183(2): Details of Resonant Amplification of the Torque between an Electric Dipole and an Electromagnetic Plane Wave.

The electric field strength of the plane wave is:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) \exp\left(i(\omega t - \kappa Z) \right) \quad -(1)$$

where ω is the angular frequency at time t and κ the wavenumber at point Z. Therefore the torque is:

$$\underline{Tq} = -\underline{\mu} \times \underline{E} \quad -(2)$$

The real and physical part of $\underline{E}$ is:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} \cos\phi + \underline{j} \sin\phi \right) \quad -(3)$$

where

$$\phi = \omega t - \kappa Z \quad -(4)$$

In general:

$$\underline{\mu} = \mu_X \underline{i} + \mu_Y \underline{j} + \mu_Z \underline{k} \quad -(5)$$

so:

$$\underline{Tq} = - \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \mu_X & \mu_Y & \mu_Z \\ E_X & E_Y & 0 \end{vmatrix} \quad -(6)$$

$$= \mu_Z E_Y \underline{i} - \mu_Z E_X \underline{j} - \left(\mu_X E_Y - \mu_Y E_X \right) \underline{k}$$

The torque is defined as:

2)

$$\underline{Tq} = \underline{r} \times \underline{F} \quad -(7)$$

where $\underline{r}$ is position vector and $\underline{F}$ is force. In general:

$$\underline{Tq} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \underline{i}(r_y F_z - r_z F_y) - \underline{j}(r_x F_z - r_z F_x) + \underline{k}(r_x F_y - r_y F_x) \quad -(8)$$

So:

$$r_y F_z - r_z F_y = \mu_z E_y \quad -(9)$$

$$r_x F_z - r_z F_x = \mu_z E_x \quad -(10)$$

$$r_x F_y - r_y F_x = -(\mu_x E_y - \mu_y E_x) \quad -(11)$$

For a torque in the X-Y plane only eq. (11) need be considered. The laser propagates perpendicular to X-Y in the Z axis. In this case the angular momentum is:

$$|\underline{L}| = L = I \frac{d\theta}{dt} \quad -(12)$$

where the moment of inertia is:

$$I = mr^2 \quad -(13)$$

where m is the mass of the molecule with dipole $\underline{\mu}$.
The torque magnitude is:

$$Tq = \frac{dL}{dt} = I \frac{d^2\theta}{dt^2} \quad -(14)$$

i.e. $$Tq = r_x F_y - r_y F_x = I \frac{d^2\theta}{dt^2} \quad -(15)$$

3) The catalyst in the nanometre mould is ←
represented by $dV/d\theta$, i.e. an extra torque.
In the linear approximation of the harmonic oscillator:

$$\frac{dV}{d\theta} = V(0)\,\theta \quad -(16)$$

So eq. (11) becomes:

$$\boxed{I\frac{d^2\theta}{dt^2} + V(0)\,\theta = -\left(\mu_x E_y - \mu_y E_x\right)}$$

$$-(17)$$

i.e. $$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = -\frac{1}{I}\left(\mu_x E_y - \mu_y E_x\right) \quad -(18)$$

where $$\omega_0^2 = \frac{V(0)}{I}. \quad -(19)$$

Therefore ω_0 is a characteristic frequency of the catalyst in the nanometre mould.

The complementary function $\theta_c(t)$ is the solution of: $$\frac{d^2\theta_c}{dt^2} + \omega_0^2\theta_c = 0 \quad -(20)$$

i.e. $$\theta_c = A_1 e^{i\omega_0 t} + A_2 e^{-i\omega_0 t} \quad -(21)$$

and describe transient effects.

By inspection, the ~~car~~ particular solution is:

$$\theta_p = D\cos\omega t \quad -(22)$$

Using eq. (22) in eq. (18):

$$-\omega^2 D\cos\omega t + \omega_0^2 D\cos\omega t = -\frac{1}{I}\left(\mu_x E_y - \mu_y E_x\right) \quad -(23)$$

so:

$$D = -\frac{1}{I}\frac{\left(\mu_x E_y - \mu_y E_x\right)}{\left(\omega_0^2 - \omega^2\right)\cos\omega t} \quad -(24)$$

and

$$\boxed{\theta_p = \frac{\mu_y E_x - \mu_x E_y}{I\left(\omega_0^2 - \omega^2\right)}} \quad -(25)$$

The complete solution is:

$$\theta = \theta_c + \theta_p \quad -(26)$$

When

$$\boxed{\omega_0 = \left(\frac{V(0)}{I}\right)^{1/2} = \omega} \quad -(27)$$

the angle θ_p is infinite:

$$\theta_p \to \infty \quad -(28)$$

and the molecule spins infinitely quickly and dissociates. This occurs because a characteristic frequency ω_0 of the catalyst is tuned to ω, the laser frequency in radians per second.