165(3): The R Spectrum for Refraction

Consider the usual approach to refraction and reflection as described in Jackson's chapter 7. This approach is valid in ECE theory for each index a. It considers

$$\underline{\nabla} \cdot \underline{B} = 0 \; , \quad \underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(1)$$

$$\underline{\nabla} \cdot \underline{D} = 0 \; , \quad \underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{0} . \quad -(2)$$

Assume a harmonic time dependence $\exp(-i\omega t)$ and a uniform, isotropic, linear system:

$$\underline{D} = \epsilon \underline{E} \; , \quad \underline{B} = \mu \underline{H} \quad -(3)$$

then

$$(\nabla^2 + \mu \epsilon \omega^2) \underline{E} = \underline{0} \quad -(4)$$

$$(\nabla^2 + \mu \epsilon \omega^2) \underline{B} = \underline{0} \quad -(5)$$

If the phase is $\exp(i(\kappa Z - \omega t))$ then:

$$\kappa = (\mu \epsilon)^{1/2} \omega \quad -(6)$$

The phase velocity from this development is:

$$v_1 = \frac{\omega}{\kappa} = \frac{c}{n} = \frac{1}{(\mu \epsilon)^{1/2}} \quad -(7)$$

where n is the refractive index. Note carefully that this is automatically a theory of special relativity because the equations used are covariant under the Lorentz transform. They are wave equations and can be related to the particle equations of special relativity by the de Broglie postulates now extended by R.

Thus:

$$E = \gamma m c^2 = \hbar \omega \quad -(8)$$

$$p = \gamma m v = \hbar \kappa \quad -(9)$$

and

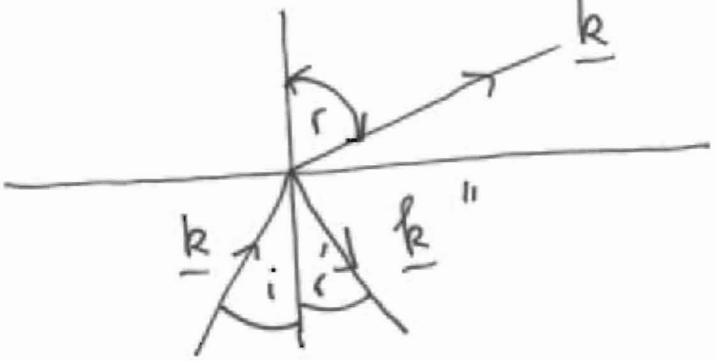
$$\frac{v}{c^2} = \frac{\kappa}{\omega} \quad -(10)$$

Therefore

$$\boxed{v = \frac{c^2}{v_1}} \quad -(11)$$

Refraction is illustrated in Fig. (1):

Fig (1)



and by Snell's law:

$$\frac{\sin i}{\sin r} = \frac{\kappa'}{\kappa} = \frac{n'}{n} \quad -(12)$$

Therefore:

$$\boxed{\frac{\sin i}{\sin r} = \frac{v'}{v}} \quad -(13)$$

From eqs. (8) and (9):

$$v'^2 = 1 - \left(\frac{x}{\omega'}\right)^2 \quad -(14)$$

$$v^2 = 1 - \left(\frac{x}{\omega}\right)^2 \quad -(15)$$

where

$$x = \frac{m c^2}{\hbar} \quad -(16)$$

5) The R factor is defined by:

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(17)$$

so

$$x^2 = c^2 R \quad -(18)$$

In eq. (13):

$$\left(\frac{\sin i}{\sin r}\right)^2 = \frac{1 - \frac{c^2 R}{\omega'^2}}{1 - \frac{c^2 R}{\omega^2}} := A \quad -(19)$$

so

$$\frac{\omega'^2 - c^2 R}{\omega^2 - c^2 R} = \left(\frac{\omega'}{\omega}\right)^2 A \quad -(20)$$

and

$$\boxed{R = \left(\frac{\omega'}{c}\right)^2 \left[\frac{1 - A}{1 - \left(\frac{\omega'}{\omega}\right)^2 A}\right] \qquad A = \left(\frac{\sin i}{\sin r}\right)^2} \quad -(21)$$

This is the R spectrum of any refraction.

It is described in terms of the experimental observables $\sin i$ and $\sin r$, ω and ω'. Therefore refraction is both particulate and undulatory. This theory can now be extended to particle scattering.