162(2): Expression for x_2 in Terms of x_1

The starting equation is:

$$x_1^2 = 2x_2^2 + \omega^2 - \omega''^2 - \omega'^2 + 2(\omega''^2 - x_2^2)^{1/2}(\omega'^2 - x_2^2)^{1/2}\cos\theta \quad -(1)$$

Let

$$x = x_2^2 \quad -(2)$$

then

$$x + (\omega''^2 - x)^{1/2}(\omega'^2 - x)^{1/2}\cos\theta = A \quad -(3)$$

where

$$A = \frac{1}{2}(x_1^2 + \omega''^2 + \omega'^2 - \omega^2) \quad -(4)$$

so

$$(\omega''^2 - x)(\omega'^2 - x)\cos^2\theta = (A - x)^2, \quad -(5)$$

$$(\omega''^2\omega'^2 - (\omega'^2 + \omega''^2)x + x^2)\cos^2\theta = A^2 - 2Ax + x^2,$$

i.e.

$$x^2(1 - \cos^2\theta) + ((\omega'^2 + \omega''^2)\cos^2\theta - 2A)x + A^2 - \omega''^2\omega'^2\cos^2\theta = 0 \quad -(6)$$

This is:

$$ax^2 + bx + c' = 0, \quad -(7)$$

$$a = 1 - \cos^2\theta,$$

$$b = (\omega''^2 + \omega'^2)\cos^2\theta - 2A,$$

$$c' = A^2 - \omega''^2\omega'^2\cos^2\theta,$$

so

$$x_1^2 = \frac{1}{2a}\left(-b \pm (b^2 - 4ac)^{1/2}\right) \quad -(8)$$