155(11): Binomial Expansion Method for Light Deflection

The angle of deflection is defined as:

$$\Delta\phi = 2\int_0^{1/R_0}\left(\frac{1}{(b^{-2}-u^2)^{1/2}} - \frac{1}{(b^{-2}-u^2+2mu^3)^{1/2}}\right)du \quad -(1)$$

$$= 1.75 \text{ arcsec} \quad -(2)$$

Here $2mu^3 << (b^{-2}-u^2)$ — (2)

Consider the binomial expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \quad -(3)$$

$$+ {}^nC_r x^r + \dots \quad -(4)$$

where $|x| < 1$.

Then: $$(1+x)^{-1/2} = 1 - \frac{x}{2} + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots \quad -(5)$$

and integral (1) is:

$$\Delta\phi = 2\int_0^{1/R_0}\frac{1}{(b^{-2}-u^2)^{1/2}}\left(\frac{a}{2} - \frac{3}{8}a^2 + \frac{5}{16}a^3 - \dots\right)du \quad -(6)$$

where $$a = \left(\frac{2mu^3}{b^{-2}-u^2}\right). \quad -(7)$$

To first order in a:

2)

$$\Delta\phi \sim 2\int_0^{1/R_0} \frac{\cancel{2}Mu^3\,du}{\left(b^{-2}-u^2\right)^{3/2}} \quad -(8)$$

Note that the result claimed by Wald in eq. (6.3.42) is different. Also, Wald uses a non-S.I. notation which causes very confusing. The ECE theory is written entirely in S.I. units.

In the correct S.I. units:

$$\boxed{M \rightarrow \frac{r_0}{2} = \frac{MG}{c^2}} \quad -(9)$$

so

$$\boxed{\Delta\phi \sim \frac{\cancel{4}^{\,2}MG}{c^2}\int_0^{1/R_0} \frac{u^3\,du}{\left(b^{-2}-u^2\right)^{3/2}}} \quad -(10)$$