155(8): Relativistic Time Delay Calculations

This is just a trivial extension of the light deflection calculation, and the latter is accepted uncritically, with the result that major errors are introduced in exactly the same way. The light deflection calculation is based on:

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} \quad -(1)$$

where

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E}, \quad r_0 = \frac{2MG}{c^2}. \quad -(2)$$

The time delay calculation is the trivial extension:

$$\frac{d\tau}{dr} = \frac{d\tau}{d\phi}\frac{d\phi}{dr}, \quad -(3)$$

using

$$L = mr^2\frac{d\phi}{d\tau} \quad -(4)$$

$$E = mc^2\frac{dt}{d\tau}\left(1-\frac{r_0}{r}\right). \quad -(5)$$

Therefore

$$\frac{dt}{dr} = \frac{dt}{d\tau}\frac{d\tau}{dr} = \frac{dt}{d\tau}\frac{d\tau}{d\phi}\frac{d\phi}{dr} \quad -(6)$$

$$\boxed{\frac{dt}{dr} = \frac{1}{cb}\left(1-\frac{r_0}{r}\right)^{-1}\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2}} \quad -(7)$$

From eq. (1), the light deflection is:

$$\Delta\phi = 2\int_{R_0}^{\infty}\frac{1}{r^2}\left(\frac{1}{b^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)\right)^{-1/2} dr \quad -(8)$$

2)

and from eq. (7) the time delay is:

$$\Delta t = \frac{1}{cb} \int \left(1 - \frac{r_0}{r}\right)^{-1} \left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr \quad -(9)$$

The two integrals (8) and (9) are the same except that one uses $1/r^2$ and the other $(1 - r/r_0)^{-1}$.

As in UFT 150 and note 150(8), the assumption used by Einstein in eq. (8) was:

$$\frac{1}{a^2} =? \; 0 \quad -(10)$$

Einstein also assumed a circular orbit so:

$$\frac{1}{b^2} =? \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right) \quad -(11)$$

in which case the denominator in eq. (8) is zero. In UFT 150 Einstein's own integral was evaluated by computer, giving a result that was different by a factor of a million.

Eq. (9) can be written as:

$$\boxed{\Delta t = \frac{1}{c} \int \left(1 - \frac{r_0}{r}\right)^{-1} \left(1 - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right) b^2\right)^{-1/2} dr} \quad -(12)$$

This is a very simple equation to evaluate by

) computer.

The Shapiro time delay is based directly on eq. (12), but again assumes zero photon mass:

$$\frac{1}{a^2} \stackrel{?}{=} 0 \qquad -(13)$$

so Shapiro evaluated:

$$\Delta t \stackrel{?}{=} \frac{1}{c}\int\left(1-\frac{r_0}{r}\right)^{-1}\left(1-\left(1-\frac{r_0}{r}\right)\frac{b^2}{r^2}\right)^{-1/2} dr \qquad -(14)$$

This is given in eq. (6.3.44) of R. M. Wald, "General Relativity" (Chicago, 1984). The notation used by Wald is:

$$\frac{r_0}{r} := \frac{2M}{r} \qquad -(15)$$

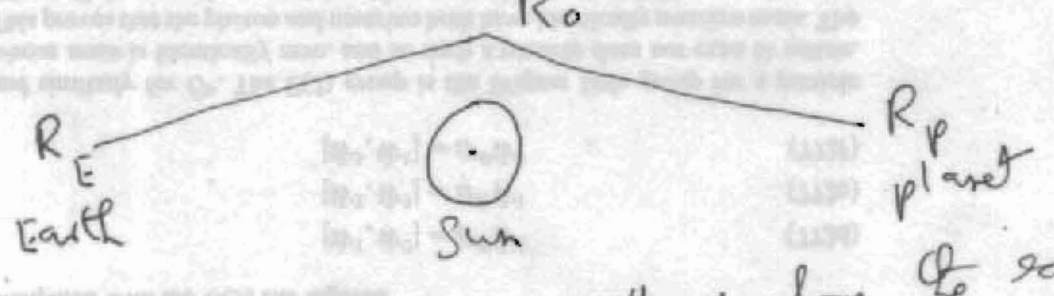

Fig (1)

A radar signal is emitted from the earth, located at the radial coordinate R_E. It grazes the sun at closest approach R_0, and is reflected off a planet at R_p.

The correct integral is therefore:

$$\Delta t = \frac{2}{c}\int_{R_E}^{R_0}\left(1-\frac{r_0}{r}\right)^{-1}\left(1-\left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)b^2\right)^{-1/2}dr$$

$$+\frac{2}{c}\int_{R_0}^{R_p}\left(1-\frac{r_0}{r}\right)^{-1}\left(1-\left(1-\frac{r_0}{r}\right)\left(\frac{1}{a^2}+\frac{1}{r^2}\right)b^2\right)^{-1/2}dr \quad -(16)$$

The factor 2 is due to the fact that the signal travels from R_E to R_p and then back from R_p to R_E.

Eq. (16) is easily evaluated by computer.

The result claimed by Wald is:

$$\Delta t = \frac{2}{c}\left(\left(R_E^2-R_0^2\right)^{1/2}+\left(R_p^2-R_0^2\right)^{1/2}\right)$$

$$+\frac{2GM}{c^3}\left(2\log_e\left(\frac{R_E+\left(R_E^2-R_0^2\right)^{1/2}}{R_0}\right)\right.$$

$$+2\log_e\left(\frac{R_p+\left(R_p^2-R_0^2\right)^{1/2}}{R_0}\right)$$

$$\left.+\left(\frac{R_E-R_0}{R_E+R_0}\right)^{1/2}+\left(\frac{R_p-R_0}{R_p+R_0}\right)^{1/2}\right) \quad -(17)$$

from

$$\frac{dt}{dr}=\left(1-\frac{2m}{r}\right)^{-1}\left(1-\left(1-\frac{2m}{r}\right)\frac{b^2}{r^2}\right)^{-1/2} \quad -(18)$$

5)

It is claimed by Wald that the result is obtained by integrating eq. (18) and then "differentiating with respect to M (holding R_0 fixed)" * ... in close analogy to the derivation of the "light bending effect".

It is claimed that null geodesics with the same R_0 are obtained by varying M (the mass of the sun) that this method

It is known from UFT 150 Einstein's own is wildly wrong. In UFT 150 integral was evaluated, i.e.:

$$\Delta \phi = 2 \int_0^{1/R_0} \left(R_0^{-2} - 2MR_0^{-3} - u^2 + 2Mu^3 \right)^{-1/2} du \quad -(19)$$

The method used by Einstein is given on Wald, page 145, and is:

$$\left. \frac{\partial(\Delta\phi)}{\partial M} \right|_{M=0} = 2 \int_0^{1/R_0} \left. \frac{(R_0^{-3} - u^3)\, du}{(R_0^{-2} - 2MR_0^{-3} - u^2 + 2Mu^3)^{3/2}} \right|_{M=0}$$

$$= 2 \int_0^{1/b} \frac{(b^{-3} - u^3)\, du}{(b^{-2} - u^2)^{3/2}}$$

$$= \frac{4}{b} \quad -(20)$$

The deflection of light to first order in M

6)

is:

$$\delta\phi = \Delta\phi - \pi \sim M \frac{\partial(\Delta\phi)}{\partial M}\Big|_{M=0}$$

$$= \frac{4M}{b} \quad -(21)$$

$$\delta\phi = \frac{4GM}{bc^2} \quad -(22)$$

Finally it is assumed that:

$$b = R_0 \quad -(23)$$

The computer evaluation of eq. (19) gives a totally different result, by six orders of magnitude. The mass of the sun, M, is constant, it does not vary, so the method used in eq. (20) is meaningless. The same method is used to evaluate eq. (18), with the same assumption of zero photon mass:

$$\frac{1}{a^2} \; ? \; 0 \quad -(24)$$

Therefore the result (17) is totally incorrect. The correct result is obtained from evaluating eq. (16) by computer, with finite photon mass.