162(8): Absorption of Massless Photon, Equivalence of Linear and Angular Momentum Descriptions.

The conservation of linear momentum means:

$$\underline{p}' + \hbar \underline{\kappa} = \underline{p}'' \quad -(1)$$

where $\hbar \underline{\kappa}$ is the momentum of the incoming photon, $\underline{p}'$ is the initial momentum of the electron in orbital 1, $\underline{p}''$ that of the electron in orbital 2. From eq. (1):

$$p^2 = p''^2 + p'^2 - 2p'p'' \cos\theta \quad -(2)$$

The orbital angular momentum on the classical level is:

$$\underline{L} = \underline{r} \times \underline{p} \quad -(3)$$

so

$$\underline{r} \times \underline{L} = \underline{r} \times (\underline{r} \times \underline{p}) = (\underline{r} \cdot \underline{p})\underline{r} - (\underline{r} \cdot \underline{r})\underline{p} \quad -(4)$$

For simplicity of development assume a planar orbit:

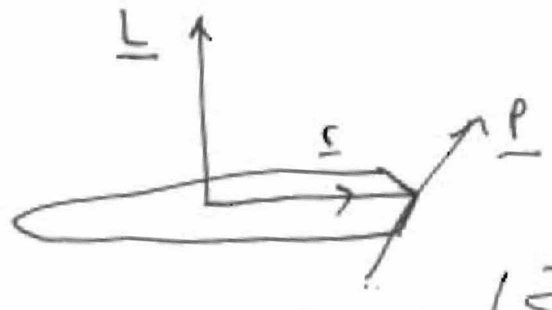

Fig (1)

Then:

$$\underline{r} \cdot \underline{p} = 0 \quad -(5)$$

so

$$\underline{p} = \frac{1}{r^2} \underline{L} \times \underline{r} \quad -(6)$$

$$p = \frac{L}{r} \sin\alpha$$

In a planar orbit:

$$\sin\alpha = 1 \quad -(7)$$

2)

so

$$L = pr \quad -(8)$$

Therefore in eq. (2):

$$L^2 = L''^2 + L'^2 - 2L'L''\cos\theta \quad -(9)$$

where L is the orbital angular momentum of the photon, and L' and L'' are the angular momenta of the electron in orbitals 1 and 2 respectively. We have:

$$L = \hbar \kappa r. \quad -(10)$$

If

$$\kappa r = 1 \quad -(11)$$

then

$$L = \hbar \quad -(12)$$

which is the minimum amount of angular momentum that can be transferred to the electron from the photon. The reduced Planck constant $\hbar$ is the spin angular momentum of the photon. In this limit of least angular momentum transfer:

$$\hbar^2 = L''^2 + L'^2 - 2L'L''\cos\theta \quad -(13)$$

i.e.

$$\boxed{\underline{\hbar} = \underline{L}'' - \underline{L}'} \quad -(14)$$

Eq. (14) is the usual description of absorption (P.W. Atkins, "Molecular Quantum Mechanics", OUP, 1983). It is seen that the usual description is an approximation of the laws of conservation of momentum (1)

3) Considering the Z component of eq. (14) then:

$$L'' - L' = \hbar \quad - (15)$$

In a massless photon the two components of $\hbar$ are:

$$m_s = \pm 1 \quad - (16)$$

Left circularly polarized light has $m_s = 1$. This is eq. (15) and the angular momentum of the electron changes by $\hbar$. This means that the electric dipole transition rule is

$$\Delta l = \pm 1 \quad - (17)$$

It is seen that these well known angular momentum descriptions are an approximation to the generally valid conservation of momentum law (1).

The electric dipole selection rule (17) cannot exist without eq. (1). However, when eq. (1) is used as in UFT 158 to UFT 161, the whole quantum theory collapses.