1)

186(1): Relation between Metric and Hamilton Jacobi Equation.

Consider the Minkowski metric in cylindrical polar coordinates:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(1)$$

For planar motion: $dZ^2 = 0.$ $-(2)$

Therefore:

$$c^2 = c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2. \quad -(3)$$

$-(4)$

Multiply by the measured mass m_0:

$$m_0 c^2 = m_0 c^2\left(\frac{dt}{d\tau}\right)^2 - m_0\left(\frac{dr}{d\tau}\right)^2 - m_0 r^2\left(\frac{d\phi}{d\tau}\right)^2$$

The constants of motion are the total energy E, the angular momentum L and linear momentum p_r. Thus:

$$E = m_0 c^2 \frac{dt}{d\tau}, \quad L = m_0 r^2 \frac{d\phi}{d\tau}, \quad p_r = m_0 \frac{dr}{d\tau} \quad -(5)$$

Therefore:

$$m_0 c^2 = \frac{E^2}{m_0 c^2} - \frac{p_r^2}{m_0} - \frac{L^2}{m_0 r^2} \quad -(6)$$

The total linear momentum p is defined as

$$p^2 = p_r^2 + \frac{L^2}{r^2} \quad -(7)$$

2)

So:

$$m_0 c^2 = \frac{E^2}{m_0 c^2} - \frac{p^2}{m_0} \quad -(8)$$

i.e

$$E^2 = c^2 p^2 + m_0^2 c^4 \quad -(9)$$

which is the Einstein energy equation generalized to account for finite angular momentum through eq. (7). When there is no angular momentum:

$$p = \frac{p}{\ell} = \gamma m_0 \underline{v} = m_0 \frac{dr}{d\tau} \quad -(10)$$

and eq. (9) reduces to eq. (10).

Eq. (1) is written in the flat spacetime without geometrical connection, the Minkowski spacetime. In the spherical spacetime with connection eq. (1) becomes:

$$ds^2 = c^2 d\tau^2 = n(r) c^2 dt^2 - m(r) dr^2 - r^2 d\phi^2 \quad -(11)$$

What effect does this have on eq. (9) which is:

$$p^\mu p_\mu = m_0^2 c^2 \quad -(12) \; ?$$

where:

3)
$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \quad p_\mu = \left(\frac{E}{c}, -\underline{p}\right) \quad -(13)$$

From eq. (11):
$$m_0 c^2 = m_0 n(r) c^2 \left(\frac{dt}{d\tau}\right)^2 - m_0 m(r) \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\phi}{d\tau}\right)^2 \quad -(14)$$

The constants of motion are:
$$E' = n(r) m_0 c^2 \frac{dt}{d\tau}, \quad p_e' = m(r) m_0 \frac{dr}{d\tau}, \quad -(15)$$
$$L = m_0 r^2 \frac{d\phi}{d\tau}$$

so we again obtain eq. (6) but with E changed to E' and p_e changed to p_e':
$$m_0 c^2 = \frac{E'^2}{m_0 c^2} - \frac{p_e'^2}{m_0} - \frac{L^2}{m_0 r^2} \quad -(16)$$

The total linear momentum p' is now:
$$p'^2 = p_e'^2 + \frac{L^2}{r^2} \quad -(17)$$

so:
$$E'^2 = c^2 p'^2 + m_0^2 c^4 \quad -(18)$$

i.e.

$$p'^{\mu} p'_{\mu} = m_0^2 c^4 \quad -(19)$$

where:

$$p'^{\mu} = \left(\frac{E'}{c}, \underline{p}' \right), \quad -(20)$$

$$p'_{\mu} = \left(\frac{E'}{c}, -\underline{p}' \right) \quad -(21)$$

The format of eqns. (12) and (19) is the same, but:

$$p^{\mu} \rightarrow p^{\mu\prime} \quad -(22)$$

$$p_{\mu} \rightarrow p'_{\mu} \quad -(23)$$

This result can be expressed in terms of a formalism similar to the minimal prescription, but note carefully that it is a formalism of general relativity:

$$p^{\mu\prime} = p^{\mu} - m_0 \underline{\Phi}^{\mu} \quad -(24)$$

$$p'_{\mu} = p_{\mu} - m_0 \underline{\Phi}_{\mu} \quad -(25)$$

where $\underline{\Phi}^{\mu}$ is the <u>gravitational</u> <u>four potential</u>.

$$\underline{\Phi}^{\mu} = \left(\underline{\Phi}, \underline{\Phi} \right) \quad -(26)$$

In electrodynamics:

5) $$p^{\mu\prime} = p^{\mu} - eA^{\mu} \quad -(27)$$

where A^{μ} is the electromagnetic four potential.

Therefore spherical spacetime is represented by the Hamilton Jacobi equation

$$(p^{\mu} - m_0 \Phi^{\mu})(p_{\mu} - m_0 \Phi_{\mu}) = m_0^2 c^2 \quad -(28)$$

or

$$(p^{\mu} - eA^{\mu})(p_{\mu} - eA_{\mu}) = m_0^2 c^2 \quad -(29)$$

This transforms the metric of spherical spacetime into equations involving A_{μ} and Φ_{μ}. In ECE theory

$$A_{\mu}^{a} = A^{(0)} q_{\mu}^{a} \quad -(30)$$

and

$$\Phi_{\mu}^{a} = \Phi^{(0)} q_{\mu}^{a} \quad -(31)$$

where q_{μ}^{a} is the Cartan tetrad.

In recent papers the Hamilton Jacobi equation has been developed with the ECE field equation:

$$(\square + R) q_{\mu}^{a} = 0 \quad -(32)$$

so spherical spacetime is transformed to field and wave equations.