1)

184(2): Amplification of the Paramagnetic Inverse Faraday Effect.

It is shown as follows that this effect is amplified at low frequency. Consider the Pauli Schroedinger equation

$$\hat{H}\phi = E\phi \quad -(1)$$

where

$$\hat{H} = \frac{1}{2m}\,\underline{\sigma}\cdot(\underline{p} - e\underline{A})\,\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*) + V \quad -(2)$$

where $\underline{A}^*$ is the complex conjugate of the vector potential $\underline{A}$. The paramagnetic IFE term is:

$$H_{IFE} = \frac{ie^2}{2m}\,\underline{\sigma}\cdot\underline{A}\times\underline{A}^* \quad -(3)$$

which follows from eq. (2) using Pauli algebra. Therefore in complex circular notation:

$$H_{IFE} = \frac{ie^2}{2m}\,\underline{\sigma}\cdot\underline{A}^{(1)}\times\underline{A}^{(2)} \quad -(4)$$

The $\underline{B}^{(3)}$ field is defined by:

$$\underline{B}^{(3)} = \underline{B}^{(3)*} = -i\frac{e}{\hbar}\,\underline{A}^{(1)}\times\underline{A}^{(2)} \quad -(5)$$

so:

$$H_{IFE} = -\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B}^{(3)} = -\frac{e}{m}\,\underline{S}\cdot\underline{B}^{(3)}$$

$$= -\underline{m}\cdot\underline{B}^{(3)} \quad -(6)$$

The vector potential is considered to be the plane wave:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \exp\left(i(\omega t - \kappa Z)\right) \quad -(7)$$

whose angular frequency is ω at instant t, and whose wave number is κ at point Z. The complex conjugate of eq. (7) is:

$$\underline{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \exp\left(-i(\omega t - \kappa Z)\right) \quad -(8)$$

so the Riemann Kinetic conjugate product is:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = \frac{A^{(0)2}}{2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ 1 & i & 0 \end{vmatrix} \quad -(9)$$

$$= i A^{(0)2}\,\underline{k} \quad -(10)$$

So:

$$\boxed{H_{IFE} = -\frac{e^2}{2m} A^{(0)2}\,\underline{\sigma}\cdot\underline{k}} \quad -(11)$$

The power density in watts per square metre of electromagnetic radiation is

$$I = \frac{c}{\mu_0} B^{(0)2} \quad -(12)$$

where $B^{(0)}$ is the magnitude of the magnetic flux density of the plane wave.

3) A method must be found to relate $A^{(0)}$ and $B^{(0)}$. One simple method is to use:

$$\underline{B}^{(1)} = \underline{\nabla} \times \underline{A}^{(1)} \quad -(13)$$

and

$$\underline{B}^{(2)} = \underline{\nabla} \times \underline{A}^{(2)} \quad -(14)$$

In a fuller development based on ECE theory, the spin connection terms in eqs. (13) and (14) are considered. In the simple theory based on eqs. (13) and (14):

$$\underline{B}^{(1)} = \underline{\nabla} \times \underline{A}^{(1)} = \kappa \underline{A}^{(1)} \quad -(15)$$

so:

$$B^{(0)} = \kappa A^{(0)} = \frac{\omega}{c} A^{(0)} \quad -(16)$$

From eqs. (12) and (16):

$$A^{(0)2} = \mu_0 c \frac{I}{\omega^2}. \quad -(17)$$

From eq. (17) in eq. (11):

$$\boxed{\begin{aligned} H_{IFE} &= -\frac{e^2 \mu_0 c}{2m} \frac{I}{\omega^2} \underline{\sigma} \cdot \underline{k} \\ &= -\underline{m} \cdot \underline{B}^{(3)} \end{aligned}} \quad -(18)$$

So the interaction Hamiltonian is proportional to I and inversely proportional to ω^2.