1)

151(2): Experimenting with New Metrics

It is simplest to begin with the Minkowski metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\underline{r} \cdot d\underline{r}, \quad -(1)$$

which is the simplest solution of the Orbital Theorem of UFT III. The orbit for this metric is:

$$\phi(\text{Minkowski}) = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2}\right)^{-1/2} dr \quad -(2)$$

where

$$a = \frac{L}{mc}, \; b = \frac{cL}{E}, \quad -(3)$$

so

$$a = \left(\frac{mc^2}{E}\right)^{-1} b = \left(\frac{E}{mc^2}\right) b \quad -(4)$$

The Minkowski metric is:

$$d\underline{r} \cdot d\underline{r} = c^2 (dt^2 - d\tau^2)$$

$$= v^2 dt^2 \quad -(5)$$

i.e.

$$v^2 dt^2 = dr^2 + r^2 d\phi^2 \quad -(6)$$

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\phi}{dt}\right)^2 \quad -(7)$$

The constants of motion are

$$E = mc^2 \frac{dt}{d\tau} = \gamma mc^2, \quad -(8)$$

$$L = mr^2 \frac{d\phi}{d\tau} = \gamma m r^2 \omega, \quad -(9)$$

$$\omega = d\phi / dt. \quad -(10)$$

Therefore:

$$a = \frac{\gamma r^2 \omega}{c} = \gamma b \quad -(11)$$

$$b = \frac{cL}{E} = r^2 \frac{\omega}{c} \quad -(12)$$

Therefore:

$$\frac{1}{b^2} - \frac{1}{a^2} = \frac{1}{b^2}\left(1 - \frac{1}{\gamma^2}\right) \quad -(13)$$

$$= \left(\frac{v}{c}\right)^2 \frac{1}{b^2} = \frac{v^2}{r^4\omega^2}$$

$$= \text{constant} \quad -(14)$$

So

$$\phi(\text{Minkowski}) = \int \frac{1}{r^2}\left(\frac{v^2}{r^4\omega^2} - \frac{1}{r^2}\right)^{-1/2} dr$$

$$\boxed{\phi = \int \left(\left(\frac{v}{\omega r}\right)^2 - 1\right)^{-1/2} dr} \quad -(15)$$

where $b = \frac{r^2\omega}{v}$ = constant of motion $-(16)$

Therefore the Minkowski metric is sufficient to give an a.S.i.t. Eq. (15) is equivalent to:

$$\frac{r_0}{r} << 1 \quad -(17)$$

This condition is true for light deflection by the sun, so:

$$\Delta\phi = 2\int_{R_0}^{\infty} \left(\left(\frac{v}{\omega r}\right)^2 - 1\right)^{-1/2} dr \quad -(18)$$

where v is the velocity of the photon.

3) If the velocity of the photon is considered to be close to c, then:

$$\Delta \phi = 2 \int_{R_0}^{\infty} \left(\left(\frac{c}{\omega r} \right)^2 - 1 \right)^{-1/2} dr \quad - (19)$$

and the orbital angular velocity of the photon can be found from the experimental $\Delta \phi$.

In a circular orbit:

$$v = \omega r \quad - (20)$$

so the integral in eq. (18) becomes singular.