1)

162(5): Zero Photon Mass Absorption with Conservation of Momentum.

This is the standard theory of absorption of the old quantum theory. Conservation of momentum has never been considered in this theory. The equations are:

$$\gamma' mc^2 + \hbar\omega = \gamma'' mc^2 \quad -(1)$$

$$\underline{p}' + \hbar \underline{\kappa} = \underline{p}'' \quad -(2)$$

where m is the electron mass. In this notation the energy of the electron in orbital 1 is:

$$E_1 = \gamma' mc^2 \quad -(3)$$

and in orbital 2:

$$E_2 = \gamma'' mc^2 \quad -(4)$$

The momentum of the electron in orbital 1 is $\underline{p}'$, and in orbital 2 is $\underline{p}''$. The momentum of the photon is $\hbar\underline{\kappa}$.

For the electron, the de Broglie postulate is:

$$E_1 = \hbar\omega' = \gamma' mc^2 \quad -(5)$$

$$E_2 = \hbar\omega'' = \gamma'' mc^2 \quad -(6)$$

so eq. (1) is:

$$\boxed{\omega' + \omega = \omega''} \quad -(7)$$

This is the same result as in note 162(1) for the photon with mass. So the usual theory, eq. (7) tells us nothing about photon mass.

2) The usual theory seems to work only because it is restricted to energy conservation.

Considering momentum conservation, eq. (2) is:

$$\hbar^2 \kappa^2 = p''^2 + p'^2 - 2p'p''\cos\theta \quad -(8)$$

If the photon is considered to be massless, as in the standard theory, then:

$$\kappa = \omega / c \quad -(9)$$

For the electron use eqs (5) and (6) to find:

$$v'^2 = c^2\left(\frac{\omega'^2 - x^2}{\omega'^2}\right), \quad v''^2 = c^2\left(\frac{\omega''^2 - x^2}{\omega''^2}\right) \quad -(10)$$

where

$$x = \frac{mc^2}{\hbar} \quad -(11)$$

So eq. (8) is:

$$\omega^2 = \omega''^2 - x^2 + \omega'^2 - x^2 - 2\left(\omega''^2 - x^2\right)^{1/2}\left(\omega'^2 - x^2\right)^{1/2}\cos\theta \quad -(12)$$

so

$$\left(\omega''^2 - x^2\right)\left(\omega'^2 - x^2\right)\cos^2\theta = \left(A - x^2\right)^2 \quad -(13)$$

where

$$A = \frac{1}{2}\left(\omega'^2 + \omega''^2 - \omega^2\right) \quad -(14)$$

Therefore:

$$x^4\left(1 - \cos^2\theta\right) + x^2\left(\left(\omega'^2 + \omega''^2\right)\cos^2\theta - 2A\right) + A^2 - \omega''^2\omega'^2\cos^2\theta = 0 \quad -(15)$$

Therefore the electron mass is given by:

$$x^2 = \frac{1}{2a}\left(-b \pm \left(b^2 - 4ac'\right)^{1/2}\right) \quad -(16)$$

where

$$a = \left(1 - \cos^2\theta\right)$$
$$b = \left(\omega'^2 + \omega''^2\right)\cos^2\theta - 2A$$
$$c' = A^2 - \omega''^2\omega'^2\cos^2\theta$$
$$A = \frac{1}{2}\left(\omega'^2 + \omega''^2 - \omega^2\right)$$

Note carefully that this is the result of the standard theory of absorption.

If the electron mass, proportional to x, is not constant, the standard theory fails. In the standard theory the angle of scatter θ is not considered. The only thing that is considered is that the electron energy increases from E_1 to E_2 by absorbing one massless photon of energy $\hbar\omega$.

It is now practical to eliminate the electron momenta p' and p'' in eq. (8) and replace them by energy E_1 and E_2 of the orbitals. This is done by:

$$E_1^2 = c^2 p'^2 + m^2 c^4 \quad -(17)$$
$$E_2^2 = c^2 p''^2 + m^2 c^4 \quad -(18)$$

This will be the subject of the next note.