Evaluation of Light Deflection

$$\boxed{\Delta\phi = 2\int_0^{1/R_0}\left(\frac{1}{b^2}-(1-r_0u)\left(\frac{1}{a^2}+u^2\right)\right)^{-1/2}du - \pi} \quad -(1)$$

The experimental value is:

$$\Delta\phi = 1.75 \text{ arc seconds} = 8.484\times10^{-6} \text{ radians} \quad -(2)$$

We have

$$\pi = 1.0 \text{ radian} \quad -(3)$$

exactly.

By definition:

$$a = \frac{L}{mc} \neq 0 \; ; \; b = \frac{cL}{E} \quad -(4)$$

because

$$\boxed{m \neq 0} \quad -(5)$$

where m is the photon mass.

For the photon assume Planck's Law:

$$E = \hbar\omega \quad -(6)$$

so

$$a = \left(\frac{\hbar\omega}{mc^2}\right)b \quad -(7)$$

If we assume that:

2)

$$\omega = \omega_0 \quad -(8)$$

where ω_0 is the rest frequency of the photon of mass m, then de Broglie's equation is

$$\hbar\omega_0 = mc^2 \quad -(9)$$

and

$$a = b. \quad -(10)$$

In this case:

$$\Delta\phi = 2\int_0^{1/R_0} \left(\frac{r_0 u}{b^2} - (1 - r_0 u) u^2 \right)^{-1/2} du. \quad -(11)$$

However, the photon grazing the sun is not at rest in the observer frame, it is travelling close to c. Its angular momentum is a constant of motion and is:

$$L = mr^2 \frac{d\phi}{d\tau}. \quad -(12)$$

Its energy is: $$E = mc^2 \left(1 - \frac{r_0}{r}\right) \frac{dt}{d\tau} \quad -(13)$$

and momentum is $$p = m\frac{dr}{d\tau}, \quad -(14)$$

so $$b = \frac{cL}{E} = \frac{r^2}{c}\left(1 - \frac{r_0}{r}\right)^{-1} \frac{d\phi}{dt} \quad -(15)$$

$$b = \frac{r^2}{c}\left(1 - \frac{r_0}{r}\right)^{-1} \frac{d\phi}{dt}$$

$$\boxed{b = \frac{r^2}{c}\left(1 - \frac{r_0}{r}\right)^{-1} \Omega} \quad -(16)$$

3) where

$$\Omega = \frac{d\phi}{dt} \quad -(17)$$

is the orbital angular velocity of the photon.

From eqs. (4):

$$a = \left(\frac{E}{mc^2}\right) b \quad -(18)$$

where

$$E = \hbar \omega \quad -(19)$$

Here ω is the spin angular velocity of the photon. So

$$a = \left(\frac{\hbar \omega}{mc^2}\right) b \quad -(20)$$

$$b = \frac{r^2}{c}\left(1 - \frac{r_0}{r}\right)^{-1} \Omega \quad -(21)$$

If $\Delta\phi$ is the experimentally measured deflection, and Δt the experimentally measured Shapiro delay then:

$$\Omega = \frac{\Delta\phi}{\Delta t} \quad -(22)$$

so the photon mass can be found experimentally.