203(6): Check on Einstein's Starting Equation

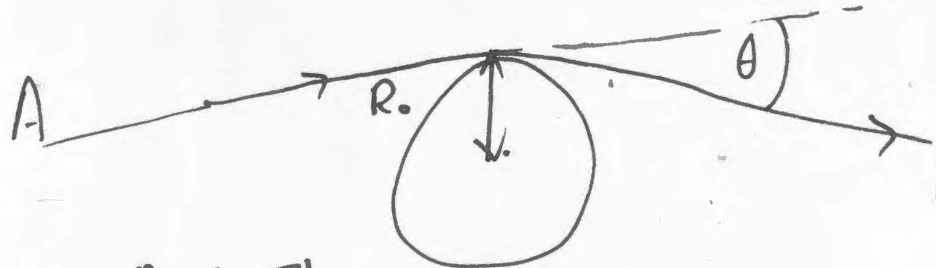


Fig (1)

Consider the identity:

$$\frac{d\theta}{dr} = \frac{d\theta}{dr} \quad \text{— (1)}$$

Then

$$d\theta = \left(\frac{d\theta}{dr}\right) dr \quad \text{— (2)}$$

So

$$\theta = \int \frac{d\theta}{dr} \, dr \quad \text{— (3)}$$

When:

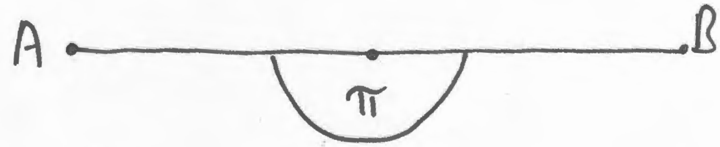
$$r = R_0 \quad \text{— (4)}$$

the light is at the sun's radius. When:

$$r = \infty \quad \text{— (5)}$$

the light is far distant from the centre of the sun.

If the light travelled from A to B in a straight line it subtends the angle π (180°).

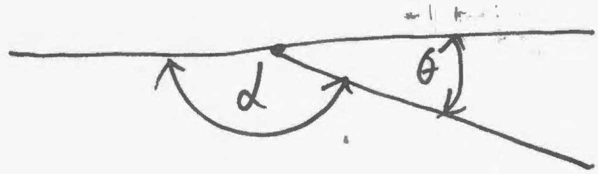


For a straight line:

$$\frac{d\theta}{dr} = 0 \quad \text{— (6)}$$

so there is no change in angle.

2)



In the case of light bending:

$$\alpha = \pi - \theta \quad - (7)$$

For a straight line:

$$\frac{d\theta}{dr} = 0 \quad - (8)$$

$$\text{So} \quad \Delta\theta = \int_{R_0}^{\infty} \frac{d\theta}{dr}\, dr = 0 \quad - (9)$$

$$- (10)$$

This means that:

$$\Delta\theta = \frac{2}{x}\left(\sin^{-1}\frac{1}{\epsilon} - \sin^{-1}\left(\frac{R_0 - \alpha}{\epsilon|R_0|}\right)\right) = 0$$

An orbit becomes a straight line when:

$$R_0 \to \infty \quad - (11)$$

So in eq. (10): $\Delta\theta = 0 \quad - (12)$

self consistently.

From eq. (3):

$$\Delta\theta = \int_{R_0}^{\infty} \left(\frac{d\theta}{dr}\right) dr \quad - (13)$$

and eqs. (11) and (13) give again:

$$\Delta\theta = 0. \quad - (14)$$

3) This is the correct way of calculating the deflection $\Delta\theta$ from the orbit:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(15)$$

The straight line is recovered from this orbit when:

$$r \to \infty \quad -(16)$$

which means that the light is infinitely away from the sun.

This means that there is another problem in the Einstein calculation.

In Wald's notation the deflection $\Delta\theta$ is given by eq. (6.3.39), page 145 of R. M. Wald, "General Relativity" (Chicago, 1984):

$$\Delta\theta = 2\int_0^{1/R_0} \frac{du}{(b^{-2} - u^2 + 2Mu^3)^{1/2}} \quad -(17)$$

and the straight line is recovered using:

$$M = ?\ 0, \quad R_0 = ?\ b \quad -(18)$$

to give

$$\Delta\theta = \pi. \quad -(19)$$

This means that:

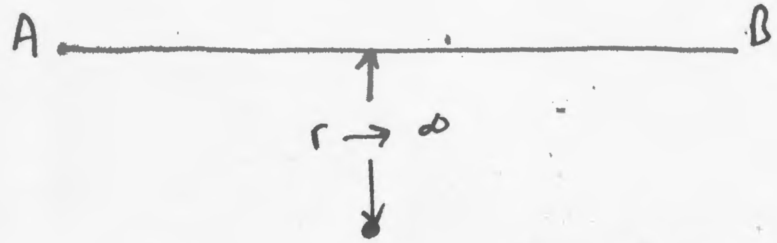
$$\frac{dr}{d\theta} \neq ?\ 0 \quad -(20)$$

for a straight line!

This is another absurd result.

4)

In the case of no deflection:



In this case:

$$\frac{dr}{d\theta} = \left(\frac{\epsilon x}{\alpha}\right) r^2 \sin(x\theta) \quad -(21)$$

so

$$\frac{d\theta}{dr} \xrightarrow[r \to \infty]{} 0 \quad -(22)$$

because

$$\frac{d\theta}{dr} = \left(\frac{\alpha}{\epsilon x}\right) \frac{1}{r^2} \cdot \frac{1}{\sin(x\theta)} \quad -(23)$$

Seen in another way:

$$\Delta\theta = \int_{\infty}^{\infty} \left(\frac{d\theta}{dr}\right) dr = 0. \quad -(24)$$

The result (22) is understandable from the fact that in a straight line there is no dependence of r on angle.

So the complete expression for deflection is:

$$\Delta\theta = \frac{2}{x}\left(\sin^{-1}\frac{1}{\epsilon} - \sin^{-1}\left(\frac{1}{\epsilon}\left(1 - \frac{\alpha}{R_0}\right)\right)\right) \quad -(25)$$