155(6): The Connection in Minkowski Spacetime

The Minkowski spacetime is defined by the metric:

$$g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(1)$$

If it is assumed that the connection is symmetric, then

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right). \quad -(2)$$

From eqs. (1) and (2):

$$\boxed{\Gamma^{\sigma}_{\mu\nu} = 0, \quad \mu = \nu} \quad -(3)$$

in Minkowski spacetime, so $R^{\rho}{}_{\sigma\mu\nu}$ and $T^{\lambda}_{\mu\nu}$ are zero.

In general:

$$[D_\mu, D_\nu] V^\rho = R^{\rho}{}_{\sigma\mu\nu} V^\sigma - T^{\lambda}_{\mu\nu} D_\lambda V^\rho \quad -(4)$$

and if

$$\mu = \nu \quad -(5)$$

then:

$$\boxed{R^{\rho}{}_{\sigma\mu\mu} V^\sigma = T^{\lambda}_{\mu\mu} D_\lambda V^\rho}. \quad -(6)$$

To see that:

$$R^{\rho}{}_{\sigma\mu\mu} = T^{\lambda}_{\mu\mu} = 0 \quad -(7)$$

additional information is required. The mathematics

2)

go only as far as eq. (6). This information is the antisymmetry laws that all quantities in eq. (4) change sign under:

$$\mu \to \nu, \quad \nu \to \mu. \quad -(8)$$

In the Minkowski spacetime, the torsion and curvature vanish, so the connection must be symmetric. From eq. (3) the symmetric connection is zero. In eq. (6):

$$R^\rho{}_{\sigma\mu\mu} V^\sigma = \left(\Gamma^\lambda_{\mu\mu} - \Gamma^\lambda_{\mu\mu}\right) D_\lambda V^\rho \quad -(9)$$

$$= 0$$

and the space is one where there is no torsion and no curvature. This space is defined by:

$$\boxed{\Gamma^\lambda_{\mu\mu} = 0.} \quad -(10)$$

The Error in the Standard Model

This is to assume:

$$T^\lambda_{\mu\mu} = 0, \quad \Gamma^\lambda_{\mu\mu} \neq ?\, 0 \quad -(11)$$

so there is no curvature and no torsion, but there is still a non-zero connection. This contradicts the fact that the space of no curvature and no torsion is defined by eq. (1), in which the symmetric connection must be zero. Therefore the symmetric connection is always zero.