205(2): Geometrical Meaning of the Theory of Photon Mass

Assume that one photon has a mass m. It is observed in astronomy that a mass m orbits a mass M in the solar system in a precessing ellipse or in general a conic section:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(1)$$

so:

$$\frac{d\theta}{dr} = \frac{\alpha}{x\epsilon}\,\frac{1}{r}\left(r^2 - \frac{1}{\epsilon^2}\left(\alpha - r\right)^2\right)^{-1/2} \quad -(2)$$

$$x\theta(r) = \frac{\alpha}{\epsilon}\int \frac{dr}{r\left(r^2 - \frac{1}{\epsilon^2}(\alpha-r)^2\right)^{1/2}} \quad -(3)$$

If

$$x = 1 \quad -(4)$$

then Newtonian gravitational theory is obtained as follows.

The structure of eq. (3) is that of Newtonian theory, in which:

$$\theta(r) = \int \frac{L}{r^2}\,\frac{dr}{\left(2m\left(E + \frac{mMG}{r} - \frac{L^2}{2mr^2}\right)\right)^{1/2}} \quad -(5)$$

Here E is the total energy, L is the total angular momentum, m is a mass orbiting a mass M, G is Newton's constant. The total energy is:

$$E = T + V \quad -(6)$$

where the kinetic energy is:

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right) \quad -(7)$$

2) and the potential energy is:

$$V = -\frac{mMG}{r} \quad -(8)$$

The total angular momentum is:

$$L = mr^2 \frac{d\theta}{dt} \quad -(9)$$

so

$$T = \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2mr^2} \quad -(10)$$

Therefore:

$$E = \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2mr^2} - \frac{mMG}{r} \quad -(11)$$

From eq. (11):

$$\frac{dr}{dt} = \left(\frac{2}{m}(E-U) - \frac{L^2}{m^2 r^2}\right)^{1/2} \quad -(12)$$

Therefore:

$$\frac{d\theta}{dr} = \frac{d\theta}{dt}\frac{dt}{dr} \quad -(13)$$

$$= \frac{L}{r^2}\left(2m\left(E - U - \frac{L^2}{2mr^2}\right)\right)^{-1/2}$$

using eqns. (9) and (12). Integrating eq. (13) gives eq. (5).

Eqs. (3) and (5) are the same equations

if:

$$\left(\frac{x\epsilon}{\alpha}\right)^2\left(r^2 - \frac{1}{\epsilon^2}(\alpha - r)^2\right) = \frac{r^2}{L^2}\left(2m\left(E + \frac{mMG}{r} - \frac{L^2}{2mr^2}\right)\right) \quad -(14)$$

3) Comparing term by term:

$$\frac{2mE}{L^2} = \left(\frac{x\epsilon}{\alpha}\right)^2\left(1-\frac{1}{\epsilon^2}\right) \quad -(15)$$

$$\frac{2m\underline{M}G}{L^2} = \left(\frac{x\epsilon}{\alpha}\right)^2\left(\frac{2\alpha}{\epsilon^2}\right) \quad -(16)$$

$$\left(\frac{x\epsilon}{\alpha}\right)^2\frac{\alpha^2}{\epsilon^2} = 1 \quad -(17)$$

So:

$$x = 1 \quad -(18)$$

$$\alpha = \frac{L^2}{m^2\underline{M}G} \quad -(19)$$

$$\frac{2mE}{L^2} = \frac{\epsilon^2}{\alpha^2}\left(1-\frac{1}{\epsilon^2}\right) \quad -(20)$$

i.e

$$\epsilon = \left(1+\frac{2EL^2}{m^3\underline{M}^2G^2}\right)^{1/2} \quad -(21)$$

From eqs. (19) and (20):

$$E = \frac{L^2}{2m}\frac{\epsilon^2}{\alpha^2}\left(1-\frac{1}{\epsilon^2}\right) \quad -(22)$$

$$= \frac{m^2\underline{M}G\alpha}{2m}\frac{\epsilon^2}{\alpha^2}\left(1-\frac{1}{\epsilon^2}\right)$$

$$\boxed{E = \frac{1}{2}m\underline{M}G\frac{\epsilon^2}{\alpha}\left(1-\frac{1}{\epsilon^2}\right)} \quad -(23)$$

The total energy is constant.

4)

The potential energy is:

$$\boxed{V = -\frac{mMG}{r} = -\frac{L^2}{md^2}\left(1+\cos\theta\right)} \quad -(24)$$

and depends on the angle θ.

The kinetic energy is:

$$T = E - V$$

$$= \frac{L^2}{2m}\frac{\epsilon^2}{d^2}\left(1-\frac{1}{\epsilon^2}\right) + \frac{L^2}{md^2}\left(1+\cos\theta\right)$$

$$= \frac{L^2}{md^2}\left(\frac{1}{2}\epsilon^2\left(1-\frac{1}{\epsilon^2}\right) + 1 + \cos\theta\right) \quad -(25)$$

$$\boxed{T = \frac{L^2}{md^2}\left(\frac{1}{2}\left(1+\epsilon^2\right) + \cos\theta\right)} \quad -(26)$$

It is seen that E, T and V are geometrical in origin. They were essentially derived by Newton by parameterizing the observed orbit. The correct (1) is the limit $x = 1$. The correct precessing ellipse is obtained by changing the angle $\theta(r)$ in eq. (5) to $x\theta(r)$. This correction is equivalent to:

$$\boxed{L \rightarrow L / x} \quad -(27)$$

5)

so:

$$\theta(r) = \int \left(\frac{L}{x}\right) \frac{1}{r^2} \frac{dr}{\left(2m\left(E + \frac{mMG}{r} - \frac{x^2 (L/x)^2}{2mr^2}\right)\right)^{1/2}} \quad -(28)$$

In the solar system:

$$x - 1 \sim 10^{-5} \quad -(29)$$

so x is very close to unity for planets.

It is not necessary to use Einstein's general relativity at all. This is not a correct theory.

Eq. (28) can be obtained from the lagrangian:

$$\boxed{\mathcal{L} = \frac{1}{2} m \left(\left(\frac{dr}{dt}\right)^2 + \frac{1}{x} r^2 \left(\frac{d\theta}{dt}\right)^2\right) - V(r)} \quad -(30)$$

so

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{m}{x} r^2 \frac{d\theta}{dt} \quad -(31)$$

$$\dot{\theta} = d\theta / dt \quad -(32)$$

The precession constant x has the property:

$$\cos(x\theta) = \cos(x\theta + 2\pi) \quad -(33)$$

If x is increased from 1 to 2π, the cosine goes through its complete range, so:

$$1 \leq x \leq 2\pi. \quad -(34)$$

6) The complete kinetic energy in cylindrical polar coordinates in a plane is:

$$T = \frac{1}{2} m \underline{v} \cdot \underline{v} \quad - (35)$$

where

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{d}{dt}\left(r \underline{e}_r\right) \quad - (36)$$

$$= \dot{r} \underline{e}_r + r \dot{\underline{e}}_r$$

where

$$\dot{\underline{e}}_r = \dot{\theta} \underline{e}_\theta, \quad - (37)$$

so

$$\underline{v} \cdot \underline{v} = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2. \quad - (38)$$

In order to describe precession:

$$\left(\frac{d\theta}{dt}\right)^2 \longrightarrow \frac{1}{x}\left(\frac{d\theta}{dt}\right)^2 \quad - (39)$$

and the angular velocity changes as follows:

$$\boxed{\omega \rightarrow x^{-1/2} \omega} \quad - (40)$$

The translational kinetic energy is

$$T_t = \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 \quad - (41)$$

and the rotational kinetic energy is:

$$T_r = \frac{1}{2} \frac{m}{x} r^2 \left(\frac{d\theta}{dt}\right)^2 \quad - (42)$$

So precessing orbits can be described classically, without Einstein's ideas.

7) It has been assumed that the photon has mass m. The de Broglie equation means:

$$\hbar \omega = \gamma m c^2 \quad - (43)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (44)$$

The classical kinetic energy is the limit of:

$$T = (\gamma - 1) m c^2 \quad - (45)$$

when

$$v << c. \quad - (46)$$

This is shown as follows:

$$T = \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) m c^2$$

$$\sim \left(1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) m c^2$$

$$= \frac{1}{2} m v^2 \quad - (47)$$

$$= \frac{1}{2} m \left(\frac{dr}{dt} \right)^2$$

So eqn. (45) is the relativistic translational kinetic energy. The Planck quantum:

$$E = \hbar \omega \quad - (48)$$

is the total relativistic translational energy.
~~kinetic energy.~~

The total energy is

$$E = \hbar\omega + \frac{L^2}{2mr^2} - \frac{mMG}{r} \quad - (49)$$

and as shown in note 203(1):

$$E = \hbar\omega \quad - (50)$$

to an excellent approximation.

The total angular momentum is:

$$L_{total} = \hbar + L \quad - (51)$$

and is $\hbar$ to an excellent approximation.

This defines photon mass theory in the Newtonian limit..