EFFECT OF GRAVITATION ON FERMION RESONANCE AND HIGHER ORDER RELATIVISTIC CORRECTIONS.

by

M. W. Evans and H. Eckardt,

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www.atomicprecision.com

ABSTRACT

An extended minimal prescription is used to evaluate the effect of gravitation on fermion resonance. The results show that such effects are pronounced in systems such as neutron stars with heavy mass and small radius, but negligible in the laboratory. Some consideration is given to the general development of tetrads and the evaluation of angular momentum from Cartan geometry. Higher order relativistic effects are evaluated from the fermion equation using the operator representation and mixed operator function representation of linear momentum. It is found that higher order relativistic corrections have a pronounced effect in theory. Some results are evaluated by computer algebra for the hydrogenic wave-functions.

Keywords: ECE theory, angular momentum, fermion equation, gravitational effects on fermion resonance, higher order relativistic corrections.

UFT 253

1. INTRODUCTION

In recent papers of this series {1 - 10} the fermion equation of ECE theory has been developed in several ways to reveal new types of spectroscopy. In Section 2 the development is continued by firstly examining the way in which Cartan geometry leads to a fundamentally new definition of angular momentum. It is shown that the tetrad is ubiquitous throughout dynamics and electrodynamics and can be defined in many ways. Rotation generators, connections and torsions elements are evaluated in the cylindrical polar basis to illustrate the methodology in the simplest possible way. Angular momentum operator theory is fundamental to all quantum mechanics {11} as is well known. A synopsis is given of spin orbit coupling theory leading to new results which are evaluated in Section 3 by computer algebra. The mixed minimal prescription is introduced for the combined effect of electromagnetism and gravitation. The mutual effect of electromagnetism on gravitation is given by a mixed term which is used to evaluate the effect of gravitation on fermion resonance spectra. The spectra are evaluated by computer algebra in Section 3. It is found that the effect of gravitation is negligible in the laboratory but becomes clearly observable in the atmosphere of a neutron star with its large mass and small radius. So these effects can be evaluated experimentally. The effect of higher order relativistic corrections in the fermion equation are evaluated systematically in the operator representation of linear momentum, and in the mixed operator / function description. It is well known that in the original Dirac equation, some results are obtained {12} with an operator representation (g factor, Landé factor, ESR, NMR, MRI) and others with a mixed operator / function description (Thomas factor, spin orbit coupling constant, Darwin effect).There is no rule which can be used to decide where a given description must be used, and in previous papers several new spectroscopies have been developed with different descriptions and novel use of Pauli algebra. This has gone further than ever before in the exploration of fundamental quantum mechanics and has shown that the Dirac equation has some arbitrariness in it that was hitherto unknown. The fermion equation is equivalent to the chiral description of the Dirac equation. For example some choices of representation may lead to unphysical results, as described in the previous paper, but almost all appear to give physically meaningful results which can be tested experimentally. If these tests prove the theory then quantum mechanics is strengthened, if not, a fundamental weakness emerges which must be rectified in future theoretical work. In Section 3 some of the key results of this papers are evaluated by computer algebra for the hydrogenic wave-functions.

2. THEORETICAL DEVELOPMENT

One of the major advances made by ECE theory is to show that a tetrad may be defined by two different descriptions of the same mathematical space, for example cylindrical polar and Cartesian. For each quantity in classical dynamics and electrodynamics a tetrad can always be defined. For example:

$$e^a = q^a_\mu \, e^\mu \qquad (1a)$$

$$r^a = q^a_\mu \, r^\mu \qquad (1b)$$

$$p^a = q^a_\mu \, p^\mu \qquad (1c)$$

$$a^a = q^a_\mu \, a^\mu \qquad (1d)$$

$$A^a = q^a_\mu \, A^\mu \qquad (1e)$$

where a and μ denote different bases. Here e^a is the unit vector, r^a is the position vector, p^a is the linear momentum vector, a^a is the linear acceleration vector, A^a is the electromagnetic potential, and so on. Eq. (1a) may be interpreted as:

$$\begin{bmatrix} e^{(1)} \\ e^{(2)} \end{bmatrix} = \begin{bmatrix} q^{(1)}_1 & q^{(1)}_2 \\ q^{(2)}_1 & q^{(2)}_2 \end{bmatrix} \begin{bmatrix} e^1 \\ e^2 \end{bmatrix} \qquad (2)$$

where attention has been restricted to two dimensions for simplicity. If one basis is the cylindrical polar and the other is the Cartesian the Eq. (3) is:

$$\begin{bmatrix} \underline{e}_r \\ \underline{e}_\theta \end{bmatrix} = \begin{bmatrix} q^{(1)}_1 & q^{(1)}_2 \\ q^{(2)}_1 & q^{(2)}_2 \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix} \quad -(3)$$

where the unit vectors are defined by:

$$\underline{e}_r = \underline{i} \cos\theta + \underline{j} \sin\theta \quad -(4)$$

$$\underline{e}_\theta = -\underline{i} \sin\theta + \underline{j} \cos\theta . \quad -(5)$$

So in this case the tetrad is the two by two matrix:

$$q^a_\mu = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad -(6)$$

which is the rotation matrix about the Z axis {12}. The infinitesimal rotation generator is well known {12} to be defined by:

$$J_Z = \frac{1}{i} \frac{d}{d\theta} q^a_\mu \Big|_{\theta = 0} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \quad -(7)$$

and within a factor $\hbar$ the infinitesimal rotation generator has the same commutator properties as the angular momentum operator of quantum mechanics.

Now note that:

$$dq^a_\mu / dX = \frac{d\theta}{dX} dq^a_\mu / d\theta = \frac{d\theta}{dX} \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \quad -(8)$$

$$dq^a_\mu / dY = \frac{d\theta}{dY} dq^a_\mu / d\theta = \frac{d\theta}{dY} \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \quad -(9)$$

and denoting:

$$X = 1, \; Y = 2 \quad -(10)$$

it follows as in note 253(1) accompanying UFT253 on www.aias.us that:

$$\partial_1 q^{(2)}_2 = \frac{1}{r}, \quad \partial_2 q^{(1)}_1 = -\frac{1}{r} \tan\theta, \quad -(11)$$

$$\partial_1 q^{(1)}_2 = -\frac{1}{r}\cot\theta , \quad \partial_2 q^{(2)}_1 = -\frac{1}{r} . \quad -(12)$$

From considerations of antisymmetry of Cartan torsion in note 253(1)it follows that the spin connections of Cartan {1 - 10} are:

$$\omega^{(1)}_{12} + \omega^{(1)}_{21} = \frac{1}{r}\left(\tan\theta + \cot\theta\right) \quad -(13)$$

and:

$$\omega^{(2)}_{12} = -\omega^{(2)}_{21} . \quad -(14)$$

They are related to the antisymmetric Christoffel connection by:

$$\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu} = \partial_\mu q^a_\nu + \omega^a_{\mu\nu} . \quad -(15)$$

This fundamental Cartan geometry can be related to angular momentum theory as follows.

Differentiating Eq. (6) with respect to θ gives:

$$\frac{\partial q^a_\mu}{d\theta} = \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} = \frac{\partial x^\mu}{\partial\theta}\partial_\mu q^a_\nu \quad -(16)$$

Define the general rotation generator matrix by:

$$J^a_\mu = \frac{\partial x^\mu}{\partial\theta}\partial_\mu q^a_\nu . \quad -(17)$$

For a Z axis rotation in cylindrical polar coordinates

$$J^a_\mu = \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \quad -(18)$$

so the infinitesimal rotation generator is:

$$J^a_\mu := \frac{1}{i} J^a_\mu(\theta = 0) = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \quad -(19)$$

The rotation generator matrix in Cartesian representation is:

$$J^{\nu}_{\mu} = q^{\nu}_{a} J^{a}_{\mu} \quad -(20)$$

where the inverse tetrad is defined by:

$$q^{\nu}_{a} q^{a}_{\nu} = 1 \quad -(21)$$

So:

$$q^{\nu}_{a} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \quad -(22)$$

Therefore:

$$J^{\nu}_{\mu} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \quad -(23)$$

From Eqs. (19) and (23):

$$i J^{\nu}_{\mu} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = \frac{1}{i} \left(\frac{\partial x^{\mu}}{\partial \theta} \partial_{\mu} q^{a}_{\nu} \right)_{\theta = 0}. \quad -(24)$$

Now lower indices using the metric $g_{\nu\rho}$ and denote:

$$J_Z = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \quad J_{\mu\nu} = g_{\mu\rho} J^{\rho}_{\nu} \quad -(25)$$

It is clear that J_Z is the vector format of the tensor J_{XY}. The angular momentum operator is:

$$\hat{J}_Z = \hbar J_Z \quad -(26)$$

In general:

$$J_{\rho} = \epsilon_{\rho}{}^{\mu}{}_{\nu} J^{\nu}_{\mu}, \quad -(27)$$

$$\epsilon_{\rho}{}^{\mu}{}_{\nu} = g^{\mu\lambda}\epsilon_{\rho\lambda\nu} \quad -(28)$$

so the angular momentum operator may be defined in any space by:

$$\hat{J}_{\rho} = i\hbar\,\epsilon_{\rho}{}^{\mu}{}_{\nu} J^{\nu}_{\mu} \quad -(29)$$

In the fermion equation in what follows the space is the Minkowski spacetime.

The angular momentum enters in to the fermion equation through the use of Pauli algebra {1 - 12}. The source of the fermion equation is the relativistic linear momentum of special relativity:

$$\underline{p} = \gamma m \underline{v} \quad -(30)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(31)$$

is the Lorentz factor and where $\underline{v}$ is the linear velocity and m the mass of a particle. Eq. (30) can be rewritten as the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(32)$$

which can be factorized as:

$$(E - mc^2)(E + mc^2) = c^2 p^2 . \quad -(33)$$

The effect of an external field is introduced with the minimal prescription. For example:

$$E \rightarrow E - e\phi \quad -(34)$$

where -e is the charge on the electron and ϕ is the electromagnetic scalar potential. The minimal prescription (34) can be extended to:

$$E \rightarrow E - e\phi + m\Phi \quad -(35)$$

where Φ the gravitational potential. The change of sign is due to the fact that m is positive but -e is negative. With the minimal prescription (34) Eq. (33) becomes:

$$E = e\phi + mc^2 + \frac{c^2 p^2}{E - e\phi + mc^2} \quad -(36)$$

Now introduce the SU(2) basis defined by the Pauli matrices, so:

$$E = e\phi + mc^2 + c^2 \underline{\sigma} \cdot \underline{p} \left(\frac{1}{E - e\phi + mc^2} \right) \underline{\sigma} \cdot \underline{p}. \quad -(37)$$

In order to linearize this equation the usual approximation {12} is to assume that the E in the denominator of the right hand side is :

$$E = \gamma mc^2 \sim mc^2 \quad -(38)$$

i.e.

$$v \ll c. \quad -(39)$$

This approximation gives:

$$E = e\phi + mc^2 + \frac{1}{2m} \underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{p} + \frac{e}{4m^2c^2} \underline{\sigma} \cdot \underline{p} \, \phi \, \underline{\sigma} \cdot \underline{p} \quad -(40)$$

The quantity E defined in this way is the total energy, so this equation quantizes to:

$$\hat{H}\psi = \left(e\phi + mc^2 + \frac{1}{2m} \underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{p} + \frac{e}{4m^2c^2} \underline{\sigma} \cdot \underline{p} \, \phi \, \underline{\sigma} \cdot \underline{p} \right) \psi \quad -(41)$$

where $\hat{H}$ is the hamiltonian operator. The linear momentum is quantized by the operator representation {1 - 12}:

$$\hat{p} = -i\hbar \underline{\nabla} \quad -(42)$$

to the linear momentum operator $\hat{p}$. It is almost never made clear in textbooks that spin orbit coupling in spectra is described as follows:

$$H_1 \phi = \frac{e}{4m^2c^2} \underline{\sigma} \cdot \underline{p}\, \phi\, \underline{\sigma} \cdot \underline{p}\, \phi = \frac{-ie\hbar}{4m^2c^2} \underline{\sigma} \cdot \underline{\nabla}\, \phi\, \underline{\sigma} \cdot \underline{p}\, \phi \quad -(43)$$

The first $\underline{p}$ is used in the operator representation, and the second $\underline{p}$ in the functional representation. There is no a priori reason for this choice. It is accepted because it seemed to give an accurate description of atomic and molecular spectra. However the immediately preceding papers have questioned the validity of this procedure. It is also important to note that the hamiltonian (43) must be interpreted as:

$$H_1 \phi = \frac{-ie\hbar}{4m^2c^2} \underline{\sigma} \cdot \underline{\nabla} \left(\phi\, \underline{\sigma} \cdot \underline{p}\, \phi \right) = \frac{-ie\hbar}{4m^2c^2} \left(\underline{\nabla} \left(\underline{\sigma} \cdot \underline{p} \right) \phi \phi + \underline{\sigma} \cdot \underline{p}\, \underline{\nabla} \cdot (\phi\phi) \right) \quad -(44)$$

where the Leibnitz Theorem has been used. The conventional description of spin orbit coupling comes from the term:

$$H_1 \phi = -\frac{ie\hbar}{4m^2c^2} \underline{\sigma} \cdot \underline{\nabla} (\phi\phi)\, \underline{\sigma} \cdot \underline{p} + \cdots \quad -(45)$$

in which the Leibnitz Theorem gives:

$$\underline{\nabla} (\phi\phi) = \phi\, \underline{\nabla} \phi + \phi\, \underline{\nabla} \phi. \quad -(46)$$

Therefore Eq. (45) gives:

$$H_1 \phi = \frac{-ie\hbar}{4m^2c^2} \left(\underline{\sigma} \cdot \underline{\nabla} \phi\, \underline{\sigma} \cdot \underline{p}\, \phi + \phi\, \underline{\sigma} \cdot \underline{\nabla} \phi\, \underline{\sigma} \cdot \underline{p} + \cdots \right) \quad -(47)$$

At this point the standard physics uses:

$$\underline{E} = -\underline{\nabla}\phi \quad -(48)$$

and ECE physics uses the same format by use of antisymmetry laws and as part of a unified field theory {1 - 10}. The Coulomb potential used to describe the interaction between an electron and proton in the H atom:

$$\phi = -\frac{e}{4\pi\epsilon_0 r} \quad -(49)$$

where ϵ_0 is the vacuum permittivity and r the distance between the electron and proton. The electric field is therefore:

$$\underline{E} = -\frac{e}{4\pi\epsilon_0}\frac{\underline{r}}{r^3} \quad -(50)$$

and the hamiltonian (47) becomes:

$$H_1\phi = \frac{-ie^2\hbar}{16\pi\epsilon_0 m^2c^2}\left(\frac{1}{r^3}\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}\,\phi - \frac{1}{r}\underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{p}\right) \quad -(51)$$

The orbital angular momentum $\underline{L}$ is introduced by Pauli algebra as follows:

$$\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p} = \underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{r}\times\underline{p} = \underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{L} \quad -(52)$$

where

$$\underline{L} = \underline{r}\times\underline{p}. \quad -(53)$$

So the real part of the first term on the right hand side of Eq. (51) is:

$$\mathrm{Re}(H_1\phi) = \frac{e^2\hbar}{16\pi\epsilon_0 m^2c^2r^3}\underline{\sigma}\cdot\underline{L}\,\phi + \cdots \quad -(54)$$

which is the usual spin orbit coupling term in the H atom {1 - 11}. Note carefully that $\underline{\sigma}\cdot\underline{L}$

in Eq. (54) is still classical. It is the expectation value:

$$\underline{\sigma} \cdot \underline{L} = \int \phi^* \underline{\hat{\sigma}} \cdot \underline{\hat{L}} \, \phi \, d\tau \qquad -(55)$$

where ^ denotes operator. In quantum mechanics {11} the Pauli matrices are quantized as follows and become operators:

$$\underline{\hat{S}} = \frac{\hbar}{2} \underline{\hat{\sigma}} \qquad -(56)$$

where $\underline{\hat{S}}$ is the spin angular momentum operator. So:

$$\underline{\hat{\sigma}} \cdot \underline{\hat{L}} = \frac{2}{\hbar} \underline{\hat{S}} \cdot \underline{\hat{L}} \qquad -(57)$$

In some representations {1-10}:

$$\underline{\hat{S}} \cdot \underline{\hat{L}} \, \phi = \frac{\hbar^2}{2} \left(j(j+1) - l(l+1) - s(s+1) \right) \phi \qquad -(58)$$

so:

$$\underline{\hat{\sigma}} \cdot \underline{\hat{L}} \, \phi = \hbar \left(j(j+1) - l(l+1) - s(s+1) \right) \phi . \qquad -(59)$$

The classical $\underline{\sigma} \cdot \underline{L}$ of Eq. (54) is therefore

$$\begin{aligned} \underline{\sigma} \cdot \underline{L} = \langle \underline{\hat{\sigma}} \cdot \underline{\hat{L}} \rangle &= \hbar \left(j(j+1) - l(l+1) - s(s+1) \right) \int \phi^* \phi \, d\tau \\ &= \hbar \left(j(j+1) - l(l+1) - s(s+1) \right) . \end{aligned} \qquad -(60)$$

From Eqs. (54) and (60):

$$Re\left(\hat{H}_1 \phi\right) = \frac{e^2 \hbar^2}{16 \pi \epsilon_0 m^2 c^2} \left(j(j+1) - l(l+1) - s(s+1) \right) \int \frac{\phi^* \phi}{r^3} \, d\tau \qquad -(61)$$

The second term on the right hand side of Eq. (51) is:

$$\hat{H}_2\phi = \frac{ie^2\hbar}{16\pi\epsilon_0 m^2 c^2 r}\,\underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{p} \quad -(62)$$

As in immediately preceding papers this can be developed with the following Pauli algebra {11}, which follows from the Pauli algebra in Eq. (52). Eq. (63) is an identity because:

$$\underline{\sigma}\cdot\underline{p} = \frac{1}{r^2}\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}\,,\quad \frac{1}{r^2}\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{r} = 1. \quad -(63)$$

Using:

$$\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p} = \underline{r}\cdot\underline{p} + i\,\underline{\sigma}\cdot\underline{L} \quad -(64)$$

Eq. (63) can be expressed as:

$$\underline{\sigma}\cdot\underline{p} = \frac{1}{r^2}\underline{\sigma}\cdot\underline{r}\left(\underline{r}\cdot\underline{p} + i\,\underline{\sigma}\cdot\underline{L}\right) \quad -(65)$$

so once more introduces the orbital angular momentum $\underline{L}$. From Eqs. (62) and (65) the real part of $\hat{H}_2\phi$ can be evaluated from:

$$\underline{\sigma}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{p} = i\,\underline{\sigma}\cdot\underline{\nabla}\phi\,\frac{\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{L}}{r^2} + \cdots \quad -(66)$$

So:

$$\text{Real}\,\hat{H}_2\phi = -\frac{e^2\hbar}{16\pi\epsilon_0 m^2 c^2 r^3}\,\underline{r}\cdot\underline{\nabla}\phi\,\underline{\sigma}\cdot\underline{L} \quad -(67)$$

where:

$$\underline{\sigma}\cdot\underline{L} = \langle\hat{\underline{\sigma}}\cdot\hat{\underline{L}}\rangle = \hbar\left(j(j+1) - \ell(\ell+1) - s(s+1)\right). \quad -(68)$$

From Eqs. (67) and (68):

$$H_2\phi = \frac{-e^2\hbar^2}{16\pi\epsilon_0 m^2 c^2 r^3}\left(j(j+1)-l(l+1)-s(s+1)\right)\underline{r}\cdot\underline{\nabla}\phi \quad -(69)$$

whose energy expectation values are:

$$E_2\phi = \frac{-e^2\hbar^2}{16\pi\epsilon_0 m^2 c^2}\left(j(j+1)-l(l+1)-s(s+1)\right)\int \phi^* \frac{\underline{r}\cdot\underline{\nabla}\phi}{r^3}\,d\tau \quad -(70)$$

In spherical polar coordinates:

$$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi \quad -(71)$$

and:

$$\underline{\nabla}\phi = \frac{\partial\phi}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\phi}\underline{e}_\phi + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\theta}\underline{e}_\theta \quad -(72)$$

Using:

$$\underline{r} = r\,\underline{e}_r \quad -(73)$$

it follows that:

$$\underline{r}\cdot\underline{\nabla}\phi = r\frac{\partial\phi}{\partial r} \quad -(74)$$

so

$$E_2\phi = \frac{-e^2\hbar^2}{16\pi\epsilon_0 m^2 c^2}\left(j(j+1)-l(l+1)-s(s+1)\right)\int \frac{\phi^*}{r^2}\frac{\partial\phi}{\partial r}\,d\tau \quad -(75)$$

and this is evaluated by computer algebra in Section 3. The result (75) is the same as

given by the second term of the type II hamiltonian of page 4 of note 252(10) on www.aias.us, thus providing a cross check on analytical correctness.

Fine structure in atoms and molecules is very accurately measurable so it is interesting to evaluate the effect of gravitation on the spectra. This is done most simply using the minimal prescription:

$$E \rightarrow E + m\Phi \quad -(76)$$

where Φ is the gravitational potential:

$$\Phi = -\frac{GM}{r} \quad -(77)$$

where G is Newton's constant and M is the mass that is gravitationally attracted to the mass m of the electron in the H atom. As in note 253(3) accompanying UFT253 on www.aias.us it follows that:

$$H\phi = \left(-m\Phi + mc^2 + \frac{1}{2m} \underline{\sigma} \cdot \underline{p} \left(1 - \frac{m\Phi}{2mc^2} \right) \underline{\sigma} \cdot \underline{p} \right) \phi \quad -(78)$$

in the approximation:

$$E \sim mc^2, \quad m\Phi \ll 2mc^2. \quad -(79)$$

The relevant spin orbit hamiltonian is:

$$H\phi = \frac{i\hbar}{4mc^2} \underline{\sigma} \cdot \underline{\nabla} \Phi \, \underline{\sigma} \cdot \underline{p} \, \phi = -\frac{i\hbar}{4mc^2} \underline{\sigma} \cdot \underline{g} \, \underline{\sigma} \cdot \underline{p} \, \phi \quad -(80)$$

where the acceleration due to gravity is

$$\underline{g} = -\underline{\nabla} \Phi. \quad -(81)$$

It is possible to develop Eq. (80) in different ways, including the following:

1) To consider the effect of the Earth's $\underline{g}$ on the electron in an H atom.

2) To evaluate the effect of the gravitational interaction between an electron and a proton in the H atom.

In both cases use the Pauli algebra:

$$\underline{\sigma} \cdot \underline{p} = \frac{1}{r^2} \underline{\sigma} \cdot \underline{r} \left(\underline{r} \cdot \underline{p} + i \underline{\sigma} \cdot \underline{L} \right) \quad -(82)$$

where

$$\underline{L} = \underline{r} \times \underline{p} \quad -(83)$$

is the classical angular momentum. From Eqs. (80) and (82):

$$\text{Real}\,\hat{H}\phi = \frac{\hbar}{4mc^2 r^2} \underline{\sigma} \cdot \underline{g}\, \underline{\sigma} \cdot \underline{r}\, \underline{\sigma} \cdot \underline{L}\, \phi . \quad -(83)$$

Now use:

$$\underline{\sigma} \cdot \underline{g}\, \underline{\sigma} \cdot \underline{r} = \underline{g} \cdot \underline{r} + i \underline{\sigma} \cdot \underline{g} \times \underline{r} . \quad -(84)$$

so:

$$\text{Real}\,\hat{H}\phi = \frac{\hbar\, \underline{g} \cdot \underline{r}}{4mc^2 r^2} \underline{\sigma} \cdot \underline{L}\, \phi . \quad -(85)$$

In spherical polar coordinates:

$$\underline{r} = r \underline{e}_r \quad -(86)$$

and from Eqs. (81) and (86):

$$\underline{g} \cdot \underline{r} = r \underline{g} \cdot \underline{e}_r \quad -(87)$$

However, from Eq. (81):

$$\underline{g} = -\frac{MG}{r^3} \underline{r} = -\frac{MG}{r^2} \underline{e}_r \quad -(88)$$

so:

$$\underline{r_g} \cdot \underline{e}_r = r_g. \quad -(89)$$

Therefore:

$$\mathrm{Re}\,\hat{H}\phi = \frac{\hbar^2 g}{4mc^2 r}\left(j(j+1) - \ell(\ell+1) - s(s+1)\right)\phi \quad -(90)$$

When considering the gravitational interaction between an electron and a proton:

$$\underline{\sigma} \cdot \underline{L} = \langle \underline{\hat{\sigma}} \cdot \underline{\hat{L}} \rangle = \hbar\left(j(j+1) - \ell(\ell+1) - s(s+1)\right) \quad -(91)$$

so

$$\mathrm{Re}\,\hat{H}\phi = \frac{\hbar^2 g}{4mc^2 r}\,\underline{\sigma} \cdot \underline{L}\,\phi \quad -(92)$$

which has the correct units of joules. The energy expectation value from Eq. (90) is:

$$E = \frac{\hbar^2 g}{4mc^2}\left(j(j+1) - \ell(\ell+1) - s(s+1)\right)\int \frac{\phi^*\phi}{r}\,d\tau \quad -(93)$$

which is evaluated by computer algebra in Section 3. The effect of the gravitational interaction between electron and proton is entirely negligible compared with the electrostatic interaction.

In evaluating the effect of the earth's gravitational field Eq. (91) can no longer be used, because $\underline{L}$ is defined as:

$$\underline{L} = \underline{R} \times \underline{p} \quad -(94)$$

where $\underline{R}$ is the distance between the electron and the centre of the earth. Therefore another method must be developed to calculate the effect on fine structure of an external mass M, for example the mass of the earth, sun or neutron star.

As in note 253(4) the usual procedure used in deriving the fine structure of atoms and molecules is to assume that:

$$x = \frac{e\phi}{2mc^2} << 1 \quad -(95)$$

so:

$$(1-x)^{-1} \sim 1 + x \quad -(96)$$

More accurately:

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots \quad -(97)$$

and if this expansion is truncated at second order:

$$(1-x)^{-1} \sim 1 + x + x^2 \quad -(98)$$

producing the following expression for total energy:

$$E = e\phi + mc^2 + \frac{1}{2m}\underline{\sigma}\cdot\underline{p}\left(1 + \frac{e\phi}{2mc^2} + \frac{e^2\phi^2}{4m^2c^4}\right)\underline{\sigma}\cdot\underline{p} \quad -(99)$$

The second order term introduces a new type of spin orbit spectroscopy described by:

$$E_1 = \frac{e^2}{8m^3c^4}\underline{\sigma}\cdot\underline{p}\,\phi^2\,\underline{\sigma}\cdot\underline{p} \quad -(100)$$

which quantizes to:

$$H_1\phi = -\frac{ie^2\hbar}{8m^3c^4}\left(\underline{\sigma}\cdot\underline{\nabla}\phi^2\,\underline{\sigma}\cdot\underline{p}\right)\phi + \cdots \quad -(101)$$

in which:

$$\phi^2 = \frac{e^2}{16\pi^2\epsilon_0^2 r^2}, \quad -(102)$$

$$\underline{\nabla}\phi^2 = -\frac{e^2}{8\pi^2\epsilon_0^2 r^4}\underline{r}. \quad -(103)$$

So:

$$H_1\phi = \frac{ie^4\hbar}{64\pi^2\epsilon_0^2 m^3 c^4 r^4}\,\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}\,\phi. \quad -(104)$$

Using the Pauli algebra:

$$\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p} = \underline{r}\cdot\underline{p} + i\,\underline{\sigma}\cdot\underline{L} \quad -(105)$$

gives:

$$\mathrm{Re}\,H_1\phi = \frac{-e^4\hbar^2}{64\pi^2\epsilon_0^2 m^3 c^4 r^4}\left(j(j+1)-\ell(\ell+1)-s(s+1)\right)\phi \quad -(106)$$

whose energy expectation values are:

$$E_1 = -\frac{e^4\hbar^2}{64\pi^2\epsilon_0^2 m^3 c^4}\left(j(j+1)-\ell(\ell+1)-s(s+1)\right)\int\frac{\phi^*\phi}{r^4}\,d\tau \quad -(107)$$

These are evaluated by computer algebra in Section 3 for the hydrogenic wavefunctions. The first order result is:

$$E_0 = \frac{e^2\hbar^2}{16\pi\epsilon_0 m^2 c^2}\left(j(j+1)-\ell(\ell+1)-s(s+1)\right)\int\frac{\phi^*\phi}{r^3}\,d\tau \quad -(108)$$

so the ratio of second to first order expectation values is:

$$\left|\frac{E_1}{E_0}\right| = \frac{e^2}{4\pi\epsilon_0 mc^2}\int\frac{\phi^*\phi}{r^4}\,d\tau \Big/ \int\frac{\phi^*\phi}{r^3}\,d\tau \quad -(109)$$

These ratios are also evaluated in Section 3 by computer algebra for the hydrogenic wavefunctions.

The effect of gravitation can now be considered by the extended minimal prescription:

$$E \rightarrow E - e\phi + m\Phi \quad -(110)$$

which produces the total energy:

$$E = e\phi - m\Phi + mc^2 + \frac{1}{2m}\underline{\sigma}\cdot\underline{p}\left(1 + \frac{e\phi - m\Phi}{2mc^2} + \frac{(e\phi - m\Phi)^2}{4m^2c^4}\right)\underline{\sigma}\cdot\underline{p} \quad -(111)$$

to second order. Eq. (111) contains the term:

$$E_2 = -\frac{e}{4m^2c^4}\underline{\sigma}\cdot\underline{p}\,\phi\,\Phi\,\underline{\sigma}\cdot\underline{p}. \quad -(112)$$

This can be used to consider the effect of external gravitation of a mass M on the fine structure of the H atom. Use the potentials:

$$\phi = -\frac{e}{4\pi\epsilon_0 r}, \quad \Phi = -\frac{GM}{R} \quad -(113)$$

where r is the distance between the proton and the electron in an H atom and where R is the distance between the electron and the centre of an external mass M, for example the earth. Quantization of Eq. (112) produces:

$$H_2\psi = \frac{ie\hbar}{4m^2c^4}\underline{\sigma}\cdot\underline{\nabla}(\phi\Phi)\underline{\sigma}\cdot\underline{p}\,\psi. \quad -(114)$$

Using the Leibnitz Theorem:

$$\underline{\nabla}(\phi\Phi) = \Phi\underline{\nabla}\phi + \phi\underline{\nabla}\Phi \quad -(115)$$

therefore:

$$H_2\psi = -\frac{ie^2\hbar MG}{16\pi\epsilon_0 m^2c^4}\left(\frac{1}{Rr^3}\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p} + \frac{1}{rR^3}\underline{\sigma}\cdot\underline{R}\,\underline{\sigma}\cdot\underline{p}\right)\psi \quad -(116)$$

where:

$$\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p} = \underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{L} \quad -(117)$$

$$\underline{\sigma} \cdot \underline{R}\,\underline{\sigma} \cdot \underline{p} = \underline{R}^{-1} \cdot \underline{p} + i\underline{\sigma} \cdot \underline{L}_1 \quad -(118)$$

and where:

$$\underline{L} = \underline{r} \times \underline{p}, \quad -(119)$$

$$\underline{L}_1 = \underline{R} \times \underline{p}. \quad -(120)$$

The term of relevance is:

$$H_2\phi = \frac{e^2\hbar^2 M G}{16\pi\epsilon_0 m^2 c^4 R r^3}\left(j(j+1) - l(l+1) - s(s+1)\right)\phi \quad -(121)$$

which gives the energy expectation values:

$$E_2 = \frac{e^2\hbar^2}{16\pi\epsilon_0 m^2 c^2}\left(\frac{MG}{Rc^2}\right)\left(j(j+1) - l(l+1) - s(s+1)\right)\int \frac{\phi^*\phi}{r^3}\,d\tau. \quad -(122)$$

Comparison of Eqs. (108) and (122) shows that:

$$\frac{E_2}{E_0} = \frac{MG}{Rc^2}. \quad -(123)$$

The effect of an external mass M a distance R away from the electron is to change the spectrum by a factor:

$$\zeta = \frac{MG}{Rc^2}. \quad -(124)$$

For the earth:

$$\zeta = 1.39 \times 10^{-9} \quad -(125)$$

For the sun:

$$\zeta = 4.242 \times 10^{-5} \quad -(126)$$

In a typical neutron star:

$$\gamma = 3.442 \text{ to } 7.867 \quad -(127)$$

so if there is an H atom in the atmosphere of a neutron star its fine structure spectrum is measurably different from the same spectrum in an earth laboratory.

Finally in this section higher order relativistic effects are considered. Consider the Einstein energy equation (33) written as:

$$E = mc^2 + \frac{c^2 p^2}{E + mc^2}. \quad -(128)$$

The g factor of the electron, ESR, NMR, MRI, Landé factor, Zeeman effects, Thomas factor, fine structure constant, Darwin term and all the new effects found in immediately preceding papers have been evaluated using:

$$E \sim mc^2 \quad -(129)$$

Eqns. (128) and (129) produce the non relativistic kinetic energy:

$$T = E - mc^2 = \frac{p^2}{2m}. \quad -(130)$$

However, the total energy is defined by:

$$E = \gamma mc^2 \quad -(131)$$

so more accurately Eq. (128) is:

$$E = mc^2 + \frac{c^2 p^2}{mc^2\left(1 - \frac{v^2}{c^2}\right)^{-1/2} + mc^2}. \quad -(132)$$

If

$$v \ll c \quad -(133)$$

then:
$$\left(1-\frac{v^2}{c^2}\right)^{-1/2} \sim 1+\frac{v^2}{2c^2} \quad -(134)$$

and in the same approximation:
$$p \sim mv. \quad -(135)$$

This quantizes to the higher order Schroedinger equation:
$$H\phi = (E-mc^2)\phi = -\frac{\hbar^2}{2m}\nabla^2\phi + \frac{\hbar^4}{8m^3c^2}\nabla^4\phi \quad -(136)$$

which is the free particle Schroedinger equation:
$$H\phi = -\frac{\hbar^2}{2m}\nabla^2\phi \quad -(137)$$

with a small relativitstic perturbation. The solutions of Eq. (137) can be found numerically using the free particle wavefunction as a starting point in an iterative procedure. The resulting wavefunctions can then be used in quantum tunnelling theory for relativistic particles. As in UFT226 to UFT231 on www.aias.us this procedure may give insight into low energy nuclear reactions.

Higher order relativistic corrections to magnetic effects such as electron spin resonance may be evaluated in a similar way, full details of which are given in note 253(7). The starting point is the usual semi-classical description for the interaction of the electron and electromagnetic field in special relativity:
$$(E-e\phi)^2 = c^2(\underline{p}-e\underline{A})^2 + m^2c^4 \quad -(138)$$

in which magnetic effects are described by the vector potential $\underline{A}$. In fully developed ECE theory the spin connection is also considered. It follows that:

$$E = e\phi + mc^2 + \frac{c^2(\underline{p} - e\underline{A})^2}{E - e\phi + mc^2} \quad -(139)$$

where the total energy is:

$$E = \gamma mc^2. \quad -(140)$$

Eq. (139) quantized to:

$$\hat{H}\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\left(\frac{2}{\gamma+1}\left(1 - \frac{e\phi}{(\gamma+1)mc^2}\right)^{-1}\right)\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi + \cdots \quad -(141)$$

which can be approximated by:

$$\hat{H}\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\frac{2}{\gamma+1}\left(1 + \frac{e\phi}{(\gamma+1)mc^2}\right)\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi + \cdots \quad -(142)$$

Now use the approximations:

$$\gamma + 1 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} + 1 \sim 2 + \frac{1}{2}\frac{v^2}{c^2} \quad -(143)$$

and:

$$\frac{2}{\gamma+1} = \left(1 + \frac{1}{4}\frac{v^2}{c^2}\right)^{-1} \sim 1 - \frac{v^2}{4c^2} \quad -(144)$$

to find that the hamiltonian in Eq. (142) can be approximated by:

$$H\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\left(1 - \frac{1}{4}\frac{v^2}{c^2} + \frac{1}{2}\left(1 - \frac{1}{4}\frac{v^2}{c^2}\right)^2\frac{e\phi}{mc^2}\right)\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi \quad -(145)$$

giving relativistic corrections to all the terms usually given by:

$$H_0\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\left(1 + \frac{e\phi}{2mc^2}\right)\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi \quad -(146)$$

Now note that:

$$v^2 = \frac{1}{m^2}(\underline{p} - e\underline{A})\cdot(\underline{p} - e\underline{A}) \quad -(147)$$

and in the SU(2) basis:

$$v^2 = \underline{\sigma}\cdot\underline{v}\,\underline{\sigma}\cdot\underline{v} = \frac{1}{m^2}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A}) \quad -(148)$$

Restricting consideration to the first term in Eq. (145) gives the relativistic corrections to ESR from:

$$H_1\phi = \frac{1}{2m}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\left(1 - \frac{1}{4m^2c^2}\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A})\right)\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi \quad -(149)$$

The first term in Eq. (149) is the usual ESR term and is worked out as in UFT248 with the operator representation:

$$\underline{p} = -i\hbar\underline{\nabla} \quad -(150)$$

So:

$$H_{11}\phi = \frac{1}{2m}\left(-\hbar^2\nabla^2\phi + e^2A^2\phi + ie\hbar\underline{\nabla}\cdot(\underline{A}\phi) - e\hbar\underline{\sigma}\cdot\underline{\nabla}\times(\underline{A}\phi) + ie\hbar\underline{A}\cdot\underline{\nabla}\phi - e\hbar\underline{\sigma}\cdot\underline{A}\times\underline{\nabla}\phi\right) \quad -(151)$$

which can be written as a sum of the well known ESR term plus other terms:

$$H_{11}\phi = -\frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{B}\phi + \cdots \quad -(152)$$

The second term in Eq. (149) is the quartic:

$$H_{12}\phi = -\frac{1}{8m^3c^2}\underline{\sigma}\cdot(\underline{p}-e\underline{A})\underline{\sigma}\cdot(\underline{p}-e\underline{A})\underline{\sigma}\cdot(\underline{p}-e\underline{A})\underline{\sigma}\cdot(\underline{p}-e\underline{A})\phi \quad -(153)$$

and can be developed in many different ways to give several entirely new types of spectra, each of which should be observable experimentally. To end this section two examples are given. First consider the function operator representation:

$$H_{12}\phi = -\frac{1}{8m^3c^2}(\underline{p}-e\underline{A})\cdot(\underline{p}-e\underline{A})\underline{\sigma}\cdot(-i\hbar\underline{\nabla}-e\underline{A})\underline{\sigma}\cdot(-i\hbar\underline{\nabla}-e\underline{A})\phi$$

$$= \frac{e\hbar}{8m^3c^2}\underline{\sigma}\cdot\underline{B}\left(p^2-2e\underline{p}\cdot\underline{A}+e^2A^2\right)\phi + \cdots \quad -(154)$$

In a uniform magnetic field:

$$\underline{A} = \frac{1}{2}\underline{B}\times\underline{r} \quad -(155)$$

and:

$$\underline{p}\cdot\underline{B}\times\underline{r} = \underline{B}\cdot\underline{r}\times\underline{p} = \underline{B}\cdot\underline{L}. \quad -(156)$$

So

$$H_{12}\phi = -\frac{e^2\hbar}{8m^3c^2}\underline{L}\cdot\underline{B}\,\underline{\sigma}\cdot\underline{B}\,\phi + \cdots \quad -(157)$$

If B is aligned in the Z axis:

$$H_{12}\phi = -\frac{e^2\hbar}{8m^3c^2}L_Z\,\sigma_Z\,B_Z^2\,\phi \quad -(158)$$

Now use:

$$\underline{S} = \frac{\hbar}{2}\underline{\sigma} \quad -(159)$$

so:

$$H_{12}\phi = -\frac{e^2}{4m^3c^2} L_Z S_Z B_Z^2 \phi = -\frac{e^2\hbar^2}{4m^3c^2} m_L m_S B_Z^2 \phi \quad -(160)$$

where the magnetic quantum numbers are given by:

$$m_L = -L, \ldots, L \quad -(161)$$

$$m_S = -S, \ldots, S \quad -(162)$$

This is a second order spin order splitting whose order of magnitude is:

$$f_{12} = 111.88 m_L B_Z^2 \quad \text{Hz} \quad -(163)$$

which compares with the well known first order resonance frequency:

$$f = 2.80 \times 10^{10} B_Z \quad \text{Hz} \quad -(164)$$

This type of relativistic correction is a small shift not very much different form the magnitude of the well known chemical shift, and therefore it ought to be observable.

In the pure operator representation Eq. (153) is:

$$H_2\phi = \frac{e\hbar}{8m^3c^2}\left(-\hbar^2\nabla^2 + ie\hbar\left(\underline{\nabla}\cdot\underline{A} + \underline{A}\cdot\underline{\nabla}\right) + e^2A^2\right)\underline{\sigma}\cdot\underline{B}\phi + \ldots \quad -(165)$$

The real part of this hamiltonian is:

$$\text{Re}\, H_2\phi = -\frac{e\hbar^3}{8m^3c^2}\underline{\nabla}\cdot\underline{\nabla}\,\underline{\sigma}\cdot\underline{B}\phi + \frac{e^3\hbar A^2}{8m^3c^2}\underline{\sigma}\cdot\underline{B}\phi \quad -(166)$$

For the uniform magnetic field (155):

$$A^2 = \frac{1}{4}\underline{B}\times\underline{r}\cdot\underline{B}\times\underline{r} = \frac{1}{4}\left(B^2r^2 - \underline{B}\cdot\underline{r}\,\underline{B}\cdot\underline{r}\right) \quad -(167)$$

For a Z axis magnetic field:

$$\underline{\sigma}\cdot\underline{B}\phi = \sigma_Z B_Z \phi \quad -(168)$$

and

$$A^2 = \frac{1}{4} B_Z^2 (r^2 - Z^2) = \frac{1}{4} B_Z^2 r^2 (1 - \cos^2\theta) \quad -(169)$$

in spherical polar coordinates. It is found that:

$$\mathrm{Re}\, H_{12}\phi = H_{11}\phi + H_{12}\phi \quad -(170)$$

where

$$H_{11}\phi = -\frac{e\hbar\,\sigma_Z B_Z}{8m^3c^2}\nabla^2\phi \quad -(171)$$

and

$$H_{12}\phi = \frac{e^3\hbar\,\sigma_Z B_Z^3}{32m^3c^2} r^2(1-\cos^2\theta)\phi. \quad -(172)$$

The Z component of the Pauli matrix is:

$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(173)$$

and resonance can occur between states of σ_Z. The energy expectation values for this type of relativistic correction are:

$$E_{11} = -\frac{e\hbar^3\sigma_Z B_Z}{8m^3c^2}\int \phi^*\nabla^2\phi\, d\tau \quad -(174)$$

and

$$E_{12} = \frac{e^3\hbar\,\sigma_Z B_Z^3}{32m^3c^2}\int \phi^* r^2(1-\cos^2\theta)\phi\, d\tau \quad -(175)$$

and are evaluated in Section 3 for the hydrogenic wavefunctions. These compare with the non relativistic result, the well known:

$$H\phi = -\frac{e\hbar}{2m}\,\underline{\sigma}\cdot\underline{B}\,\phi \quad -(176)$$

The resonance frequencies are:

$$\omega_0 = eB_Z/m \quad -(177)$$

$$\omega_{11} = \frac{e\hbar^2 B_Z}{4m^3c^2}\int \phi^* \nabla^2 \phi \, d\tau \quad -(178)$$

$$\omega_{12} = \frac{e^3 B_Z^3}{16m^3c^2}\int \phi^* r^2 (1-\cos^2\theta)\phi \, d\tau \quad -(179)$$

The relativistic corrections depend on the orbital in which the electron is situated, so interesting spectra are expected.

3. COMPUTATION FOR THE HYDROGENIC WAVEFUNCTIONS

Section by Dr. Horst Eckardt

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Effect of gravitation on fermion resonance and higher order relativistic corrections

M. W. Evans[*], H. Eckardt[†]
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

3 Computation for the Hydrogenic wavefunctions

As in the preceding papers, the expectation values of energy eigenvalues defined in section 2 (Eqs.(75, 93, 107-109)) are evaluated by computer algebra. The constant factors of the energy eigenvalues have been compiled in the following list, extended by their numerical values in electron Volts (eV). To obtain the full energy expectation values, the results listed in the tables have to be multiplied by these values.

$$E_0 = -\frac{e^2\ \hbar^2}{16\ \pi\ \epsilon_0\ m^2\ c^2} \frac{Z^3}{a_0^3} = 0.0143015\, Z^3, \tag{180}$$

$$E_1 = -\frac{e^4\, h_{bar}^2}{64\, \pi^2\, \epsilon_0^2\, m^3\, c^4} \frac{Z^4}{a_0^4} = -7.61574\, 10^{-7}\, Z^4, \tag{181}$$

$$E_2 = -E_0, \tag{182}$$

$$E_{grav} = -\frac{g\, h_{bar}^2}{4\, m\, c^2} \frac{Z}{a_0} = 1.55071\, 10^{-24}\, Z. \tag{183}$$

The ratio between E_1 and E_0 is

$$\frac{E_1}{E_0} = -\frac{e^2}{4\, \pi\, \epsilon_0\, m\, c^2} = -5.32514\, 10^{-5}\, Z. \tag{184}$$

Obviously the corrections are small, in particular for the splitting in the gravitational field of the earth (Eq.(93)). The contributions of the integration are listed in Table 1. The quantum number factor is again

$$F_j := j(j+1) - l(l+1) - s(s+1). \tag{185}$$

It can be seen that in general

$$E_2 = 0, \tag{186}$$

[*]email: emyrone@aol.com
[†]email: mail@horst-eckardt.de

i.e. there is no correction of anglular momentum of type Eq.(75) or (62) respectively. S orbitals do not contribute as found in previous papers. The modulus of the corrections only depends on the angular quantum number l. The ratio E_1/E_0 decreases with increasing quantum numbers.

Finally we try an estimation of the fourth-order corrections to the kinetc energy given in Eq.(136). The radial part of the fourth-order nabla operator in spherical coordinates is

$$\nabla_r^4 \, \psi = \frac{4}{r} \, \frac{d^3}{d\,r^3} \, \psi + \frac{d^4}{d\,r^4} \, \psi. \tag{187}$$

The expectation value of this operator is

$$E_4 := \frac{\hbar^4}{8m^3c^2} \cdot 4\pi \int \psi^* \; \nabla_r^4 \; \psi \; r^2 \; dr \tag{188}$$

where the angular integration gives 4π. Evaluating this integral with the $1s$ orbital of Hydrogen gives the result

$$E_4 = -\frac{\hbar^4}{8m^3c^2} \cdot 4\pi \frac{3\,Z^4}{4\,\pi\,a_0^4}, \tag{189}$$

and with the factors worked out in eV units it is

$$E_4 = -0.846900 \; Z^4 \; eV. \tag{190}$$

This value is quite high compared to the binding energies of the electron in Hydrogen.

n	l	m_l	j	s	m_s	m_j	F_j	$E_0[\frac{Z^2}{a_0^3}]$	$E_1[\frac{Z^4}{a_0^4}]$	$E_1/E_0[\frac{Z}{a_0}]$	$E_2[\frac{Z^2}{a_0^3}]$	$E_{grav}[\frac{Z}{a_0}]$
1	0	0	1/2	1/2	-1/2	-1/2	0	—	0	0	0	0
1	0	0	1/2	1/2	1/2	1/2	0	—	0	0	0	0
2	0	0	1/2	1/2	-1/2	-1/2	0	—	0	0	0	0
2	0	0	1/2	1/2	1/2	1/2	0	—	0	0	0	0
2	1	-1	3/2	1/2	-1/2	-3/2	1	$\frac{1}{24}$	$\frac{1}{24}$	1	0	$\frac{1}{4}$
2	1	-1	3/2	1/2	1/2	-1/2	1	$\frac{1}{24}$	$\frac{1}{24}$	1	0	$\frac{1}{4}$
2	1	0	1/2	1/2	-1/2	-1/2	-1	$-\frac{1}{24}$	$-\frac{1}{24}$	1	0	$-\frac{1}{4}$
2	1	0	3/2	1/2	1/2	1/2	1	$\frac{1}{24}$	$\frac{1}{24}$	1	0	$\frac{1}{4}$
2	1	1	1/2	1/2	-1/2	1/2	-1	$-\frac{1}{24}$	$-\frac{1}{24}$	1	0	$-\frac{1}{4}$
2	1	1	3/2	1/2	1/2	3/2	1	$\frac{1}{24}$	$\frac{1}{24}$	1	0	$\frac{1}{4}$
3	0	0	1/2	1/2	-1/2	-1/2	0	—	0	0	0	0
3	0	0	1/2	1/2	1/2	1/2	0	—	0	0	0	0
3	1	-1	3/2	1/2	-1/2	-3/2	1	$\frac{1}{81}$	$\frac{10}{729}$	$\frac{10}{9}$	0	$\frac{1}{9}$
3	1	-1	3/2	1/2	1/2	-1/2	1	$\frac{1}{81}$	$\frac{10}{729}$	$\frac{10}{9}$	0	$\frac{1}{9}$
3	1	0	1/2	1/2	-1/2	-1/2	-1	$-\frac{1}{81}$	$-\frac{10}{729}$	$\frac{10}{9}$	0	$-\frac{1}{9}$
3	1	0	3/2	1/2	1/2	1/2	1	$\frac{1}{81}$	$\frac{10}{729}$	$\frac{10}{9}$	0	$\frac{1}{9}$
3	1	1	1/2	1/2	-1/2	1/2	-1	$-\frac{1}{81}$	$-\frac{10}{729}$	$\frac{10}{9}$	0	$-\frac{1}{9}$
3	1	1	5/2	1/2	1/2	3/2	1	$\frac{1}{81}$	$\frac{10}{729}$	$\frac{10}{9}$	0	$\frac{1}{9}$
3	2	-2	5/2	1/2	-1/2	-5/2	2	$\frac{2}{405}$	$\frac{4}{3645}$	$\frac{2}{9}$	0	$\frac{2}{9}$
3	2	-2	5/2	1/2	1/2	-3/2	2	$\frac{2}{405}$	$\frac{4}{3645}$	$\frac{2}{9}$	0	$\frac{2}{9}$
3	2	-1	3/2	1/2	-1/2	-3/2	-2	$-\frac{2}{405}$	$-\frac{4}{3645}$	$\frac{2}{9}$	0	$-\frac{2}{9}$
3	2	-1	5/2	1/2	1/2	-1/2	2	$\frac{2}{405}$	$\frac{4}{3645}$	$\frac{2}{9}$	0	$\frac{2}{9}$
3	2	0	3/2	1/2	-1/2	-1/2	-2	$-\frac{2}{405}$	$-\frac{4}{3645}$	$\frac{2}{9}$	0	$-\frac{2}{9}$
3	2	0	5/2	1/2	1/2	1/2	2	$\frac{2}{405}$	$\frac{4}{3645}$	$\frac{2}{9}$	0	$\frac{2}{9}$
3	2	1	3/2	1/2	-1/2	1/2	-2	$-\frac{2}{405}$	$-\frac{4}{3645}$	$\frac{2}{9}$	0	$-\frac{2}{9}$
3	2	1	5/2	1/2	1/2	3/2	2	$\frac{2}{405}$	$\frac{4}{3645}$	$\frac{2}{9}$	0	$\frac{2}{9}$
3	2	2	3/2	1/2	-1/2	3/2	-2	$-\frac{2}{405}$	$-\frac{4}{3645}$	$\frac{2}{9}$	0	$-\frac{2}{9}$
3	2	2	5/2	1/2	1/2	5/2	2	$\frac{2}{405}$	$\frac{4}{3645}$	$\frac{2}{9}$	0	$\frac{2}{9}$

Table 1: Energies E_0, E_1, E_1/E_0, E_2, E_{grav} with $F_j = j(j+1) - l(l+1) - s(s+1)$.