192(11): Precession of the Perihelion

By observation the orbit of a planet is the precessing ellipse:

$$r = \frac{\alpha}{1+\cos(x\theta)} \quad -(1)$$

In a revolution of 2π radians:

$$x\theta \rightarrow x\theta + 2\pi x \quad -(2)$$

The earth for example precesses in one year by 0.05 arc seconds. If an initial measurement is made at some point in the orbit, then the earth advances by:

$$x\theta \rightarrow x\theta + \left(2\pi + \frac{0.05}{3600}\right) \quad -(3)$$

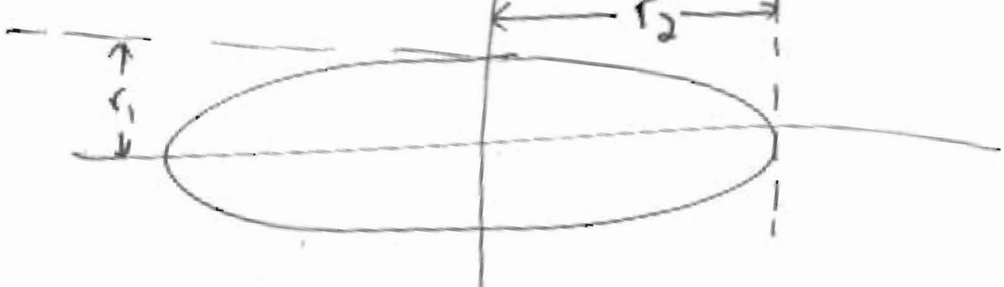
in radians. So:

$$2\pi x = 2\pi + \frac{0.05}{3600} \quad -(4)$$

$$x = 1 + \frac{0.05}{2\pi \times 3600}$$

$$\boxed{x = 1 + 2.21 \times 10^{-6}} \quad -(5)$$

Fig (1)



With reference to Fig. (1):

$$r_1 = \frac{\alpha}{(1-\epsilon^2)^{1/2}}, \quad r_2 = \frac{\alpha}{1-\epsilon^2} \quad -(6)$$

2) From eq. (1):

$$\theta_1 = \frac{1}{x}\cos^{-1}\left(\frac{d}{r_1} - 1\right), \quad \theta_2 = \frac{1}{x}\cos^{-1}\left(\frac{d}{r_2} - 1\right) \qquad -(7)$$

So:

$$\Delta\theta = \theta_2 - \theta_1 = \int_{r_1}^{r_2} \frac{1}{r^2}\left(\frac{1}{b^2} - m(r)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr \qquad -(8)$$

It follows that:

$$\cos^{-1}\left(\frac{d}{r_2} - 1\right) - \cos^{-1}\left(\frac{d}{r_1} - 1\right) = x\int_{r_1}^{r_2} \frac{1}{r^2}\left(\frac{1}{b^2} - m(r)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr \qquad -(9)$$

where:

$$m(r) = \frac{1}{b^2} - \left(\frac{x\epsilon}{d}\right)^2 + \left(\frac{x}{d}\right)^2\left(1 - \frac{d}{r}\right)^2 \qquad -(10)$$

$$\sim \left(\frac{E}{mc^2}\right)^2\left(1 - \frac{1}{2}\left(\frac{\omega r}{c}\right)^2\right) \qquad -(11)$$

The distances r_1 and r_2 can be observed, so x can be found from eq. (9), using increasingly accurate approximations.

It should be the experimental value (5).