1)

194(2): Time Delay in Lagrangian Dynamics and General Relativity.

In Lagrangian dynamics the time delay is calculated from the observation of an orbit. Similarly the linear velocity of an elliptical orbit that is precessing is:

$$v = \frac{dr}{dt} = \left(\frac{Lx}{m\alpha}\right)\frac{1}{r}\left(\epsilon^2 r^2 - (\alpha - r)^2\right)^{1/2} \quad -(1)$$

where:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)}, \quad -(2)$$

and where

$$L = mr^2\frac{d\theta}{dt} = \text{constant} \quad -(3)$$

In general relativity the same linear velocity is:

$$v = \frac{dr}{dt} = c\,b\,n(r)\left(\frac{1}{b^2} - n(r)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{1/2} \quad -(4)$$

In the usual pseudoscience it is claimed that:

$$n(r) = 1 - \frac{r_0}{r} \quad -(5)$$

The difference between eqs (1) and (4) is that the former is obtained from:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad -(6)$$

using the Euler Lagrange equation:

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$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(7)$$
$$= 0$$

with the lagrangian:

$$\mathcal{L} = \frac{1}{2} m \left( \dot{r}^2 + r^2 \dot{\theta}^2 \right) - V(r) \quad -(8)$$

Eq. (4) is obtained from the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\theta^2 \quad -(9)$$

and lagrangian:

$$\mathcal{L}_{GR} = \frac{1}{2} mc^2 = \frac{1}{2} m \left( m(r) c^2 \left(\frac{dt}{d\tau}\right)^2 - \frac{1}{m(r)} \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\theta}{d\tau}\right)^2 \right)$$
$$-(10)$$

and the angular momentum from eq. (10) is:

$$L_{GR} = mr^2 \frac{d\theta}{d\tau} = \text{constant} \quad -(11)$$

The two constant angular momenta (3) and (11) are worked out with different hypotheses, respectively from the lagrangians (8) and (10). The former contains the potential $V(r)$, but the latter does not contain a potential energy at all. From eq. (7):

$$L = \frac{\partial \mathcal{L}}{\partial \left(\frac{d\theta}{dt}\right)} \quad -(12)$$

3) but from eq. (10):

$$L_{GR} = -\frac{\partial \mathcal{L}_{GR}}{\partial\left(\frac{d\theta}{d\tau}\right)} \quad -(13)$$

If the line element (9) is written as:

$$ds^2 = -n(r)c^2 dt^2 + \frac{dr^2}{n(r)} + r^2 d\theta^2 \quad -(14)$$

the lagrangian is:

$$\mathcal{L}_{GR} = -\frac{1}{2}m\left(n(r)c^2\left(\frac{dt}{d\tau}\right)^2 - \frac{1}{n(r)}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\theta}{d\tau}\right)^2\right) \quad -(15)$$

and

$$L_{GR} = mr^2\frac{d\theta}{d\tau} = \frac{\partial \mathcal{L}_{GR}}{\partial\left(\frac{d\theta}{d\tau}\right)} \quad -(16)$$

So:

$$\boxed{L_{GR} = \left(\frac{dt}{d\tau}\right) L} \quad -(17)$$

because

$$\frac{d\theta}{d\tau} = \left(\frac{dt}{d\tau}\right)\frac{d\theta}{dt} \quad -(18)$$

However, both $L_{GR}$ and $L$ are constants of motion, so:

$$\frac{dt}{d\tau} = ? \text{ constant} \quad -(19)$$

This leads to a fundamental self-inconsistency because:

4)

$$ds^2 = c^2 d\tau^2 = m(r)c^2 dt^2 - d\underline{r} \cdot d\underline{r} \quad -(20)$$

where

$$d\underline{r} \cdot d\underline{r} = v^2 dt^2 \quad -(21)$$

so

$$c^2 d\tau^2 = m(r)c^2 dt^2 - v^2 dt^2$$

$$= \left(m(r)c^2 - v^2\right) dt^2 \quad -(22)$$

so

$$\frac{dt}{d\tau} = \left(m(r) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(23)$$

and this is not a constant.