155(9): Change of Angle with Time

The change of angle with time is given by

$$\boxed{\frac{d\phi}{dt} = \frac{cb}{r^2}\left(1 - \frac{r_0}{r}\right)} \quad -(1)$$

from:

$$\frac{d\phi}{dt} = \frac{d\phi}{dr}\frac{dr}{dt} \quad -(2)$$

where:

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} \quad -(3)$$

$$\frac{dr}{dt} = cb\left(1 - \frac{r_0}{r}\right)\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{1/2} \quad -(4)$$

Therefore light deflection is given by:

$$\Delta\phi = cb\int \frac{1}{r^2}\left(1 - \frac{r_0}{r}\right) dt \quad -(5)$$

i.e.

$$\boxed{\Delta\phi = \frac{cb}{r^2}\left(1 - \frac{r_0}{r}\right)\Delta t} \quad -(6)$$

Eq (1) defines the relativistic angular velocity.

2)

$$\omega = \frac{d\phi}{dt} = \frac{cb}{r^2}\left(1 - \frac{r_0}{r}\right) \quad -(7)$$

At distance of closest approach:

$$r = R_0 \quad -(8)$$

then

$$\omega = \frac{cb}{R_0^2}\left(1 - \frac{r_0}{R_0}\right) \quad -(9)$$

Eq (6) is a relation between the angle of deflection $\Delta\phi$ and the time delay Δt

At distance of closest approach:

$$\Delta\phi = \frac{cb}{R_0^2}\left(1 - \frac{r_0}{R_0}\right)\Delta t \quad -(10)$$

This gives an experimental method of determining:

$$b = \frac{cL}{E} = \text{constant} \quad -(11)$$