow 0 \qquad (9)$$

i.e.

$$\frac{d\theta}{dt} \xrightarrow[r \rightarrow \infty]{} 0 \qquad (10)$$

and

$$\frac{d\theta}{dt} \xrightarrow[\theta \rightarrow 0]{} 0 \qquad (11)$$

so

$$v^2 \xrightarrow[r \rightarrow \infty]{} \text{indeterminate} = \frac{0}{0} \qquad (12)$$

It is known experimentally that v becomes constant as $r \rightarrow \infty$. This velocity curve is explained by the hyperbolic spiral alone. As r becomes infinite, the two torsions vanish.