184(1): Definition of Cross Product of the Pauli Matrix vector and Cartesian Vector.

The Pauli matrices are defined by:

$$\sigma_X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad -(1)$$

and the Pauli vector by:

$$\underline{\sigma} = \sigma_X \underline{i} + \sigma_Y \underline{j} + \sigma_Z \underline{k}, \quad -(2)$$

in which the Pauli matrices are treated in a scalar like fashion.

Therefore:

$$\underline{\sigma} \cdot \underline{r} = \sigma_X X + \sigma_Y Y + \sigma_Z Z \quad -(3)$$

where

$$\underline{r} = X \underline{i} + Y \underline{j} + Z \underline{k} \quad -(4)$$

i.e.

$$\underline{\sigma} \cdot \underline{r} = \begin{bmatrix} Z & X - iY \\ X + iY & -Z \end{bmatrix}. \quad -(5)$$

In developing the interaction of the $\underline{B}^{(3)}$ field with the spin magnetic dipole moment of the electron:

$$\underline{m} = \frac{e}{m} \underline{S} = \frac{e\hbar}{2m} \underline{\sigma} \quad -(6)$$

2) it is necessary to consider the energy:

$$E = -\underline{m} \cdot \underline{B}^{(3)} \quad -(7)$$

and the torque:

$$\underline{Tq} = -\underline{m} \times \underline{B}^{(3)} \quad -(8)$$

The energy (7) gives RFR (radiatively induced fermion resonance), i.e. high resolution ESR, NMR and MRI without magnets.

The torque (8) has not been considered hitherto and is defined by its magnitude:

$$Tq = mB^{(3)} \sin\theta \quad -(9)$$

where $\theta = \omega t. \quad -(10)$

If a torque of type (9) is used in a nanometric catalyst, the following Euler equation results:

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = \left(\frac{mB^{(3)}}{I}\right)\sin(\omega t) \quad -(11)$$

whose particular solution is:

$$\boxed{\theta = \left(\frac{mB^{(3)}}{I}\right)\left(\frac{\sin(\omega t)}{\omega_0^2 - \omega^2}\right)} \quad -(12)$$

3)

At resonance:

$$\omega = \omega_0 \quad -(13)$$

it is found that:

$$\theta \rightarrow \infty \quad -(14)$$

and dissociation takes place.

The experimental design relies on the ability to define ω and ω_0, the latter being a characteristic frequency of the catalyst.

There are other torques present such as:

$$\underline{Tq} = -\underline{m} \times \underline{B}^{(1)} \quad -(15)$$

where

$$\underline{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}} \left(i\underline{i} + \underline{j} \right) e^{i\phi} \quad -(16)$$

where

$$\phi = \omega t - \kappa Z. \quad -(17)$$

The torque (15) is:

$$\boxed{\underline{Tq} = -\frac{e\hbar}{2m} \underline{\sigma} \times \underline{B}^{(1)}} \quad -(18)$$

In general the torque $\underline{\sigma} \times \underline{B}^{(1)}$ is worked out with the vector definition of cross product:

4)

$$\underline{\sigma} \times \underline{B}^{(1)} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \sigma_X & \sigma_Y & \sigma_Z \\ B_X^{(1)} & B_Y^{(1)} & 0 \end{vmatrix} \quad -(19)$$

$$= \underline{i}\left(-\sigma_Z B_Y^{(1)}\right) - \underline{j}\left(-\sigma_Z B_X^{(1)}\right) + \underline{k}\left(\sigma_X B_Y^{(1)} - \sigma_Y B_X^{(1)}\right) \quad -(20)$$

The torque about the Z axis is:

$$Tq_Z = \sigma_X B_Y^{(1)} - \sigma_Y B_X^{(1)} \quad -(21)$$

i.e.

$$Tq_Z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & B_Y^{(1)} - iB_X^{(1)} \\ B_Y^{(1)} + iB_X^{(1)} & 0 \end{bmatrix} \quad -(22)$$

i.e.

$$Tq_Z = 0 \quad -(23)$$

There are two types of torque about the X axis:

$$Tq_X \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -B_Y^{(1)} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(24)$$

and two about the Y axis:

$$Tq_Y \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B_X^{(1)} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(25)$$