1) 194(4): Energy in General Relativity

The total energy is a constant of motion and is defined by:

$$E = n(r) m c^2 \frac{dt}{d\tau} \quad -(1)$$

$$= n(r) m c^2 \left(n(r) - \frac{v^2}{c^2} \right)^{-1/2}$$

$$= \text{constant}$$

Solving for $n(r)$:

$$n^2(r) m^2 c^4 = E^2 \left(n(r) - \frac{v^2}{c^2} \right) \quad -(2)$$

i.e.

$$n(r) = \frac{1}{2}\left(\frac{E}{mc^2}\right)\left[1 \pm \left(1 - 4\frac{v^2}{c^2}\left(\frac{mc^2}{E}\right)^2 \right)^{1/2} \right] \quad -(3)$$

which is the same result as in note 194(3)

Therefore for a spherically symmetric spacetime defined by:

$$ds^2 = c^2 d\tau^2 = n(r) c^2 dt^2 - \frac{dr^2}{n(r)} - r^2 d\theta^2 \quad -(4)$$

the function $n(r)$ must be given by eq. (3) otherwise E and L are not constant. The velocity v is defined by:

2) $d\underline{r} \cdot d\underline{r} = v^2 dt^2 = \frac{dr^2}{n(r)} + r^2 d\theta^2$ — (5)

in the plane $dZ^2 = 0.$ — (6)

In special relativity v is a constant, but in general relativity v changes with time because one frame can accelerate with respect to another. Therefore in general $n(r)$ is also a function of velocity:

$$n(r) = n(r, v) \quad — (7)$$

where $v^2 = n^{-1}(r, v)\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2$ — (8).

The coordinate system being used is (r, θ), the cylindrical polar coordinates in the plane defined by eq. (6). So r cannot be a function of v. The assertion of EGR is:

$$n(r) = 1 - \frac{r_0}{r} \quad — (9)$$

which is incompatible with eq. (3) because r would have to be a function of v if eqs (3) and (9) were equated

Conclusion

EGR contains a fundamental contradiction because it does not give a constant E and L.