154(5): Effect of Gravitation on the Electromagnetic Four Potential.

For the sake of argument consider the gravitational metric

$$ds^2 = c^2 dt^2 \left(1 - \frac{r_0}{r}\right) - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\phi^2 - dZ^2 \quad -(1)$$

where

$$r_0 = \frac{2MG}{c^2} \quad -(2)$$

The metric in Cartan geometry is:

$$g_{\mu\nu} = \begin{bmatrix} (1 - r_0/r) & 0 & 0 & 0 \\ 0 & -(1 - r_0/r)^{-1} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(3)$$

The electromagnetic four potential is:

$$A^\mu = (\phi, c\underline{A}) \quad -(4)$$

and without gravitation is written in the Minkowski metric:

$$\eta_{(a)(b)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(5)$$

in the standard model.

In ECE theory, the metric (5) is replaced by the tetrad:

$$\underline{V}^{(1)} = \underline{e}^{(1)} e^{i\Phi} \quad -(6)$$

2) where $\underline{e}^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j})$ — (7)

and $\Phi = \omega t - \kappa Z$. — (8)

The electromagnetic plane wave is, in ECE theory:

$$\underline{A}^{(1)} = A \underline{q}^{(1)} \quad -(9)$$

where $A$ is a scalar valued magnitude. The metric of electromagnetism in ECE theory is:

$$g_{\mu\nu} = q^{(a)}_{\mu} q^{(b)*}_{\nu} \eta_{(a)(b)} \quad -(10)$$

For a tetrad of type (6) eq. (10) produces:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(11)$$

but this is not the Minkowski metric. The metric (11) has been built up from the more fundamental tetrad (6) using:

$$\underline{q}^{(2)} = \underline{q}^{(1)*} = \underline{e}^{(2)} e^{-i\Phi} \quad -(12)$$

The phase $\Phi$ appears in the tetrad but not in the metric.

The effect of gravitation of type (1)

) on the electromagnetic plane wave is to change the metric (11) to the metric (1).

Eq. (1) is written in cylindrical polar coordinates, in which the vector potential is:

$$\underline{A}^{(1)} = \left( A_r^{(1)} \underline{e}_r + A_\phi^{(1)} \underline{e}_\phi \right) e^{i\Phi} \quad (13)$$

In Cartesian coordinates:

$$\underline{A}^{(1)} = \left( A_x^{(1)} \underline{i} + A_y^{(1)} \underline{j} \right) e^{i\Phi} \quad (14)$$

and in complex circular coordinates

$$\underline{A}^{(1)} = A \underline{e}^{(1)} e^{i\Phi} \quad -(15)$$

So:
$$A^2 = \underline{A}^{(1)} \cdot \underline{A}^{(2)} = A_x^{(1)} A_x^{(2)} + A_y^{(1)} A_y^{(2)} \quad -(16)$$
$$= A_r^{(1)} A_r^{(2)} + A_\phi^{(1)} A_\phi^{(2)}$$

Now use:

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad -(17)$$

and
$$A_x^{(1)} = A/\sqrt{2}, \quad A_y^{(1)} = -iA/\sqrt{2}, \quad -(18)$$
$$A_x^{(2)} = A/\sqrt{2}, \quad A_y^{(2)} = iA/\sqrt{2},$$

to find that:

4)

$$A_r^{(1)} = \frac{A}{\sqrt{2}} \left( \cos\phi - i \sin\phi \right) \quad -(19)$$

$$A_\phi^{(1)} = \frac{A}{\sqrt{2}} \left( - \sin\phi - i \cos\phi \right) \quad -(20)$$

In Cartan geometry, the effect of gravitation is to change

$$g_{rr} = e_r e_r \rightarrow \left(1 - \frac{r_0}{r}\right)^{-1} g_{rr} \quad -(21)$$

so:

$$A_r^{(1)} A_r^{(2)} = \frac{A^2}{2} \rightarrow \frac{A^2}{2} \left(1 - \frac{r_0}{r}\right)^{-1} \quad -(22)$$

so from eq (16):

$$A^2 \rightarrow \frac{A^2}{2} \left( \left(1 - \frac{r_0}{r}\right)^{-1} + 1 \right) \quad -(23)$$

and

$$A_r^{(1)} \rightarrow A_r^{(1)} \left(1 - \frac{r_0}{r}\right)^{-1/2} \quad -(24)$$

Therefore:

$$\boxed{\begin{aligned} \underline{A}^{(1)} &= \frac{A}{2} \left( 1 + \left(1 - \frac{r_0}{r}\right)^{-1} \right)^{1/2} \left( \underline{i} - i \underline{j} \right) e^{i\Phi} \\ &= \left( A_r^{(1)} \left(1 - \frac{r_0}{r}\right)^{-1/2} \underline{e}_r + A_\phi^{(1)} \underline{e}_\phi \right) e^{i\Phi} \end{aligned}}$$

$-(25)$

5)

Note that there is a change of polarization due to gravitation when the cylindrical polar unit vectors are used. When the Cartesian unit vectors are used there is a change of magnitude of the plane wave.

The electric and magnetic fields are found from ECE theory as usual, using the anti-symmetry laws and spin connection. The reflection of light due to gravitation is found as in UFT 150.

The change in scalar potential is:

$$\boxed{\phi^{(1)} \rightarrow \left(1 - \frac{r_0}{r}\right)^{1/2} \phi^{(1)}} \quad -(26)$$

The link between (3) and (5) is:

$$g_{\mu\nu} = q_\mu^{(a)} q_\nu^{(b)} \eta_{(a)(b)} \quad -(27)$$

The reverse effect of electromagnetism on gravitation is found by

$$\eta_{(a)(b)} = q^\mu_{(a)} q^\nu_{(b)} g_{\mu\nu} \quad -(28)$$

using the ECE theory of gravitation based on tetrads.

6)

The combined tetrad elements in the two coordinate systems of eq. (25) are:

$$A^{(1)}_{x} = \frac{A}{2}\left(1+\left(1-\frac{r_0}{r}\right)^{-1}\right)^{1/2} e^{i\Phi} \quad -(29)$$

$$A^{(1)}_{y} = -i A^{(1)}_{x} \quad -(30)$$

$$A^{(1)\prime}_{r} = A^{(1)}_{r}\left(1-\frac{r_0}{r}\right)^{-1/2} e^{i\Phi} \quad -(31)$$

$$A^{(1)\prime}_{\phi} = A^{(1)}_{\phi} e^{i\Phi} \quad -(32)$$

Solving eq. (31):

$$\boxed{\left(1-\frac{r_0}{r}\right)^{-1/2} = \frac{A^{(1)\prime}_{r}}{A^{(1)}_{r}} e^{-i\Phi}} \quad -(33)$$

Solving eq. (29):

$$\boxed{\left(1-\frac{r_0}{r}\right)^{-1} = \left(\frac{2A^{(1)}_{x}}{A} e^{-i\Phi}\right)^{2} - 1} \quad -(34)$$

In principle, changing the electromagnetic parameters will change the gravitational attraction.