SELF CONSISTENCY OF THE ECE2 COVARIANT ORBITAL EQUATIONS.

by

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ABSTRACT

The relativistic orbital equations of ECE2 theory are derived and checked for self consistency using three methods: the kinematic, Lagrangian and Hamiltonian. Having derived the equations in the observer frame the de Sitter rotation method is applied to find the spin connection and vacuum force. The orbital equations with and without de Sitter rotation are solved numerically. The infinitesimal line element of the ECE2 theory is used to derive an orbit equation, and this is shown to be the trajectory of a free particle. More generally, the infinitesimal line element can be developed in the most general spherically symmetric spacetime, the m theory of previous UFT papers.

Keywords: ECE2 covariant orbital equations, frame rotation and m theories

UFT 414

1. INTRODUCTION

In immediately preceding UFT papers (www.aias.us) {1 - 41}frame rotation has been shown to result in several advances in understanding, for example frame rotation produces the spin connection, the vacuum force, precessing and retrograde orbits. The spin connection can be expressed in terms of isotropically averaged vacuum fluctuations of the type used in the well known Lamb shift theory. These advances go well beyond the standard model's Einsteinian general relativity (EGR), which cannot produce retrograde precession and which has recently been refuted experimentally by an order of magnitude in S star systems. ECE2 is able to describe S star systems to any degree of accuracy by using the relevant angular velocity of frame rotation. In Section 2 of this paper the ECE2 covariant orbital equations are derived in three ways, giving the same result. This is a triple cross check of the theory using kinematic, Lagrangian and Hamiltonian methods, both in the observer frame and rotated frame. The resulting equations of motion are solved numerically to give the relativistic orbit. In UFT413, the orbit was derived in a well defined classical limit. The ECE2 covariant infinitesimal line element corresponding to the orbital equations is used to derive the relativistic equation of motion of a free particle. This method is checked using the relativistic hamiltonian of a free particle, giving the same result and a double cross check. Section 3 is a discussion of the numerical results accompanied by graphics.

This paper is a brief synopsis of extensive calculations posted in the background notes accompanying UFT414 on www.aias.us. Note 414(1) is a summary of orbital equations derived in UFT413 on the classical level. Eq. 414(2) describes the hamiltonian method on the classical level. Eq. 414(3) develops the relativistic hamiltonian method. Note 414(4) is the basis for Section 2 and uses the fundamental kinematic and Lagrangian methods to give the same relativistic orbital equations, providing a cross check on all concepts and calculations. Note 414(5) uses the hamiltonian method to give a triple cross check of the derivation of the relativistic orbital equations, with and without frame rotation. Note 414(6) summarizes the relativistic orbital equations. Note 414(7) calculates the spin connection due to the transition from the classical to the relativistic theory. Note 414(8) calculates the relativistic spin connection due to frame rotation, and Note 414(9) calculates the relativistic trajectory of a free particle in two ways: using the ECE2 covariant infinitesimal line element and using the relativistic hamiltonian of a free particle. Both methods give precisely the same result, giving another double cross check.

2. SELF CONSISTENT DERIVATIONS THE RELATIVISTIC ORBITAL EQUATIONS.

Consider the relativistic velocity in any coordinate system {1 - 41}:

$$\underline{v} = \gamma \dot{\underline{r}} \qquad -(1)$$

where γ is the Lorentz factor and where $\underline{r}$ is the position vector. The relativistic acceleration is

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\gamma \dot{\underline{r}}\right) = \frac{d\gamma}{dt}\dot{\underline{r}} + \gamma \ddot{\underline{r}} \qquad -(2)$$

In the plane polar system (r , ϕ):

$$\dot{\underline{r}} = \dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi \qquad -(3)$$

and

$$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\phi}^2\right)\underline{e}_r + \left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right)\underline{e}_\phi \qquad -(4)$$

in which $\underline{e}_r$ and $\underline{e}_\phi$ are the unit vectors of the plane polar system.

It follows that the relativistic orbital equation for a mass m orbiting a mass M is:

$$\underline{g} = \frac{d\gamma}{dt}\left(\dot{r}\,\underline{e}_r + r\dot{\phi}\,\underline{e}_\phi\right) + \gamma\left(\left(\ddot{r} - r\dot{\phi}^2\right)\underline{e}_r + \left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right)\underline{e}_\phi\right)$$

$$= -\frac{MG}{r^2}\,\underline{e}_r \quad -(5)$$

where G is Newton's constant. The relativistic force equation is:

$$\underline{F} = m\underline{g} = -\frac{mMG}{r^2}\,\underline{e}_r. \quad -(6)$$

Eq. (5) gives two simultaneous equations:

$$\frac{d\gamma}{dt}\dot{r} + \gamma\left(\ddot{r} - r\dot{\phi}^2\right) = -\frac{MG}{r^2} \quad -(7)$$

and

$$\frac{d\gamma}{dt} r\dot{\phi} + \gamma\left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right) = 0 \quad -(8)$$

which can be solved numerically using the methods developed in previous UFT papers. The Lorentz factor in these equations is:

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(9)$$

in which the Newtonian velocity is:

$$v_N^2 = \dot{r}^2 + r^2\dot{\phi}^2. \quad -(10)$$

Eq. (7) is the relativistic Leibniz equation and Eq. (8) is the conservation of relativistic angular momentum L:

$$\frac{dL}{dt} = 0 \quad -(11)$$

where

$$L = \gamma m r^2 \dot{\phi} \quad -(12)$$

is a constant of motion. The other constant of motion is the relativistic hamiltonian H.

Using the frame rotation of immediately preceding UFT papers:

$$\phi' = \phi + \omega_1 t \quad -(13)$$

Eqs. (7) and (8) become:

$$\frac{d\gamma}{dt}\dot{r} + \gamma\left(\ddot{r} - r\dot{\phi}'^2\right) = -\frac{MG}{r^2} \quad -(14)$$

and

$$\frac{d\gamma}{dt} r\dot{\phi}' + \gamma\left(r\ddot{\phi}' + 2\dot{r}\dot{\phi}'\right) = 0 \quad -(15)$$

and the Lorentz factor becomes:

$$\gamma = \left(1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\phi}'^2\right)\right)^{-1/2} \quad -(16)$$

These relativisitc orbits go well beyond the standard model's EGR.

The orbital equations can also be derived using the ECE2 covariant Lagrangian:

$$\mathcal{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r} \quad -(17)$$

where the Lorentz factor is given by Eq. (9). Use the Lagrange variables r and ϕ to find the two relevant Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(18)$$

and

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \quad -(19)$$

As shown in Note 414(4) Eq. (18) produces

$$\frac{d\gamma}{dt}\dot{r} + \gamma\left(\ddot{r} - r\dot{\phi}^2\right) = -\frac{mG}{r^2} \quad -(20)$$

which is Eq. (7) Q. E. D.

The Lagrangian and kinematic methods give the same results, giving a double cross check on concepts, Q. E. D.

Eq. (19) gives

$$\frac{dL}{dt} = 0 \quad -(21)$$

where

$$L = \gamma m r^2 \dot{\phi} \quad -(22)$$

i.e. Eq. (19) gives Eq. (8), Q. E. D. This is another double cross check. It can be shown straightforwardly as in Note 414(4) that:

$$\frac{d}{dt}\left(\gamma m r^2 \dot{\phi}\right) = r^2\dot{\phi}\frac{d\gamma}{dt} + \gamma\left(2r\dot{r}\dot{\phi} + r^2\ddot{\phi}\right) = 0 \quad -(23)$$

which is Eq. (8), Q. E. D.

Eqs. (14) and (15) are obtained with the Lagrange variables r and ϕ :

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(24)$$

and

$$\frac{\partial \mathcal{L}}{\partial \phi'} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}'} \quad -(25)$$

giving another cross check, Q. E. D.

In Eq. (7)

$$\dot{r} = v \quad -(26)$$

and

$$\frac{dv}{dt} = \ddot{r} - r\dot{\phi}^2 \quad -(27)$$

so Eq. (7) is:

$$F = \frac{d}{dt}(\gamma m v) = m\left(v\frac{d\gamma}{dt} + \gamma\frac{dv}{dt}\right) = -\frac{mMG}{r^2}. \quad -(28)$$

Write Eq. (28) as:

$$F = m\frac{dv}{dt}\left(v\frac{d\gamma}{dv} + \gamma\right) \quad -(29)$$

where

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv}\frac{dv}{dt} \quad -(30)$$

has been used. In Eq. (29):

$$\frac{d\gamma}{dv} = \gamma^3\frac{v}{c^2} \quad -(31)$$

so the magnitude F of the orbital force is:

$$F = m\gamma\frac{dv}{dt}\left(1 + \gamma^2\frac{v^2}{c^2}\right)$$

$$= m\gamma \frac{dv}{dt}\left(1 + \frac{v^2}{c^2\left(1-\frac{v^2}{c^2}\right)}\right)$$

$$= m\gamma^3 \frac{dv}{dt} \quad -(32)$$

which is the relativistic second law of Newton. Therefore the relativistic Leibniz equation is:

$$F = m\gamma^3 \frac{dv}{dt} = -\frac{mMG}{r^2}. \quad -(33)$$

It has been shown that:

$$\frac{d\gamma}{dt}\dot{r} + \gamma\left(\ddot{r} - r\dot{\phi}^2\right) = \gamma^3 \frac{dv}{dt} \quad -(34)$$

where:

$$v = \dot{r} \quad -(35)$$

and

$$\frac{dv}{dt} = \ddot{r} - r\dot{\phi}^2. \quad -(36)$$

Therefore the relativistic orbit in frame (r , ϕ) is given by simultaneous solution of:

$$F = m\gamma^3 \frac{dv}{dt} = -\frac{mMG}{r^2} \quad -(37)$$

and

$$L = \gamma m r^2 \dot{\phi}, \quad \frac{dL}{dt} = 0. \quad -(38)$$

The results are discussed in Section 3.

A triple cross check of the orbital equations is possible using the relativistic

hamiltonian:

$$H = \gamma m c^2 - \frac{mMG}{r}. \quad -(39)$$

This is a constant of motion, so:

$$\frac{dH}{dt} = 0. \quad -(40)$$

It follows as in Note 414(5) that:

$$c^2 \frac{d\gamma}{dt} = -v \frac{MG}{r^2}. \quad -(41)$$

The relativistic force magnitude is:

$$F = \frac{d}{dt}(\gamma m v) = m v \frac{d\gamma}{dt} + m \gamma \frac{dv}{dt} \quad -(42)$$

so

$$v \frac{d\gamma}{dt} + \gamma \frac{dv}{dt} = -\frac{MG}{r^2}. \quad -(43)$$

Using Eq. (41):

$$-\frac{v^2}{c^2} \frac{MG}{r^2} + \gamma \frac{dv}{dt} = -\frac{MG}{r^2} \quad -(44)$$

and

$$\gamma \frac{dv}{dt} = -\frac{MG}{r^2}\left(1 - \frac{v^2}{c^2}\right) \quad -(45)$$

so

$$\gamma^3 \frac{dv}{dt} = -\frac{MG}{r^2} \quad -(46)$$

which is Eq. (37), Q. E. D.

With reference to Note 414(8) consider the effect of the frame rotation:

$$\phi' = \phi + \omega_1 t \quad -(47)$$

on the relativistic orbit equations (37) and (38). From Eq. (47):

$$\dot{\phi}' = \dot{\phi} + \omega_1 + t\frac{d\omega_1}{dt} \quad -(48)$$

and the orbit equations in frame (r , ϕ') are:

$$F = m\gamma'^3\left(\ddot{r} - r\dot{\phi}'^2\right) = -\frac{mMG}{r^2} \quad -(49)$$

and

$$\frac{dL}{dt} = 0, \quad L = \gamma' m r^2 \dot{\phi}' \quad -(50)$$

in which the cube of the rotated Lorentz factor is:

$$\gamma'^3 = \left(1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\phi}'^2\right)\right)^{-3/2} \quad -(51)$$

It follows as in Note 414(8) that the orbital field equations are

$$F = m\gamma^3\left(\ddot{r} - r\dot{\phi}^2\right) = -\frac{mMG}{r^2} + \Omega_r \Phi \quad -(52)$$

and

$$\frac{dL}{dt} = \frac{d}{dt}\left(\gamma m r^2\left(\dot{\phi} + \phi_1 + t\frac{d\phi_1}{dt}\right)\right) = 0 \quad -(53)$$

where the spin connection produced by the frame rotation (47) is:

$$\Omega_r = \frac{1}{r}\left(1+\gamma^2 A\right)^{3/2} - \frac{c^2}{MG}\gamma^3 A - \frac{1}{r}. \quad -(54)$$

Here:

$$A = \frac{r^2}{c^2}\left(\omega_1 + t\frac{d\omega_1}{dt}\right)\left(\omega_1 + t\frac{d\omega_1}{dt} + 2\dot{\phi}\right) \quad -(55)$$

and

$$\gamma = \left(1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\phi}^2\right)\right)^{-1/2}. \quad -(56)$$

The orbit produced by Eqs. (52) and (53) is discussed in Section 3. The vacuum force due to frame rotation is:

$$F(\text{vac}) = \Omega_r \Phi = -\frac{mMG}{r}\Omega_r \quad -(57)$$

and the correctly covariant total force is:

$$F = -\frac{mMG}{r^2} + F(\text{vac}) \quad -(58)$$

as in immediately preceding UFT papers.

The ECE2 covariant infinitesimal line element corresponding to the orbit equations (37) and (38) is

$$ds^2 = c^2 d\tau^2 = (c^2 - v_N^2)\, dt^2 \quad -(59)$$

where τ is the proper time and v_N is the Newtonian velocity. It follows as in papers such as UFT106 and UFT192 that:

$$mc^2 = mc^2\left(\frac{dt}{d\tau}\right)^2 - mv_N^2\left(\frac{dt}{d\tau}\right)^2. \quad -(60)$$

The Lorentz factor follows directly from Eq. (59):

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2}. \quad -(61)$$

Therefore the infinitesimal line element immediately gives the Einstein energy equation:

$$E^2 = c^2 p^2 + E_0^2 \quad -(62)$$

where

$$E = \gamma m c^2, \; \underline{p} = \gamma m \underline{v}, \; E_0 = m c^2 \quad -(63)$$

Q. E. D.

In plane polar coordinates the infinitesimal line element (59) is

$$m c^2 = \frac{E^2}{m c^2} - m\left(\frac{dr}{d\tau}\right)^2 - m r^2 \left(\frac{d\phi}{d\tau}\right)^2 \quad -(64)$$

from which it follows as in Note 414(9) that:

$$m\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{m c^2} - \frac{L^2}{m r^2} - m c^2 \quad -(65)$$

where E and L are respectively the relativistic total energy and relativistic angular momentum. Both are constants of motion. Using:

$$\frac{dr}{d\tau} = \frac{dr}{d\phi}\frac{d\phi}{d\tau} = \frac{L}{m r^2}\frac{dr}{d\phi} \quad -(66)$$

it follows that:

$$\left(\frac{dr}{d\phi}\right)^2 = r^4 \left(\frac{1}{b^2} - \frac{1}{a^2} - \frac{1}{r^2}\right) \quad -(67)$$

where a and b are constants of motion defined by:

$$a = \frac{L}{mc}, \; b = \frac{Lc}{E}. \quad -(68)$$

Therefore the ECE2 covariant infinitesimal line element (59) gives the orbit equation (67) without any consideration of potential energy. It follows that the orbit equation (67) must be given by the free particle relativistic hamiltonian:

$$H = E = \gamma mc^2, \quad -(69)$$
$$F = \gamma^3 \, dv/dt = 0. \quad -(70)$$

It follows from Eq. (69) that:

$$\gamma = \frac{E}{mc^2} = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(71)$$

so

$$\frac{v_N^2}{c^2} = 1 - \frac{m^2c^4}{E^2}. \quad -(72)$$

The Newtonian velocity is:

$$v_N^2 = \dot{r}^2 + r^2\dot{\phi}^2 \quad -(73)$$

Using:

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \dot{\phi}\frac{dr}{d\phi} \quad -(74)$$

the Newtonian velocity can be expressed as:

$$v_N^2 = \left(r^2 + \left(\frac{dr}{d\phi}\right)^2\right)\dot{\phi}^2 \quad -(75)$$

In Newtonian dynamics the constant angular momentum is:

$$L_0 = mr^2\dot{\phi} \quad -(76)$$

so

$$v_N^2 = \frac{L_0^2}{m^2}\left(\frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\phi}\right)^2\right) \quad -(77)$$

From Eqs. (71) and (77)

$$\left(\frac{dr}{d\phi}\right)^2 = r^4\left(\frac{1}{b^2} - \frac{m^2c^4}{E^2}\cdot\frac{m^2c^2}{L_0^2}\right) - r^2 \quad -(78)$$

In this equation:

$$\frac{m^2c^4}{E^2} = \frac{1}{\gamma^2}, \quad -(79)$$

$$L_0^2 = \frac{L^2}{\gamma^2} \quad -(80)$$

so:

$$\left(\frac{dr}{d\phi}\right)^2 = r^4\left(\frac{1}{b^2} - \frac{1}{a^2}\right) - r^2 \quad -(81)$$

which is Eq. (67), Q. E. D.

Therefore the infinitesimal line element (59) and the Einstein energy equation and orbit (81) are those of a relativistic free particle.

Eq. (81) can be integrated using:

$$\phi = \int \frac{1}{r^2}\left(A - \frac{1}{r^2}\right)^{-1/2} dr \quad -(82)$$

where:

$$A := \frac{1}{b^2} - \frac{1}{a^2} \quad -(83)$$

The Wolfram online integrator gives:

$$\phi = -\tan^{-1}\left((Ar^2 - 1)^{-1/2}\right) \quad -(84)$$

so it follows as in Note 414(8) that:

$$r^2 = \frac{L^2}{m^2c^2(\gamma^2 - 1)}\left(\frac{1}{\tan^2\phi} - 1\right). \quad -(85)$$

This the relativistic trajectory of a free particle and is graphed in Section 3. In the non relativistic limit:

$$v = r\dot{\phi}\left(\frac{1}{\tan^2\phi} - 1\right)^{1/2} \quad -(86)$$

as in Note 414(9).

In order to describe the relativistic orbit of m about M the infinitesimal line element is needed of the most general spherically symmetric spacetime:

$$ds^2 = c^2 d\tau^2 = m(r)\, c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad -(87)$$

where m is a function of r. In order to introduce the potential energy into the infinitesimal line element the infinitesimal line element (87) must be used together with the rotating frame theory. This will be the subject of UFT415.

3. NUMERICAL ANALYSIS AN DISCUSSION

Section by Dr. Horst Eckardt

Self consistency of the ECE2 covariant orbital equations

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3 Numerical analysis and discussion

3.1 Comparison with Euler-Lagrange equations in cartesan coordinates

In earlier work we had derived the relativistic Euler-Lagrange orbital equations in cartesian coordinates which are

$$\ddot{\mathbf{r}} = \frac{M\,G}{\gamma\,r^3}\left(\frac{\dot{\mathbf{r}}\,(\dot{\mathbf{r}}\cdot\mathbf{r})}{c^2} - \mathbf{r}\right). \tag{88}$$

We show that these are equivalent to those in plane polar coordinates used in this work. For this, we transform Eq. (88) into polar coordinates, using the transformations

$$X = r\cos(\phi), \tag{89}$$
$$Y = r\sin(\phi). \tag{90}$$

Then the velocity components are

$$v_X = \dot{X} = \dot{r}\cos(\phi) - r\dot{\phi}\sin(\phi), \tag{91}$$
$$v_Y = \dot{Y} = \dot{r}\sin(\phi) + r\dot{\phi}\cos(\phi) \tag{92}$$

and the accelerations are

$$a_X = \dot{v}_X = \ddot{r}\cos(\phi) - 2\dot{\phi}\dot{r}\sin(\phi) - \ddot{\phi}r\sin(\phi) - \dot{\phi}^2 r\cos(\phi), \tag{93}$$
$$a_X = \dot{v}_X = \ddot{r}\sin(\phi) + 2\dot{\phi}\dot{r}\cos(\phi) + \ddot{\phi}r\cos(\phi) - \dot{\phi}^2 r\sin(\phi). \tag{94}$$

The scalar product $\dot{\mathbf{r}}\cdot\mathbf{r}$ in (88) simplifies to

$$\dot{\mathbf{r}}\cdot\mathbf{r} = v_X X + v_Y Y = r\dot{r}. \tag{95}$$

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Inserting (91-95) into (88) then gives after some trigonometric reductions and resolution to $\ddot{\phi}$ and $\ddot{r}$:

$$\ddot{\phi}\,\mathbf{e}_X = \left(\frac{GM\,\dot{\phi}\,\dot{r}}{\gamma c^2\,r^2} - \frac{2\dot{\phi}\,\dot{r}}{r}\right)\mathbf{e}_X, \tag{96}$$

$$\ddot{r}\,\mathbf{e}_Y = \left(\frac{GM\,\dot{r}^2}{\gamma c^2\,r^2} + \dot{\phi}^2 r - \frac{GM}{\gamma r^2}\right)\mathbf{e}_Y \tag{97}$$

where

$$\mathbf{e}_X = \begin{bmatrix}1\\0\end{bmatrix}, \quad \mathbf{e}_Y = \begin{bmatrix}0\\1\end{bmatrix} \tag{98}$$

are the cartesian unit vectors. Because these are the same at both sides of (96, 97), it follows directly

$$\ddot{\phi} = \frac{GM\,\dot{\phi}\,\dot{r}}{\gamma c^2\,r^2} - \frac{2\dot{\phi}\,\dot{r}}{r}, \tag{99}$$

$$\ddot{r} = \frac{GM\,\dot{r}^2}{\gamma c^2\,r^2} + \dot{\phi}^2 r - \frac{GM}{\gamma r^2}. \tag{100}$$

These equations are identical to those derived from the Lagrangian

$$\mathscr{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r} \tag{101}$$

with the γ factor

$$\gamma = \left(1 - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{c^2}\right)^{-1/2}, \tag{102}$$

Q.E.D.

Applying frame rotation consists in the replacement

$$\phi \to \phi' = \phi + \omega_1 t. \tag{103}$$

For relativistic motion, the relativistic γ factor (102) is to be replaced by

$$\gamma = \left(1 - \frac{\dot{r}^2 + r^2\left(\frac{d}{dt}(\phi + \omega_1 t)\right)^2}{c^2}\right)^{-1/2} \tag{104}$$

$$= \left(1 - \frac{\dot{r}^2 + r^2\left(\dot{\phi} + \omega_1 + \dot{\omega}_1 t)^2\right)}{c^2}\right)^{-1/2}. \tag{105}$$

Inserting this in the Lagrangian (101) and evaluating the Euler-Lagrange equations for ϕ and r, we obtain the equation set

$$\ddot{\phi} = -\ddot{\omega}_1 t - \frac{2\dot{\omega}_1\,\dot{r}t}{r} + \frac{\dot{\omega}_1 GM\,\dot{r}t}{\gamma c^2\,r^2} + \frac{\omega_1 GM\,\dot{r}}{\gamma c^2\,r^2} \tag{106}$$
$$- 2\dot{\omega}_1 - \frac{2\omega_1\,\dot{r}}{r} + \frac{GM\,\dot{\phi}\,\dot{r}}{\gamma c^2\,r^2} - \frac{2\dot{\phi}\,\dot{r}}{r},$$

$$\ddot{r} = \dot{\omega}_1^2 r\,t^2 + 2\dot{\omega}_1\,\dot{\phi}rt + 2\omega_1\,\dot{\omega}_1 rt \tag{107}$$
$$+ \omega_1{}^2 r + 2\omega_1\,\dot{\phi}r + \dot{\phi}^2 r + \frac{GM\,\dot{r}^2}{\gamma c^2\,r^2} - \frac{GM}{\gamma r^2}.$$

These equations have to be solved simultaneously for a given function ω_1. This equation set becomes identical to (99, 100) for $\omega_1 \to 0$ as required for consistency.

3.2 Numerical solution without rotation

The equation set (99, 100) has been solved numerically for a demo system with parameters near to unity. There is a forward precession of the orbit which is graphed in Fig. 1. This is as already known from previous papers. The relativistic angular momentum with rotation is defined by

$$L = \gamma m r^2 \left(\dot{\phi} + \omega_1 + \dot{\omega}_1 t \right) \tag{108}$$

and comes out as a constant of motion from the Euler-Lagrange equations. It is compared with its non-relativistic counterpart

$$L_N = m r^2 \left(\dot{\phi} + \omega_1 + \dot{\omega}_1 t \right) \tag{109}$$

in Fig. 2, showing the required constancy in the relativistic case. The γ factor is graphed in Fig. 3, indicating that we are in a significant relativistic case. The total energy

$$E = mc^2(\gamma - 1) - \frac{mMG}{r} \tag{110}$$

and the corresponding Newtonian expression

$$E = \frac{1}{2} m \left(\dot{r}^2 + r^2 (\dot{\phi} + \omega_1 + \dot{\omega}_1 t)^2 \right) - \frac{mMG}{r} \tag{111}$$

are graphed in Fig. 4. Only at apastron positions, where the orbital velocity is minimal, the Newtonian values are roughly equal to the relativistic values.

3.3 Numerical solution with rotation

For the solution of the frame-rotated equations (106, 107) the rotation function

$$\omega_1 = a \exp(-b\, t) \tag{112}$$

was used with a negative a and positive b parameter. Due to frame rotation, there is a much higher precession effect in the orbit (Fig. 5). Besides this, the orbital extension (X and Y width) is smaller as an effect of rotation. The angular momenta, γ factor and total energies (Figs. 6-8) look very similar to the case without rotation (Figs. 2-4), however the rotation period has shortened and there is a larger distance between relativistic and non-relativistic values, indicating that the higher rotation frequency increases orbital velocity and thereby relativistic effects. The γ factor has a higher minimum, underpinning the same result.

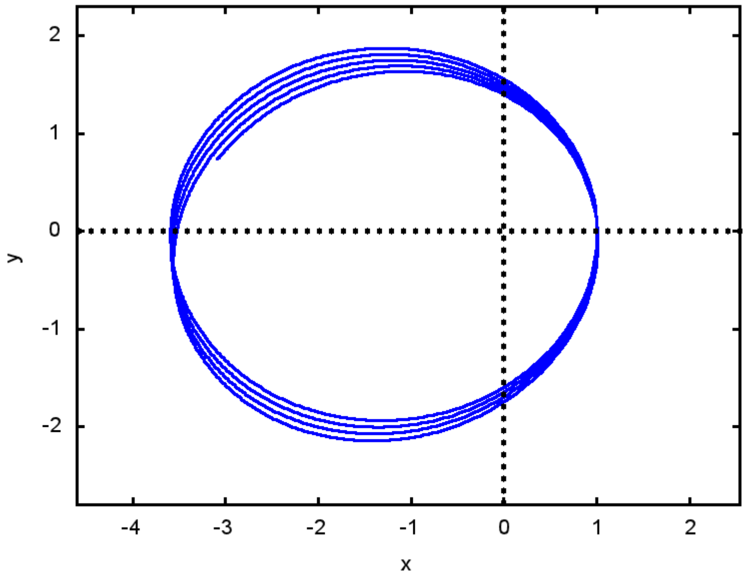

Figure 1: Orbit of relativistic motion.

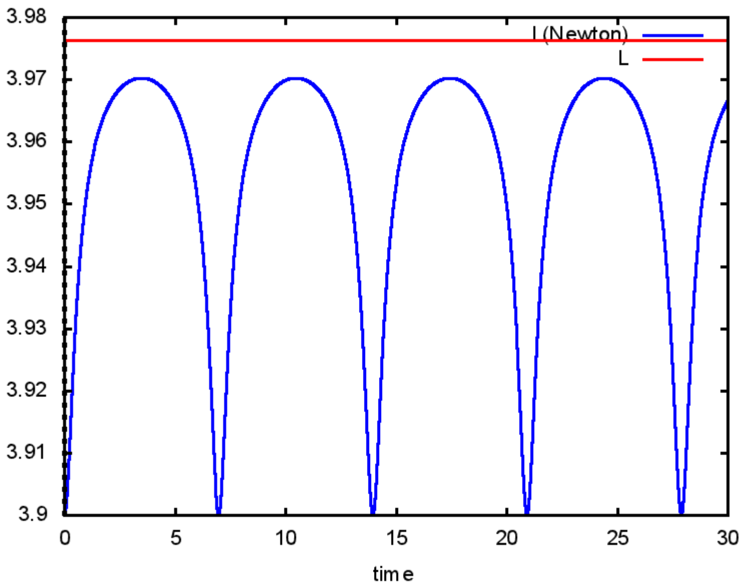

Figure 2: Angular momenta of relativistic orbit.

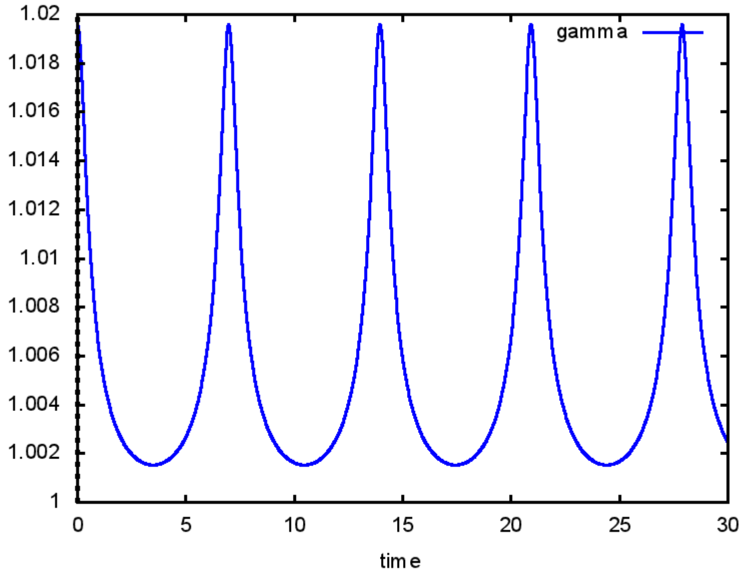

Figure 3: γ factor of relativistic orbit.

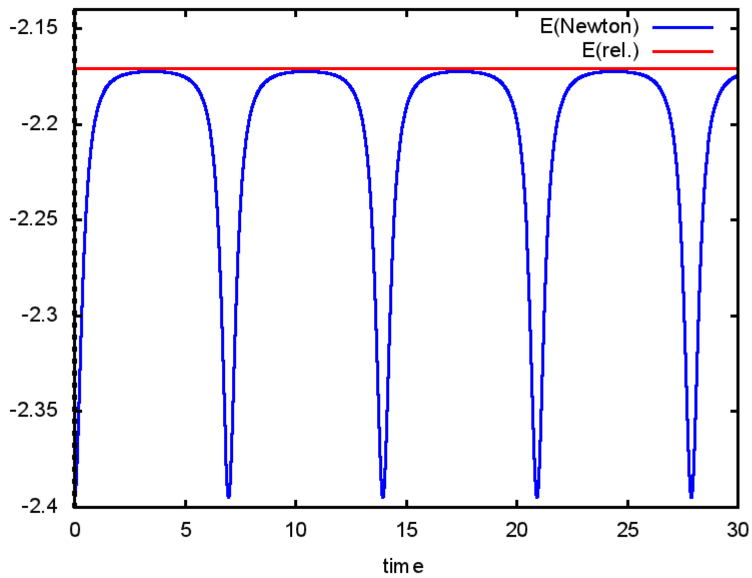

Figure 4: Total energies of relativistic orbit.

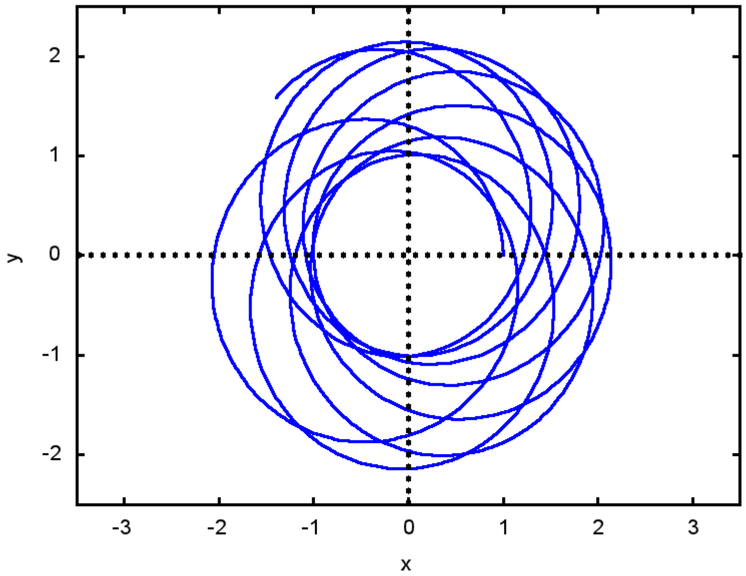

Figure 5: Orbit of relativistic motion with frame rotation.

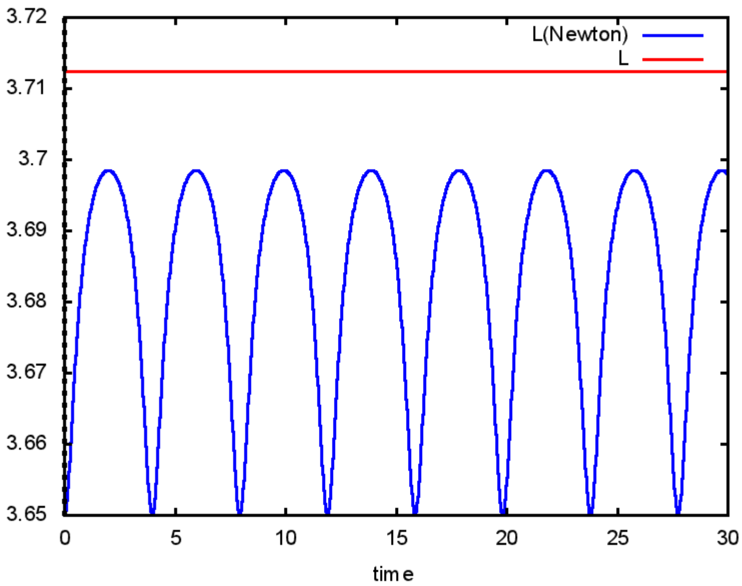

Figure 6: Angular momenta of relativistic orbit with frame rotation.

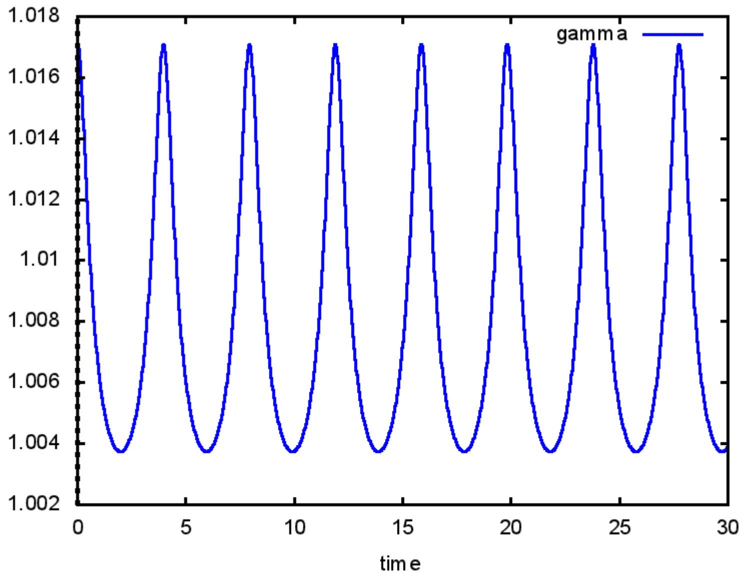

Figure 7: γ factor of relativistic orbit with frame rotation.

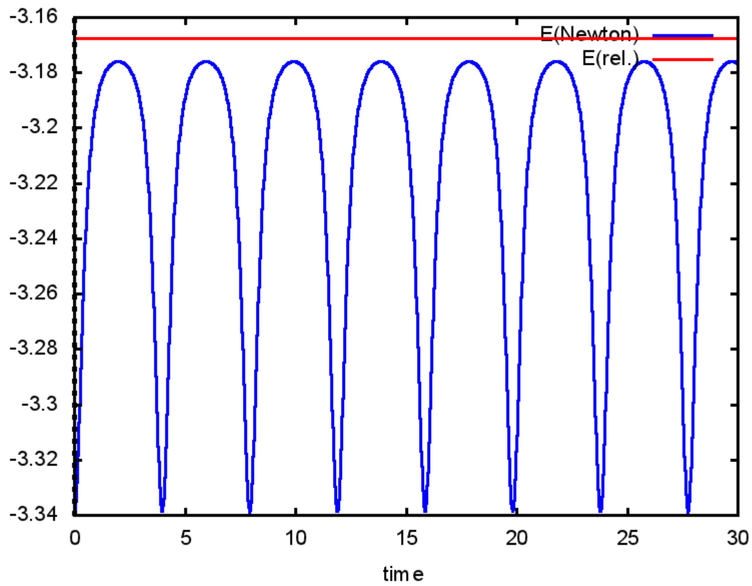

Figure 8: Total energies of relativistic orbit with frame rotation.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for voluntary posting, site maintenance and feedback maintenance. Alex Hill is thanked for many translations, and Robert Cheshire nd Michael Jackson for broadcasting and video preparation.

REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom, D. J. Crothers and U. E. Bruchholtz, "Principles of ECE Theory, Volume Two" (ePubli, Berlin 2017).

{2} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory, Volume One" (New Generation, London 2016, ePubli Berlin 2017).

{3} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (UFT301 on www.aias.us and Cambridge International 2010).

{4} M. W. Evans, H. Eckardt and D. W. Lindstrom "Generally Covariant Unified Field Theory" (Abramis 2005 - 2011, in seven volumes softback, open access in various UFT papers, combined sites www.aias.us and www.upitec.org).

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007, open access as UFT302, Spanish translation by Alex Hill).

{6} H. Eckardt, "The ECE Engineering Model" (Open access as UFT203, collected equations).

{7} M. W. Evans, "Collected Scientometrics" (open access as UFT307, New Generation, London, 2015).

{8} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001, open access in the Omnia Opera section of www.aias.us).

{9} M . W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997 and 2001) in two editions and six volumes, hardback, softback and e book.

{10} M. W. Evans and J. - P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 1999) in five volumes hardback and five volumes softback, open source in the Omnia Opera Section of www.aias.us).

{11} M. W. Evans, Ed. "Definitive Refutations of the Einsteinian General Relativity" (Cambridge International Science Publishing, 2012, open access on combined sites).

{12} M. W. Evans, Ed., J. Foundations of Physics and Chemistry (Cambridge International Science Publishing).

{13} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory (World Scientific 1974).

{14} G. W. Robinson, S. Singh, S. B. Zhu and M. W. Evans, "Water in Biology, Chemistry and Physics" (World Scientific 1996).

{15} W. T. Coffey, M. W. Evans, and P. Grigolini, "Molecular Diffusion and Spectra" (Wiley Interscience 1984).

{16} M. W. Evans, G. J. Evans, W. T. Coffey and P. Grigolini", "Molecular Dynamics and the Theory of Broad Band Spectroscopy (Wiley Interscience 1982).

{17} M. W. Evans, "The Elementary Static Magnetic Field of the Photon", Physica B, 182(3), 227-236 (1992).

{18} M. W. Evans, "The Photon's Magnetic Field: Optical NMR Spectroscopy" (World Scientific 1993).

{19} M. W. Evans, "On the Experimental Measurement of the Photon's Fundamental Static Magnetic Field Operator, B(3): the Optical Zeeman Effect in Atoms", Physica B, 182(3), 237 - 143 (1982).

{20} M. W. Evans, "Molecular Dynamics Simulation of Induced Anisotropy: I Equilibrium Properties" , J. Chem. Phys., 76, 5473 - 5479 (1982).

{21} M. W. Evans, "A Generally Covariant Wave Equation for Grand Unified Theory" Found. Phys. Lett., 16, 513 - 547 (2003).

{22} M. W. Evans, P. Grigolini and P. Pastori-Parravicini, Eds., "Memory Function Approaches to Stochastic Problems in Condensed Matter" (Wiley Interscience, reprinted 2009).

{23} M. W. Evans, "New Phenomenon of the Molecular Liquid State: Interaction of Rotation and Translation", Phys. Rev. Lett., 50, 371, (1983).

{24} M .W. Evans, "Optical Phase Conjugation in Nuclear Magnetic Resonance: Laser NMR Spectroscopy", J. Phys. Chem., 95, 2256-2260 (1991).

{25} M. W. Evans, "New Field induced Axial and Circular Birefringence Effects" Phys. Rev. Lett., 64, 2909 (1990).

{26} M. W. Evans, J. - P. Vigier, S. Roy and S. Jeffers, "Non Abelian Electrodynamics", "Enigmatic Photon V olume 5" (Kluwer, 1999)

{27} M. W. Evans, reply to L. D. Barron "Charge Conjugation and the Non Existence of the Photon's Static Magnetic Field" , Physica B, 190, 310-313 (1993).

{28} M. W. Evans, "A Generally Covariant Field Equation for Gravitation and Electromagnetism" Found. Phys. Lett., 16, 369 - 378 (2003).

{29} M. W. Evans and D. M. Heyes, "Combined Shear and Elongational Flow by Non Equilibrium Electrodynamics", Mol. Phys., 69, 241 - 263 (1988).

{30} Ref. (22), 1985 printing.

{31} M. W. Evans and D. M. Heyes, "Correlation Functions in Couette Flow from Group Theory and Molecular Dynamics", Mol. Phys., 65, 1441 - 1453 (1988).

{32} M. W. Evans, M. Davies and I. Larkin, Molecular Motion and Molecular Interaction in the Nematic and Isotropic Phases of a Liquid Crystal Compound", J. Chem. Soc. Faraday II, 69, 1011-1022 (1973).

{33} M. W. Evans and H. Eckardt, "Spin Connection Resonance in Magnetic Motors", Physica B., 400, 175 - 179 (2007).

{34} M. W. Evans, "Three Principles of Group Theoretical Statistical Mechanics", Phys. Lett. A, 134, 409 - 412 (1989).

{35} M. W. Evans, "On the Symmetry and Molecular Dynamical Origin of Magneto Chiral Dichroism: "Spin Chiral Dichroism in Absolute Asymmetric Synthesis" Chem. Phys. Lett., 152, 33 - 38 (1988).

{36} M. W. Evans, "Spin Connection Resonance in Gravitational General Relativity", Acta Physica Polonica, 38, 2211 (2007).

{37} M. W. Evans, "Computer Simulation of Liquid Anisotropy, III. Dispersion of the Induced Birefringence with a Strong Alternating Field", J. Chem. Phys., 77, 4632-4635 (1982).

{38} M. W. Evans, "The Objective Laws of Classical Electrodynamics, the Effect of Gravitation on Electromagnetism" J. New Energy Special Issue (2006).

{39} M. W. Evans, G. C. Lie and E. Clementi, "Molecular Dynamics Simulation of Water from 10 K to 1273 K", J. Chem. Phys., 88, 5157 (1988).

{40} M. W. Evans, "The Interaction of Three Fields in ECE Theory: the Inverse Faraday Effect" Physica B, 403, 517 (2008).

{41} M. W. Evans, "Principles of Group Theoretical Statistical Mechanics", Phys. Rev., 39, 6041 (1989).