1) 186(7): Calculation of the Connection Directly from Metric Compatibility.

The metric compatibility condition is:

$$\partial_\rho g_{\mu\nu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda} = 0 \quad -(1)$$

Consider a metric of the type:

$$g_{\mu\nu} = \begin{bmatrix} 1-\frac{r_0}{r} & 0 & 0 & 0 \\ 0 & \left(1-\frac{r_0}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\phi \end{bmatrix} \quad -(2)$$

$$g^{\mu\nu} = \begin{bmatrix} \left(1-\frac{r_0}{r}\right)^{-1} & 0 & 0 & 0 \\ 0 & \left(1-\frac{r_0}{r}\right) & 0 & 0 \\ 0 & 0 & -1/r^2 & 0 \\ 0 & 0 & 0 & -1/(r^2 \sin^2\phi) \end{bmatrix} \quad -(3)$$

For $\mu = \nu = 0 \quad -(4)$

eq. (1) becomes:

$$\partial_\rho g_{00} - \Gamma^{0}_{\rho 0} g_{00} - \Gamma^{0}_{\rho 0} g_{00} \quad -(5)$$

i.e.

$$\Gamma^{0}_{10} = \frac{1}{2 g_{00}} \partial_1 g_{00} \quad -(6)$$

$$= \frac{1}{2\left(1-\frac{r_0}{r}\right)} \frac{\partial}{\partial r}\left(1-\frac{r_0}{r}\right)$$

$$\boxed{\Gamma^0_{10} = \frac{r_0}{2r^2\left(1-\frac{r_0}{r}\right)}} \quad -(7)$$

for

$$\mu = \nu = 1 .$$

$$\partial_\rho g_{11} - \Gamma^1_{\rho 1} g_{11} - \Gamma^1_{\rho 1} g_{11} = 0 \quad -(8)$$

i.e. $$\partial_1 g_{11} = 2\Gamma^1_{11} g_{11} . \quad -(9)$$

For $$\mu = \nu = 2 \quad -(10)$$

$$\partial_1 g_{22} = 2\Gamma^2_{12} g_{22} \quad -(11)$$

$$\Gamma^2_{12} = \frac{1}{2g_{22}} \partial_1 g_{22} \quad -(12)$$

$$\boxed{\Gamma^2_{12} = \frac{1}{r}} \quad -(13)$$

For $$\mu = \nu = 3 \quad -(14)$$

$$\partial_\rho g_{33} = 2\Gamma^3_{\rho 3} g_{33} \quad -(15)$$

and $$\Gamma^3_{13} = \frac{1}{2g_{33}} \partial_1 g_{33}$$

$$\boxed{\Gamma^3_{13} = \frac{1}{r}} \quad -(16)$$

3) $$\boxed{\Gamma^3_{23} = \frac{1}{2g_{33}} \partial_2 g_{33} = \frac{\sin\phi}{\cos\phi}} \quad -(17)$$

In the limit:

$$r_0 << r \quad -(18)$$

$$\Gamma^0_{10} \rightarrow \frac{mG}{c^2 r^2} \quad -(19)$$

and

$$\boxed{F = -mc^2 \Gamma^0_{10} = -\frac{mMG}{r^2}} \quad -(20)$$

From eq. (9) it is seen that Γ^1_{11} exists by metric compatibility but the commutator method shows that for Γ^1_{11} both the torsion and curvature vanish. So Γ^1_{11} is a property of a spacetime in which there is no curvature or torsion.

In gravitational theory, only Γ^0_{10} is significant, and for this connection:

$$T^0_{10} = 2\Gamma^0_{10} \quad -(21)$$

and the curvature is zero.