192(4): Severe Criticism of the "Schwarzschild" Metric.

In a letter of 22nd Dec. 1915 to Albert Einstein, Karl Schwarzschild proposed the m function:

$$m(R) = 1 - \frac{\gamma}{(R^3 + r_0^3)^{1/3}} \quad -(1)$$

where

$$R \neq r \quad -(2)$$

and

$$r_0 = 2MG/c^2. \quad -(3)$$

Schwarzschild severely criticized errors in a paper published by Einstein on Nov. 22nd. 1915 on the precession of the perihelion.

An unknown individual later changed eq. (1) to:

$$m(r) = 1 - \frac{r_0}{r} \quad -(4)$$

and falsely attributed eq. (4) to Schwarzschild. It is claimed in the standard model of physics that eq. (4) gives a precessing ellipse. This note shows that this claim is incorrect.

Consider the so called "Schwarzschild" metric:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{r_0}{r}\right) c^2 dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \quad -(5)$$

More accurately this is the infinitesimal line element in plane:

$$dZ^2 = 0 \quad -(6)$$

cylindrical polar coordinates.

From eq. (5):

2) $$\frac{dr}{d\theta} = r^2\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{1/2} \quad -(7)$$

In order for this to be a precessing ellipse:

$$\frac{dr}{d\theta} = \frac{x\epsilon}{\alpha} r^2 \quad -(8)$$

So $$\left(\frac{x\epsilon}{\alpha}\right)^2 = \frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right) \quad -(9)$$

In this notation:

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E} \quad -(10)$$

where E is the conserved total energy and L the conserved angular momentum. So

$$\frac{a}{b} = \frac{E}{mc^2}. \quad -(11)$$

In eq. (8), ϵ is the eccentricity, 2α is the latus rectum, and x a constant. The gravity r_0 of the standard model is:

$$r_0 = \frac{2MG}{c^2} \quad -(12)$$

where M is the attracting mass, G is Newton's constant and c the vacuum speed of light.

Define the constant:

$$A := \frac{1}{b^2} - \left(\frac{x\epsilon}{\alpha}\right)^2 \quad -(13)$$

then from eq. (9):

$$\left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right) = A \quad -(14)$$

This is a cubic equation in r:

3)

$$\left(a^2 A - 1\right) r^3 + r_0 r^2 - a^2 r + r_0 a^2 = 0 \quad - (15)$$

This can be solved by computer to give three roots, which are values of r in terms of constants. So the so called "Schwarzschild metric" gives only three values of r. This makes no sense at all.

As shown in note 192(3) the function $n(r)$ for a precessing ellipse defined by eq. (8) must be:

$$n(r) = \frac{A}{\frac{1}{a^2} + \frac{1}{r^2}} \quad - (16)$$

$$= \left(\frac{1}{b^2} - \left(\frac{x\epsilon}{d}\right)^2\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)^{-1}$$

The ellipse becomes static if

$$x \rightarrow 1. \quad - (17)$$

For the Minkowski limit:

$$n(r) \rightarrow 1 \quad - (18)$$

which can only be reached when:

$$\left.\begin{array}{l} \epsilon \rightarrow 0, \\ r \rightarrow \infty \end{array}\right\} \quad - (19)$$

and

$$n(r) \rightarrow \left(\frac{a}{b}\right)^2 = \left(\frac{E}{mc^2}\right)^2 \quad - (20)$$

i.e.

$$E \rightarrow mc^2 \quad - (21)$$

in which case the orbit is a circle ($\epsilon = 0$) at infinite r, meaning a free particle limit.