183(3): Resonance Amplification of Torque due to the $B^{(3)}$ Field.

As shown in 00347 on www.aias.us the torque is between the induced magnetic dipole moment $\underline{m}^{ind}$ and the magnetic field of the laser:

$$\underline{B}_L^+ = \frac{B_0}{\sqrt{2}}\left(\underline{j} + i\underline{i}\right) e^{i\phi_L} \quad -(1)$$

$$\underline{B}_L^- = \frac{B_0}{\sqrt{2}}\left(\underline{j} - i\underline{i}\right) e^{-i\phi_L} \quad -(2)$$

$$\underline{B}_R^+ = \frac{B_0}{\sqrt{2}}\left(\underline{j} - i\underline{i}\right) e^{i\phi_R} \quad -(3)$$

$$\underline{B}_R^- = \frac{B_0}{\sqrt{2}}\left(\underline{j} + i\underline{i}\right) e^{-i\phi_R} \quad -(4)$$

If $\underline{e}_1$, $\underline{e}_2$ and $\underline{e}_3$ are unit vectors in axes 1, 2 and 3 of the moment of inertia frame then the components of the induced magnetic dipole moment in the molecule fixed frame are:

$$m_1 = -E_0^2 e_{1Z}\left(b_{123}'' - b_{132}''\right) \quad -(5)$$

$$m_2 = -E_0^2 e_{2Z}\left(b_{213}'' - b_{231}''\right) \quad -(6)$$

$$m_3 = -E_0^2 e_{3Z}\left(b_{312}'' - b_{321}''\right) \quad -(7)$$

where b'' is the imaginary part of a magnetic electric electric hyperpolarizability.

The components in the laboratory frame are:

2)

$$\begin{bmatrix} m_X \\ m_Y \\ m_Z \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \quad -(8)$$

so
$$m_X = R_{11} m_1 + R_{12} m_2 + R_{13} m_3 \quad -(9)$$
$$m_Y = R_{21} m_1 + R_{22} m_2 + R_{23} m_3 \quad -(10)$$
$$m_Z = R_{31} m_1 + R_{32} m_2 + R_{33} m_3 \quad -(11)$$

and
$$\underline{m}^{ind} = m_X \underline{i} + m_Y \underline{j} + m_Z \underline{k} \quad -(12)$$

In 00347 the matrix R was evaluated with field applied molecular dynamics computer simulation.

The torque is therefore:

$$\underline{Tq} = m_Z B_Y \underline{i} - m_Z B_X \underline{j} - (m_X B_Y - m_Y B_X) \underline{k} \quad -(13)$$

As in note 183(2):

$$r_X F_Y - r_Y F_X = -(m_X B_Y - m_Y B_X) \quad -(14)$$

$$= \frac{dL}{dt} = I \frac{d^2\theta}{dt^2} \quad -(15)$$

Taking account of the catalyst in the nanomotor model:

3)

$$I \frac{d^2\theta}{dt^2} + V(0)\theta = -(m_x B_y - m_y B_x) \quad -(16)$$

whose particular solution is:

$$\boxed{\theta_p = \frac{m_y B_x - m_x B_y}{I(\omega_0^2 - \omega^2)}} \quad -(17)$$

where

$$\omega_0 = \left(\frac{V(0)}{I}\right)^{1/2} \quad -(18)$$

and

$$\theta_p = D\cos\omega t \quad -(19)$$

In order to find an expression for ω, consider

$$\ddot{\theta} + \omega_0^2\theta = A\cos\omega t \quad -(20)$$

then

$$\theta_p = \frac{A\cos\omega t}{\omega_0^2 - \omega^2} \quad -(21)$$

so

$$\cos\omega t = \frac{1}{AI}(m_y B_x - m_x B_y) \quad -(22)$$

therefore ω is of order approximating to the laser frequency.