1)

194(3): Angular Momentum in Relativity

In relativity the angular momentum is a conserved quantity:

$$L = mr^2 \frac{d\theta}{d\tau} \quad -(1)$$

$$= mr^2 \frac{d\theta}{dt} \cdot \frac{dt}{d\tau} \quad -(2)$$

In a spherical spacetime the line element is defined by:

$$ds^2 = c^2 d\tau^2 = m(r)c^2 dt^2 - d\underline{r} \cdot d\underline{r} \quad -(3)$$

where

$$d\underline{r} \cdot d\underline{r} = v^2 dt^2 = \frac{dr^2}{m(r)} - r^2 d\theta^2 \quad -(4)$$

in the plane

$$dZ^2 = 0. \quad -(5)$$

So in general:

$$c^2 d\tau^2 = \left(m(r)c^2 - v^2\right) dt^2 \quad -(6)$$

and

$$d\tau^2 = \left(m(r) - \frac{v^2}{c^2}\right) dt^2 \quad -(7)$$

so

$$\frac{dt}{d\tau} = \left(m(r) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(8)$$

From eqs. (2) and (8):

$$L = mr^2 \frac{d\theta}{dt}\left(m(r) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(9)$$

However, L is a constant of motion so the right hand side of eq. (9) must be a constant. From note 19.(9):

$$\frac{d\theta}{dt} = \frac{cbn(r)}{r^2} \quad -(10)$$

so:

$$\boxed{L = mcbn(r)\left(n(r) - \frac{v^2}{c^2}\right)^{-1/2}} \quad -(11)$$

In Einsteinian general relativity (EGR):

$$n(r) =? \ 1 - \frac{r_0}{r} \quad -(12)$$

so L is not a constant, and EGR is an internally self inconsistent theory.

From eq. (11):

$$L^2 = (mcbn(r))^2\left(n(r) - \frac{v^2}{c^2}\right)^{-1} \quad -(13)$$

so:

$$L^2\left(n(r) - \frac{v^2}{c^2}\right) = m^2c^2b^2n^2(r) \quad -(14)$$

which is a quadratic for n(r) in terms of L and v. This means that n(r) depends on v:

$$(mcb)^2 n^2(r) - L^2 n(r) + \frac{L^2 v^2}{c^2} = 0 \quad -(15)$$

so eq. (12) can never be correct.

In the limit of special relativity:

3) $$m(r) = 1 \quad -(16)$$

but this means that:

$$L^2\left(1 - \frac{v^2}{c^2}\right) = (mcb)^2 \quad -(17)$$

and
$$1 - \frac{v^2}{c^2} = \left(\frac{mcb}{L}\right)^2 \quad -(18)$$
$$= \left(\frac{mc^2}{E}\right)^2$$

Now use
$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(19)$$

to find
$$\boxed{E = \gamma mc^2} \quad -(20)$$

for the total energy E, another constant of motion.

1) In special relativity, eq. (20) is the correct result, and γ is constant because v is constant.

2) In general relativity, eq. (11) is a contradiction unless $m(r)$ is not a constant. because [illegible] is again constant from eq. (15).

Therefore in order to save the hypothesis of general relativity, v must be related to n(r) by eq. (15), in which L is a constant. From eq. (15), which is the quadratic:

$$ax^2 + bx + c = 0 \quad -(21)$$

$x = n(r)$, $a = (mcb)^2$, $b = -L^2$, $c = \left(\frac{Lv}{c}\right)^2$,

$$n(r) = \frac{1}{2a}\left(-b \pm \left(b^2 - 4ac\right)^{1/2}\right) \quad -(22)$$

$$\boxed{n(r) = \frac{1}{2}\left(\frac{E}{mc^2}\right)^2\left[1 \pm \left(1 - 4\frac{v^2}{c^2}\left(\frac{mc^2}{E}\right)^2\right)^{1/2}\right]} \quad -(23)$$

Therefore n(r) must depend on a non-constant v as in eq. (23). Obviously, eq. (12) of EGR is completely wrong.

Eq. (23) is valid for any spherically symmetric spacetime.