192(9): The $m(r)$ Function of a Binary Pulsar

If the orbit is considered to be a spiralling and precessing ellipse then:

$$r = e^{y\theta} \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(1)$$

and $$\frac{dr}{d\theta} = yr + \left(\frac{x\epsilon}{\alpha}\right) e^{y\theta} r^2 \sin(x\theta) \quad -(2)$$

i.e. $$e^{y\theta} \sin(x\theta) = \left(\frac{\alpha}{\epsilon x}\right)\left(\frac{dr}{d\theta} - yr\right) \quad -(3)$$

where $$\frac{dr}{d\theta} = r^2\left(\frac{1}{b^2} - m(r)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{1/2} \quad -(4)$$

In cylindrical polar coordinates:

$$r^2 = X^2 + Y^2 \quad -(5)$$

$$\theta = \tan^{-1}\frac{Y}{X}$$

If we define:

$$r_1 = r_0 e^{y\theta} \quad -(6)$$

$$r_2 = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(7)$$

then $$e^{y\theta} = \frac{r_1}{r_0}, \quad 1+\epsilon\cos(x\theta) = \frac{\alpha}{r_2} \quad -(8)$$

$$\cos(x\theta) = \frac{1}{\epsilon}\left(1 - \frac{\alpha}{r_2}\right) \quad -(9)$$

so: $$r^2\left(\frac{1}{b^2} - m(r)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{1/2} = \frac{r_1}{r_0}\left[1 - \cos^2(x\theta)\right]^{1/2}$$

$$= \frac{r_1}{r_0}\left[1 - \frac{1}{\epsilon^2}\left(1 - \frac{\alpha}{r_2}\right)^2\right]^{1/2}$$