186(4): Riemann Torsion of the Solar System

The connection of the solar system is described by:

$$\Gamma^{\sigma}_{\rho\mu} = \frac{1}{4} g^{\sigma\alpha} \partial_\rho g_{\mu\alpha} \quad -(1)$$

with

$$\sigma = \alpha = \mu \quad -(2)$$

If the so called "Schwarzschild metric" is used for the sake of illustration:

$$g_{00} = 1 - \frac{r_0}{r}, \quad g_{11} = -\left(1 - \frac{r_0}{r}\right)^{-1} \quad -(3)$$

$$g^{00} = \left(1 - \frac{r_0}{r}\right)^{-1}, \quad g^{11} = -\left(1 - \frac{r_0}{r}\right) \quad -(4)$$

There are only two non-zero connections:

$$\Gamma^{0}_{10} = \frac{1}{4} g^{00} \partial_1 g_{00} \quad -(5)$$

and

$$\Gamma^{1}_{11} = \frac{1}{4} g^{11} \partial_1 g_{11} \quad -(6)$$

However:

$$\Gamma^{1}_{11} = 0 \quad -(7)$$

because of antisymmetry. So there is only one connection for the solar system:

$$\Gamma^{0}_{10} = -\Gamma^{0}_{01} = \frac{1}{4} g^{00} \frac{\partial g_{00}}{\partial r} \quad -(8)$$

$$= \frac{1}{4} \left(1 - \frac{r_0}{r}\right)^{-1} \frac{\partial}{\partial r} \left(1 - \frac{r_0}{r}\right)$$

i.e.

$$\Gamma^{0}_{10} = -\frac{r_0}{4r^2}\left(1-\frac{r_0}{r}\right)^{-1}$$

$$\boxed{\Gamma^{0}_{10} = -\frac{r_0}{4r(r-r_0)}} \quad -(9)$$

and the torsion of the solar system is:

$$T^{0}_{10} = \Gamma^{0}_{10} - \Gamma^{0}_{01}$$

$$\boxed{T^{0}_{10} = -\frac{r_0}{2r(r-r_0)}} \quad -(10)$$

If

$$r_0 << r \quad -(11)$$

then

$$\boxed{T^{0}_{10} \rightarrow -\frac{MG}{r^2c^2}} \quad -(12)$$

where M is the mass of the sun and r the distance from the sun.

This is a simple and powerful result, much less complicated than the older description by Einstein and others.
