1)

183(1) : Resonant Amplification of Electromagnetically Induced Torque.

Consider the rotational motion in two dimensions of an electric dipole moment subjected to a time varying electric field. The number of dipoles whose axes lie in an element of angle $d\theta$ on the circumference of a circle is $f(\theta, t)\, d\theta$, where $f(\theta, t)$ is the distribution function. In the absence of friction the motion is described by

$$I \frac{d^2\theta}{dt^2} + \frac{\partial V}{\partial \theta} = -|\underline{\mu} \times \underline{E}| \quad -(1)$$

where $\underline{\mu}$ is the electric dipole moment and $\underline{E}$ the electric field. As shown in the animation by Pelkie and myself on www.aias.us the circularly polarized laser is able to spin the dipole $\underline{\mu}$.

In eq. (1) I is the moment of inertia of the spun molecule or ion and V is a potential well of the type generated in a catalyst or molecular liquid. Assuming a linear harmonic oscillator:

$$\frac{\partial V}{\partial \theta} = V(0)\,\theta \quad -(2)$$

where $V(0)$ is a potential energy magnitude.

2) For the sake of analytical simplicity and illustration consider:

$$|\underline{\mu} \times \underline{E}| := \mu E \sin\left(\theta - n\frac{\pi}{2}\right) \quad -(3)$$

$$= -\mu E \cos\theta$$

so:

$$I\frac{d^2\theta}{dt^2} + V(0)\,\theta = \mu E \cos\theta \quad -(4)$$

Define:

$$\theta = \omega t. \quad -(5)$$

Therefore eq. (4) becomes:

$$\frac{d^2\theta}{dt^2} + \omega_0^2\,\theta = A\cos\omega t \quad -(6)$$

where:

$$\omega_0^2 = \frac{V(0)}{I} \quad -(7)$$

and

$$A = \frac{\mu E}{I} \quad -(8)$$

The solution of eq. (6) is:

$$\boxed{\theta(t) = \frac{\mu E}{I}\,\frac{\cos\omega t}{\omega_0^2 - \omega^2}} \quad -(9)$$

Here ω is an angular frequency of

3) the electromagnetic field and ω_0 is a natural angular frequency of the catalyst.

At resonance:

$$\omega_0 = \omega \quad -(10)$$

the angle $\theta(t)$ goes to infinity and the molecule dissociates by centrifugal force.

More accurately, in a circularly polarized beam:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(11)$$

where

$$\phi = \omega t - \kappa Z. \quad -(12)$$

So:

$$\text{Real}\ \underline{E} = \text{Real}\left(\frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)\left(\cos\phi + i\sin\phi\right)\right) \quad -(13)$$

$$= \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i}\cos\phi + \underline{j}\sin\phi\right)$$

and

$$\underline{Tq} = -\underline{\mu} \times \underline{E} \quad -(14)$$

$$= -\frac{E^{(0)}}{\sqrt{2}}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & \mu_Z \\ \cos\phi & \sin\phi & 0 \end{vmatrix}$$

$$= \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i}\,\mu_Z\sin\phi - \underline{j}\,\mu_Z\cos\phi\right)$$