192(7): New Approach to Newtonian Dynamics

In view of the shortcomings of the Einsteinian GR it is proposed that Newtonian gravitation be defined as

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\theta^2 \quad -(1)$$

with:

$$m(r) = \frac{a^2 A}{1 + \left(\frac{a}{r}\right)^2} \quad -(2)$$

and

$$A = \frac{1}{b^2} - \left(\frac{x\epsilon}{\alpha}\right)^2 \quad -(3)$$

$$x = 1. \quad -(4)$$

Eq (1) gives the ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(5)$$

which is a solution of:

$$\frac{d^2 u}{d\theta^2} + u = \frac{1}{\alpha} \quad -(6)$$

$$u = \frac{1}{r}. \quad -(7)$$

where

If

$$\alpha = \frac{L^2}{m^2 M G} \quad -(8)$$

then

$$\frac{d^2 u}{d\theta^2} + u = -\frac{m}{L^2 u^2} F(u) \quad -(9)$$

2)

i.e

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2} F(r) \quad -(10)$$

where

$$F(r) = -\frac{mMG}{r^2} \quad -(11)$$

Eq (10) is the well known result of classical lagrangian dynamics, and the function $F(r)$ is the Hooke / Newton inverse square law.

When

$$x \neq 1 \quad -(12)$$

observed corrections to Newtonian orbits are given, i.e. precession of ellipses.

Here

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E}, \quad -(13)$$

$$E = mc^2 n(r)\left(n(r) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(14)$$

$$L = mr^2\frac{d\theta}{d\tau} \quad -(15)$$

$$\frac{dt}{d\tau} = \left(n(r) - \frac{v^2}{c^2}\right)^{-1/2} \quad -(16)$$

In the $r \rightarrow \infty$ limit, eq. (5) gives:

$$n(r) \xrightarrow[r\to\infty]{} a^2\left(\frac{1}{b^2} - \left(\frac{x\epsilon}{d}\right)^2\right) \quad -(17)$$

In this case eq (1) approaches the Minkowski metric

3)
$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(18)$$
which is equivalent to:
$$E^2 = p^2 c^2 + m^2 c^4 \quad -(19)$$
for a free particle. So for a static ellipse:
$$\frac{a^2}{b^2} - \left(\frac{a\epsilon}{\alpha}\right)^2 = 1 \quad -(20)$$
i.e. $$\left(\frac{a}{b}\right)^2 = \left(\frac{E}{mc^2}\right)^2 = 1 + \left(\frac{\epsilon L}{mc\alpha}\right)^2 \quad -(21)$$
i.e $$E^2 = \frac{m^2 c^4 \epsilon^2 L^2}{m^2 c^2 \alpha^2} + m^2 c^4 \quad -(22)$$
$$= \left(\frac{\epsilon c L}{\alpha}\right)^2 + m^2 c^4$$
Comparing eqs. (19) and (22):
$$\boxed{p = \frac{\epsilon L}{\alpha}} \quad -(23)$$
The Michaelson limit is reached when:
$$r \to \alpha, \quad p \to \frac{\epsilon L}{\alpha} \quad -(24)$$