21) 155(15): Time Delay Calculation for Finite Deflection with $m \neq 0$, $M \neq 0$.

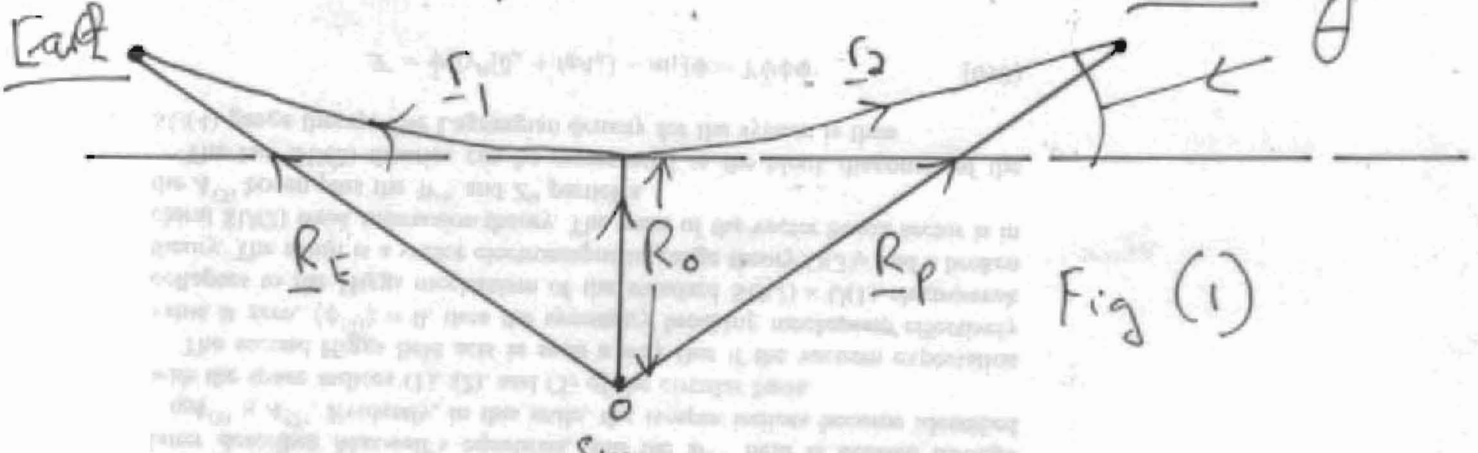

Fig (1)

The angle of deflection is θ (not to be confused with ϕ of the cylindrical polar system). In fig (1) the angle θ has been greatly exaggerated. It is only 1.75 arc seconds (8.484×10^{-6} radians), and much too small to be seen by the eye in a drawing.

From the base line calculation:

$$\boxed{b = R_0} \quad — (1)$$

is a constant of motion. Therefore the time taken to go from the earth to the planet and back is:

$$t = \frac{2}{c}\left(\int_0^{R_E} f(r)\,dr + \int_0^{R_P} f(r)\,dr\right) \quad — (2)$$

where:

$$f(r) = \left(1 - \frac{r_0}{r}\right)^{-1}\left(1 - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)R_0^2\right)^{-1/2} dr \quad — (3)$$

2) where

$$r_0 = \frac{2MG}{c^2} \quad -(4)$$

where M is the mass of the sun:

$$M = 1.989 \times 10^{30} \, kg. \quad -(5)$$

the Newton constant is:

$$G = 6.6726 \times 10^{-11} \, m^3 \, kg^{-1} \, s^{-2}, \quad -(6)$$

and to its first six decimal places:

$$c = 2.997925 \times 10^{8} \, ms^{-1} \quad -(7)$$

the constant of motion a is:

$$a = \frac{L}{mc} \quad -(8)$$

where L is the conserved orbital angular momentum, m is the mass of the photon. The angular momentum is:

$$L = \frac{R_0 E}{c} \quad -(9)$$

where E is the conserved total energy of the photon. From Planck's theory:

$$E = \hbar \omega \quad -(10)$$

for one photon. So:

$$\boxed{a = \left(\frac{\hbar \omega}{mc^2}\right) R_0} \quad -(11)$$

the measured time delay is the difference

3) between t of eq. (2) and the base line time for no deflection given in note 155(14).

For one visible frequency photon:

$$\omega \sim 10^{16} \text{ radians per second} \quad -(12)$$

The only unknown in eq. (2) is m, the mass of the photon. Clearly, the beam is made up of many photons, so a more accurate theory would need the well known Planck distribution.

Baseline Calculation with Finite m.

In this case t_0 is given by eq. (2) but with

$$r_0 = 0, \quad -(13)$$

$$m = 0 \quad -(14)$$

i.e.

So:

$$f(r) = \left(1 - \left(\frac{1}{a^2} + \frac{1}{r^2}\right)R_0^2\right)^{-1/2}$$

$$= \left(1 - \left(\frac{R_0}{a}\right)^2 - \frac{R_0^2}{r^2}\right)^{-1/2} \quad -(15)$$

$$= \left(1 - \left(\frac{R_0}{a}\right)^2\right)^{-1/2}\left(1 - \frac{d^2}{r^2}\right)^{-1/2} \quad -(16)$$

where

$$d^2 = R_0^2\left(1 - \left(\frac{R_0}{a}\right)^2\right)^{-1}$$

Here,

$$\int\left(1 - \frac{d^2}{r^2}\right)^{-1/2} dr = \left(r^2 - d^2\right)^{1/2} \quad -(17)$$