24

The Fundamental Invariants of the Evans Field Theory

Summary. The fundamental invariants of the Evans field theory are constructed by using the Stokes formula in the structure equations and Bianchi identities of differential geometry. The structure invariants are independent of the base manifold, and can be used to define the rotation and translation generators in general relativity. The identity invariants are defined by integrating the Bianchi identities. In so doing a new theorem of differential geometry is obtained and given the appellation "inverse structure theorem". The latter defines the integrated tetrad and integrated spin connection. These methods of differential geometry are used to explain the class of all Aharonov Bohm effects and to provide a rigorous geometrical basis for the Heisenberg equation of motion and uncertainty principle. The origin of the Planck constant is discussed in terms of differential geometry.

Key words: Evans field theory, fundamental invariants, inverse structure theorem, Aharonov Bohm effects; origin of the Planck constant in differential geometry.

24.1 Introduction

It was shown by Wigner [1] in 1939 that there are two fundamental invariants that characterise any particle in special relativity. These are the mass and spin invariants, the first and second Casimir invariants of the Poincaré group. Thus, any particle is characterized by its mass and spin. The Poincare group of special relativity has ten generators. In order to extend Wigner's analysis to general relativity the sixteen generators of the Einstein group need to be considered in order to define the fundamental invariants of general relativity. In Section 24.2 the structure equations and Bianchi identities of differential geometry are integrated with the Stokes formula of differential geometry in order to identify two types of invariants, the structure and identity invariants. The latter convey new meaning to familiar quantities such as the displacement four-vector x^a and the rotation generator $\theta^a{}_b$.

In the original work by Wigner [1] the first Casimir invariant is:

$$C_1 = \boldsymbol{P}^\mu \boldsymbol{P}_\mu \tag{24.1}$$

and the second Casimir invariant is:

$$C_2 = W^\mu W_\mu. \tag{24.2}$$

Here:

$$\boldsymbol{P}_\mu = i\frac{\partial}{\partial x^\mu} \tag{24.3}$$

is the spacetime translation generator, which is the energy-momentum in special relativity within a factor $\hbar$ [2]. Spacetime translation is defined by:

$$x^\mu \to x^\mu + a^\mu. \tag{24.4}$$

The second Casimir invariant C_2 is defined by the Pauli Lubanski vector [2]:

$$W_\mu = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}J^{\nu\rho}\boldsymbol{P}^\sigma \tag{24.5}$$

It is seen that C_1 and C_2 are the two Casimir invariants that characterize the Poincaré group of special relativity. They are the mass and spin invariants respectively and are invariant under Lorentz transform. For example:

$$p^\mu p_\mu = m^2c^2 = E_0^2/c^2 \tag{24.6}$$

and the rest energy E_0 is the same in all frames of reference, being a fundamental scalar. Similarly it may be shown [2] that:

$$C_2 = m^2 S(S+1) \tag{24.7}$$

in reduced units, and this is also a scalar invariant. In Section 24.2 Wigner's analysis is extended to general relativity using methods based on differential geometry rather than group theory, and the fundamental structure and identity invariants defined. In Section 24.3 the inverse structure theorem is proven from integration of the Bianchi identities of differential geometry. In Section 4 the class of Aharonov Bohm effects is explained with the theory of Sections 24.2 and 24.3. Finally in Section 24.5 the Heisenberg equation of motion is deduced from the invariants and the origin of the fundamental constants such as the Planck constant discussed.

24.2 The Structure and Identity Invariants of Differential Geometry

The Maurer Cartan structure equations of differential geometry are [3]:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{24.8}$$

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{24.9}$$

Here T^a is the torsion form, q^a is the tetrad form, $R^a{}_b$ is the curvature or Riemann form, $\omega^a{}_b$ is the spin connection one-form and $D\wedge$ denotes the covariant exterior derivative:

$$D\wedge = d\wedge + \omega^a{}_b \wedge \tag{24.10}$$

where $d\wedge$ is the exterior derivative.

The Bianchi identities of differential geometry are:

$$D\wedge T^a = d\wedge T^a + \omega^a{}_b \wedge T^b = R^a{}_b \wedge q^b \tag{24.11}$$

$$D\wedge R^a{}_b = d\wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b = 0. \tag{24.12}$$

These equations [3] have been used recently to develop the Evans field theory [4]–[35], generally accepted [36] as the first objective unified field theory of physics. The structure equations and Bianchi identities are augmented in all manifolds and tangent spacetimes by the tetrad postulate:

$$D_\mu q^a{}_\nu = 0 \tag{24.13}$$

and the Evans Lemma:

$$\Box q^a = R q^a. \tag{24.14}$$

Here R is the scalar curvature. The Einstein postulate is:

$$R = -kT \tag{24.15}$$

where T is the index contracted canonical energy momentum tensor and k is the Einstein constant. In the Evans field theory the Einstein postulate is extended to all radiated and matter fields, and not restricted to the gravitational field.

All these equations retain their form under general coordinate transformation and are therefore rigorously objective equations of general relativity.

The rest curvature is defined by:

$$|R_0| = \left(\frac{mc}{\hbar}\right)^2 = p^\mu p_\mu / \hbar^2 = \boldsymbol{P}^\mu \boldsymbol{P}_\mu \tag{24.16}$$

where

$$p_\mu = \hbar \boldsymbol{P}_\mu \tag{24.17}$$

is the energy momentum vector in special relativity. So it is seen that R_0 is a Casimir invariant of differential geometry and of the Evans field theory. The important conclusion is reached that any particle in unified field theory is characterized by the eigenvalues R of the Evans Lemma. We therefore achieve a quantization of general relativity, an objective and causal quantization of unified field theory. The eigenfunction or wavefunction is the tetrad. Thus eigenvalues of the tetrad are fundamental invariants and observables of unified field theory.

In special relativity $\boldsymbol{p}_\mu$ is the generator of spacetime translations. In special relativity and Minkowski spacetime:

$$x^\mu x_\mu = c^2t^2 - X^2 - Y^2 - Z^2 \tag{24.18}$$

is the square of the line element, and is invariant under Lorentz transformation. The corresponding Casimir invariant is the mass invariant, C_1, of the Poincaré group. Therefore we look for the objective generalization of Eq. (24.18) using differential geometry, having already found that:

$$|R_0| = C_1. \tag{24.19}$$

To proceed use the Stokes formula of differential geometry:

$$\int_s d \wedge f^a = \oint f^a \tag{24.20}$$

where f^a is any differential form [3]. The Stokes formula is true for any manifold enclosed by any surface. On the left hand side of Eq. (24.20) appears a surface integral over the surface S. On the right hand side appears a contour integral over the boundary enclosing the surface. Therefore we find results such as:

$$\int_s d \wedge q^a = \oint q^a \tag{24.21}$$

$$\int_s d \wedge \omega^a{}_b = \oint \omega^a{}_b \tag{24.22}$$

$$\int_s d \wedge T^a = \oint T^a \tag{24.23}$$

$$\int_s d \wedge R^a{}_b = \oint R^a{}_b. \tag{24.24}$$

There are two fundamental structure invariants, which are found by surface integration of the structure equations (24.8) and (24.9):

$$x^a = \int_s T^a \tag{24.25}$$

$$\theta^a{}_b = \int_s R^a{}_b. \tag{24.26}$$

It is seen that x^a has the units of metres and that $\theta^a{}_b$ is unitless and antisymmetric in its indices a and b. The latter are indices of the tangent spacetime, a Minkowski spacetime [3]. The details of the base manifold have been integrated out in Eqs. (24.25) and (24.26). Therefore x^a and $\theta^a{}_b$ are invariants in the sense that they are independent of the base manifold by surface integration.

It is now possible to define the scalar invariants:

$$E_1 = x^a x_a \tag{24.27}$$

$$E_2 = \theta^a{}_b \theta^b{}_a \tag{24.28}$$

by index contraction. These play a similar role to the Casimir invariants of special relativity. and E_1 and E_2 are the fundamental structure invariants of differential geometry and thus of the Evans field theory. They are invariant under the general coordinate transformation and thus the same to any observer.

From Eqs. (24.8) and (24.21)

$$x^a = \oint q^a + \int_s \omega^a{}_b \wedge q^b \tag{24.29}$$

and from Eqs. (24.9) and (24.22)

$$\theta^a{}_b = \oint \omega^a{}_b + \int_s \omega^a{}_c \wedge \omega^c{}_b . \tag{24.30}$$

Eqs. (24.29) and (24.30) give considerable insight to the fundamental meaning of translation and rotation in general relativity and can be regarded as the integral form of the fundamental Maurer Cartan structure equations. Thus, they provide considerable insight to unified field theory from differential geometry [4]–[36].

The two identity invariants originate in integration of the two Bianchi identities , Eqs. (24.11) and (24.12). Using the Stokes formula:

$$\theta^a = \int_s D \wedge T^a = \oint T^a + \int_s \omega^a{}_b \wedge T^b = \int_s R^a{}_b \wedge q^b \tag{24.31}$$

$$\kappa^a{}_b = \int_s D \wedge R^a{}_b = \oint R^a{}_b + \int_s \omega^a{}_c \wedge R^c{}_b - \int_s R^a{}_c \wedge \omega^c{}_b \tag{24.32}$$

The first identity invariant θ^a is unitless an the second identity invariant $\kappa^a{}_b$ has the units of inverse metres. The identity invariants, unlike the structure invariants, depend on an index of the base manifold, i.e. :

$$Q^a{}_\mu = \theta^a{}_\mu = \left(\int_s D \wedge T^a \right)_\mu \tag{24.33}$$

$$\Omega^a{}_{b\mu} = \kappa^a{}_{b\mu} = \left(\int_s D \wedge R^a{}_b \right)_\mu . \tag{24.34}$$

The first identity invariant has the same index structure as the tetrad, and is also unitless, so it is given the appellation "integrated tetrad" $Q^a{}_\mu$. Similarly the second identity invariant may be identified as the "integrated spin connection" $\Omega^a{}_\mu$.

24.3 The Inverse Structure Theorem

The inverse structure theorem is true for all manifolds and tangent spacetimes, and states that if

$$T^a = D \wedge q^a \tag{24.35}$$

$$R^a{}_b = D \wedge \omega^a{}_b \tag{24.36}$$

then

$$Q^a = \int_s D \wedge T^a \tag{24.37}$$

$$\Omega^a{}_b = \int_s D \wedge R^a{}_b . \tag{24.38}$$

Using the second Bianchi identity it is seen that:

$$\Omega^a{}_b = 0 \tag{24.39}$$

i.e. the integrated spin connection vanishes for all manifolds and tangent spacetimes. In a differential geometry where:

$$D \wedge T^a = R^a{}_b \wedge q^b \neq 0 \tag{24.40}$$

i.e. the geometry of a unified field theory, then the integrated tetrad is non-zero:

$$Q^a = \int_s D \wedge T^a \neq 0. \tag{24.41}$$

An objective unified field theory is therefore always defined by:

$$\begin{aligned} Q^a &\neq 0, \\ \Omega^a{}_b &= 0. \end{aligned} \tag{24.42}$$

In a differential geometry where:

$$D \wedge T^a = R^a{}_b \wedge q^b = 0 \tag{24.43}$$

then:

$$\begin{aligned} Q^a &= 0, \\ \Omega^a{}_b &= 0. \end{aligned} \tag{24.44}$$

Eq. (24.44) defines the geometry when electromagnetism and gravitation have become independent.

24.4 The Aharonov Bohm Effects

These fundamental results may now be applied to give a straightforward explanation of the class of Aharonov Bohm (AB) effects [2]. Proceed by multiplying both sides of Eq. (24.29) by the fundamental C negative potential $A^{(0)}$ in volts [4]–[36]:

$$A^{(0)}x^a = A^{(0)} \oint q^a + A^{(0)} \int_s \omega^a{}_b \wedge q^b \tag{24.45}$$

and rewrite Eq. (24.45) as:

$$\Phi^a = \oint A^a + \int_s \omega^a{}_b \wedge A^b. \tag{24.46}$$

In Eq. (24.46) Φ^a has the units of magnetic flux (weber = volt meter). We therefore obtain:

$$\Phi^a = \int_s F^a = \oint A^a + \int_s \omega^a{}_b \wedge A^b. \tag{24.47}$$

This is the equation of the Evans field theory that explains the class of all AB effects in terms of differential geometry. The AB effects are observed when it is arranged experimentally that:

$$\oint A^a = 0 \tag{24.48}$$

For example, in the region enclosed by the electron beams in the Chambers experiment [2] condition (24.48) is true outside the iron whisker. In the Chambers experiment (and all other AB effects), the magnetic flux being observed is therefore:

$$\Phi^a = A^{(0)} \int_s \omega^a{}_b \wedge q^b. \tag{24.49}$$

It is seen that the class of AB effects is due to a structure invariant of differential geometry. The presence of the covariant exterior derivative means that if a magnetic flux is excluded from a region of the base manifold it is nonetheless observed in another region. The AB effects depend only on $A^{(0)}$, (which is a scalar voltage), and on differential geometry. They are therefore due to "spacetime geodynamics". The AB effects can be explained in causal general relativity and are effects of the Evans unified field theory. Conversely the AB effects all provide quantitative experimental evidence for the Evans unified field theory.

The integrated potential for all manifolds and surfaces is:

$$\alpha^a = \int_s D \wedge F^a = \oint F^a + \int_s \omega^a{}_b \wedge F^b = \int_s R^a{}_b \wedge A^b \tag{24.50}$$

where

$$\int_s F^a = \oint A^a + \int_s \omega^a{}_b \wedge A^b \tag{24.51}$$

In general therefore, the class of AB effects is described by Eqs. (24.50) and (24.51) for all manifolds and surfaces. When:

$$d \wedge F^a = R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b = 0 \tag{24.52}$$

then:

$$\oint F^a + \int_s \omega^a{}_b \wedge F^b = \int_s R^a{}_b \wedge A^b \tag{24.53}$$

Eqs. (24.51) and (24.53) are the integrated counterparts of:

$$F^a = D \wedge A^a \tag{24.54}$$

$$D \wedge F^a = R^a{}_b \wedge A^b \tag{24.55}$$

i.e. of:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{24.56}$$

$$d \wedge F^a + \omega^a{}_b \wedge F^b = R^a{}_b \wedge A^b. \tag{24.57}$$

In regions where it is arranged experimentally that:

$$\oint A^a = \int_s d \wedge A^a = 0 \tag{24.58}$$

then Eqs. (24.56) and (24.57) reduce to:

$$F^a = \omega^a{}_b \wedge A^b \tag{24.59}$$

$$d \wedge F^a = 0. \tag{24.60}$$

The integrated counterparts of Eqs. (24.59) and (24.60) are:

$$\Phi^a = \int_s F^a = \int_s \omega^a{}_b \wedge A^b. \tag{24.61}$$

$$\oint F^a = 0 \tag{24.62}$$

Eqs. (24.59) to (24.62) describe the class of all Aharonov Bohm effects, and more generally the class of all "non-local" effects in general relativity and quantum mechanics.

All these effects are due to spacetime geodynamics.

Objective physics, or general relativity, is needed to describe the AB effects, which are therefore evidence for the Evans field theory. In a non-objective theory of special relativity such as the Maxwell-Heaviside field theory:

$$F = d \wedge A \tag{24.63}$$

$$d \wedge F = 0 \tag{24.64}$$

or:

$$\int_s F = \oint A \tag{24.65}$$

$$\oint F = 0. \tag{24.66}$$

So if $\oint A$ is zero experimentally the magnetic flux disappears:

$$\Phi = \int_s F = 0 \tag{24.67}$$

and there are no Aharonov Bohm effects in the Maxwell Heaviside field theory. This is conclusive evidence in favor of the Evans field theory and general relativity over Maxwell Heaviside field theory and special relativity. The fundamental reason for the failure of the Maxwell Heaviside field theory is that it is a theory of Minkowski spacetime, (flat spacetime), and uses the exterior derivative (e.g. as in Eq. (24.63)). The correct description of electrodynamics uses the covariant exterior derivative in the Evans field theory. The use of the covariant derivative introduces the spin connection and provides a description of the AB effects. In the last analysis the AB effects indicate that spacetime is not the Minkowski spacetime. This inference had of course been arrived at by Einstein and Hilbert long before the Chambers experiment, but the Maxwell Heaviside theory was adhered to in trying to explain the experiment. This caused confusion for over forty years. It has been shown lately [4]–[35] that textbook explanations [2] of the AB effect in terms of gauge transformation are incorrect mathematically as well as conceptually.

The only quantity apart from differential geometry that enters into the Evans field theory is the fundamental potential $A^{(0)}$ (units of volts = $JsC^{-1}m^{-1}$). The unified theory of matter fields, gravitation, electrodynamics, and the weak and strong nuclear fields is then built up entirely from the structure equations and Bianchi identties, together with the Evans Lemma and Einstein postulate. The Evans Lemma is based directly on the tetrad postulate of differential geometry and leads to objective quantization in physics: and to all the fundamental wave and quantum equations of physics, the wavefunction being the tetrad. This is a consequence of the fact that physics must be objective and causal both on philosophical and on experimental grounds. Using the units of $A^{(0)}$ this fundamental quantity can be written as:

$$A^{(0)} = \frac{\hbar}{er_0} \tag{24.68}$$

where $\hbar$ is Planck's constant, e the proton charge and r_0 a fundamental length in metres. If this fundamental length is identified with the Compton wavelength:

$$r_0 = \lambda_0 = \frac{\hbar}{mc} \tag{24.69}$$

it follows that $A^{(0)}$ may be expressed as:

$$A^{(0)} = \frac{mc}{e} = \frac{En_0}{ec}, \tag{24.70}$$

$$En_0 = mc^2, \tag{24.71}$$

and thus:

$$A^{(0)} = \frac{mc}{e} = \frac{\hbar}{e\lambda_0}. \tag{24.72}$$

If we now identify the fundamental momentum

$$p_0 = eA^{(0)} = \hbar\kappa_0 = \hbar\omega_0 c \tag{24.73}$$

Eq. (24.72) becomes the de Broglie postulate [4]–[36]:

$$En_0 = \hbar\omega_0 = mc^2 \tag{24.74}$$

In differential geometry, Eq. (24.74) can be expressed as

$$\kappa_0 = \frac{1}{\lambda_0} = \frac{m_0 c}{\hbar} = \frac{e}{\hbar}A^{(0)} = gA^{(0)}, \tag{24.75}$$

the factor $gA^{(0)}$ entering into the analysis [4]–[36] through the fact that the covariant derivative has entered into the theory as:

$$D\wedge = d\wedge + gA\wedge . \tag{24.76}$$

In Eq. (24.76) a "barebones" notation has been used which suppresses all indices for clarity of presentation and to reveal the fundamental structure of the theory. Therefore the factor $eA^{(0)}/\hbar$ originates in the spin connection of spacetime geodynamics. Fundamentally:

$$mc = eA^{(0)} \tag{24.77}$$

and so the rest energy divided by c has been factorized into the product of two C negative quantities, e (the fundamental charge) and $A^{(0)}$ (the fundamental voltage). This procedure means that we may always write:

$$mc = eA^{(0)} = e(\frac{\hbar\kappa_0}{e}) \tag{24.78}$$

and for two signs of charge there is only one sign of mass. This inference is observed experimentally in particles and antiparticles. The latter are predicted by the Evans wave equation, which in a well defined limit reduces to the Dirac equation of special relativity. The origin of charge and voltage is the same as that of mass (i.e. rest energy), and this origin is spacetime geodynamics.

24.5 Derivation of the Heisenberg Equation of Motion and Origin of the Planck Constant

Due to the antisymmetry of the unitless structure invariant ${\theta^a}_b$ it can always be written for all base manifolds as a commutator in the tangent spacetime:

$$ {\theta^a}_b = [x^a, \kappa_b] \tag{24.79} $$

where κ_b has the units of inverse metres or wavenumber. The commutator is the basis for the Heisenberg equation of motion and therefore this equation has its origin in spacetime geodynamics, in common with all equations of physics. Using the de Broglie postulate in the form:

$$ p_b = \hbar \kappa_b \tag{24.80} $$

we obtain the Heisenberg equation directly and in an objective or generally covariant format:

$$ [x^a, p_b] = \hbar {\theta^a}_b. \tag{24.81} $$

It is seen that the structure invariants are fundamental to the Heisenberg equation. The de Broglie postulate or wave particle duality in Eq. (24.80) expresses momentum in terms of a geometric quantity κ_b. The latter is the result of the existence of the two structure invariants of differential geometry, invariants which are always related through the Bianchi identity:

$$ D \wedge T^a = {R^a}_b \wedge q^b. \tag{24.82} $$

In the particular case:

$$ D \wedge T^a = {R^a}_b \wedge q^b = 0 \tag{24.83} $$

the curvature with symmetric connection:

$$ {R^a}_b \wedge q^b = 0 \tag{24.84} $$

can always be expressed as the torsion with anti-symmetric connection:

$$ D \wedge T^a = 0. \tag{24.85} $$

In other words Eq. (24.84) is true if and only if the connection is symmetric, and Eq. (24.85) is true if and only if the connection is anti-symmetric. More generally (Eq. (24.82)) the connection must therefore be asymmetric. The Bianchi identity is therefore the geometrical reason why the Heisenberg equation is a relation between the structure invariants. In general differential geometry is at the root of everything in physics, and this inference is an obvious development to unified field theory of the fact that Riemann geometry is at the root of everything in gravitation. The particular type of Riemann geometry used in the Einstein theory of gravitation is one with a Riemann tensor and symmetric Christoffel connection. The relation between x^a and $\theta^a{}_b$ can also be made clear by integrating both sides of the Bianchi identity:

$$\int_s D \wedge T^a = \int_s R^a{}_b \wedge q^b. \tag{24.86}$$

In the special cases:

$$\int_s D \wedge T^a = D \wedge \int_s T^a \tag{24.87}$$

$$\int_s R^a{}_b \wedge q^b = \left(\int_s R^a{}_b \right) \wedge q^b \tag{24.88}$$

we obtain the following tangent spacetime equation for all base manifolds:

$$D \wedge x^a = \theta^a{}_b \wedge q^b. \tag{24.89}$$

Eq. (24.89) is similar in structure to the Heisenberg equation because both sides are commutators, and because $D\wedge$ also has the units of inverse metres.

The origin and fundamental meaning of the Planck constant may be elucidated by comparing the de Broglie duality equation (24.80) with Einstein postulate (24.15). Thereby we obtain a relation between $\hbar$ and k using:

$$R = \kappa_b \kappa^b = \frac{1}{\hbar} p_b \kappa^b = -kT, \tag{24.90}$$

a relation which implies:

$$\hbar k = -\frac{1}{T} p_b \kappa^b. \tag{24.91}$$

The origin of $\hbar$ and k is therefore essentially the same. We may write:

$$|T| = \frac{1}{k} |R| \tag{24.92}$$

and since $|R|$ is quantized by the Evans Lemma, Eq. (24.92) is a Planck relation. From the Lemma we are able to define [4]–[36] the fundamental volume:

$$V_0 = \frac{k\hbar^2}{mc^2} = \frac{k\hbar^2}{En_0} \tag{24.93}$$

and therefrom the fundamental frequency and wavenumber:

$$\omega_0 = \frac{En_0}{\hbar}. \tag{24.94}$$

It therefore follows that the Planck constant and the Einstein constant are inversely related:

$$\hbar k = V_0 \omega_0 \tag{24.95}$$

through the fundamental volume V_0 and the fundamental frequency ω_0. Expressing Eqs. (24.95) as

$$\frac{\hbar k}{c} = V_0 \kappa_0 \tag{24.96}$$

suggests the existence of a fundamental curvature (in inverse square metres):

$$R_u = \frac{c}{\hbar k} \left(= 1.52348 \times 10^{52} m^{-2}\right) \tag{24.97}$$

Using the following values of the universal constants:

$$\left.\begin{aligned} c &= 2.997925 \times 10^{8} m s^{-1} \\ k &= 1.86595 \times 10^{-26} N s^{2} kg^{-2} \\ \hbar &= 1.05459 \times 10^{-34} J s \end{aligned}\right\} \tag{24.98}$$

it is found that:

$$R_u = 1.52348 \times 10^{52} m^{-2} \tag{24.99}$$

is a universal constant. Using the Lemma:

$$\Box q^{a}{}_{\mu} = R_u q^{a}{}_{\mu} \tag{24.100}$$

it is suggested that R_u be interpreted as the initial curvature of the universe, its maximum possible curvature, associated with its minimum possible volume. This curvature and volume define the dimensions of the universe at an initial event in spacetime. This conclusion is similar to the Big Bang model, except insofar as the initial event is not a mathematical singularity. A mathematical singularity is incompatible with objective physics.

The minimum curvature of the universe is a universal constant which defines and is defined by the values of $c, \hbar$ and k observed experimentally. It should not be confused with the rest curvature of a particle [4]–[36]:

$$|R_0| = \frac{m^2 c^2}{\hbar^2} = \frac{m}{\hbar^2} E n_0. \tag{24.101}$$

The latter is the eigenvalue of the Evans Lemma in the limit when there is no interaction between particles and so when the gravitational field is infinitesimally different from zero. In this sense R_0 is a "least curvature", i.e. the curvature attained by a particle when the gravitational field is minimized. The minimum possible unit of action in this limit (the limit of special relativity) is a universal constant, the Planck constant. Action is quantized into multiples of the minimum possible action $\hbar$ because of the existence of the Evans Lemma in differential geometry, more accurately described as spacetime geodynamics. In this same limit the rest volume of the particle is defined by Eq. (24.93). Thus $\hbar$ is defined by spacetime geodynamics.

Arguing by analogy, the rest curvature of the universe is defined by that initial event in spacetime when the net gravitational field in the universe is zero: there is no net gravitational attraction or repulsion in the universe. The latter has contracted to this initial volume and curvature because of gravitational attraction, but the equilibrium thus attained is an unstable equilibrium, so expansion occurs once more as gravitational repulsion builds up. The eigenvalue of the oscillatory universe is the tetrad and the process is a dynamical one, it is governed by the Evans Lemma and wave equation of the universe.

The following terms are therefore introduced to distinguish these two types of fundamental curvature, the universal curvature, Eq. (24.97), and the rest curvature, defined in terms of the particle rest energy by:

$$R_0 = -\frac{m}{\hbar^2} E n_0. \tag{24.102}$$

The standard form of the Heisenberg equation is obtained if the following definition is used:

$$\theta^a{}_b = \frac{i}{\hbar} J^a{}_b, \tag{24.103}$$

a definition which implies:

$$[x^a, \kappa_b] = \frac{i}{\hbar} J^a{}_b. \tag{24.104}$$

Using the de Broglie wave particle duality:

$$[x^a, \hbar\kappa_b] = [x^a, p_b] = \frac{i}{\hbar} J^a{}_b, \tag{24.105}$$

we obtain the well known position / momentum form of the Heisenberg equation:

$$[x^a, p_b] = \frac{i}{\hbar} J^a{}_b. \tag{24.106}$$

The units of $J^a{}_b$ are those of action or angular momentum. The least action principle of Hamilton asserts that the classical action is minimized in the universe. The last possible action in the universe is $\hbar$, so:

$$[x^a, p_b]_{\min} = i\hbar\epsilon^a{}_b \tag{24.107}$$

where $\epsilon^a{}_b$ is the two dimensional anti-symmetric unit tensor of the tangent spacetime for all base manifolds. The fundamental reason why $\hbar$ occurs in quanta is the equation governing the evolution of the tetrad [4]–[36]:

$$q^a{}_\mu(t, \boldsymbol{r}) = \exp\left(i\frac{S}{\hbar}\right) q^a{}_\mu(0, \mathbf{0}) \tag{24.108}$$

The tetrad is quantized, so x^a, p_b and $\theta^a{}_b$ are also quantized, being always related to the tetrad through an equation such as (24.89) of spacetime geodynamics.

Finally, the origin and meaning of mass may be elucidated using the proportionality:

$$|R_0| = \xi |R_u|, \tag{24.109}$$

to obtain an identity of spacetime geodynamics:

$$m^2 = \frac{\hbar}{ck} \frac{|R_0|}{|R_u|}. \tag{24.110}$$

It is concluded that the square of the mass of any particle is the ratio of two fundamental curvatures within the universal constant $\hbar/(ck)$. The fundamental reason for the occurrence of particles is the Evans least curvature principle [4]–[36], one of whose ramifications is Eq. (24.108). Charge e is then the factorization of mass as discussed in Eq. (24.78), which shows that there are two signs of charge for one sign of mass.

Acknowledgements

The Ted Annis Foundation and Craddock Inc. are thanked for funding and the Fellows and Emeriti of AIAS for many interesting discussions.

References

1. E. P. Wigner, Ann. Math., **40**, 149 (1939).
2. L. H. Ryder, *Quantum Field Theory*, (Cambridge Univ Press, 1996, 2nd ed.).
3. S. P. Carroll, *Lecture Notes in General Relativity*, (a graduate course at Harvard, UC Santa Barbara and U Chicago, arXiv: gr-gq / 973019 v1 Dec 1997, public domain).
4. M. W. Evans, A Unified Field Theory for Gravitation and Electromagnetism. *Found. Phys. Lett.*, **16**, 367 (2003).
5. M. W. Evans, A Generally Covariant Wave Equation for Grand Unified Field Theory, Found. Phys. Lett, **16**, 507 (2003).
6. M. W. Evans, The Equations of Grand Unified Field Theory in terms of the Maurer Cartan Structure Relations of Differential Geometry, *Found. Phys. Lett.*, **17**, 25 (2003).
7. M. W. Evans, Derivation of Dirac's Equation from the Evans Wave Equation, *Found. Phys. Lett.*, **17**, 149 (2004).
8. M. W. Evans, Unification of the Gravitational and Strong Nuclear Fields, *Found. Phys. Lett.*, **17**, 267 (2004).
9. M. W. Evans, The Evans Lemma of Differential Geometry, *Found. Phys. Lett.*, **17**, 433 (2004).
10. M. W. Evans, Derivation of the Evans Wave Equation from the Lagrangian and Action, Origin of the Planck Constant in General Relativity, *Found. Phys. Lett.*, **17**, 535 (2004).
11. M. W. Evans et alii (AIAS Author Group), Development of the Evans Wave Equation in the Weak Field Limit: the Electrogravitic Equation, *Found. Phys. Lett.*, **17**, 497 (2004).
12. M. W. Evans, Physical Optics, the Sagnac Effect and the Aharonov Bohm Effect in the Evans Unified Field Theory, *Found. Phys. Lett.*, **17**, 301 (2004).
13. M. W. Evans, Derivation of the Geometrical Phase from the Evans Phase Law of Generally Covariant Unified Field Theory, *Found. Phys. Lett.*, **17**, 393 (2004).
14. M. W. Evans, Derivation of the Lorentz Boost from the Evans Wave Equation, *Found. Phys. Lett.*, **17**, 663 (2004).
15. M. W. Evans, The Electromagnetic Sector of the Evans Field Theory, *Found. Phys. Lett.*, **18**, 37 (2005), preprint on www.aias.us.
16. M. W. Evans, New Concepts from the Evans Field Theory Part One: The Evolution of Curvature, Oscillatory Universe without Singularity, and Gen-

eral force and Field Equations, *Found. Phys. Lett.*, in press (2005); preprint on www.aias.us.

17. M. W. Evans, New Concepts from the Evans Field Theory Part Two: Derivation of the Heisenberg Equation and Reinterpretation of the Heisenberg Uncertainty Principle, *Found. Phys. Lett.*, in press (2005), preprint on www.aias.us.
18. M. W. Evans, Derivation of O(3) Electrodynamics from Generally Covariant Unified Field Theory, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
19. M. W. Evans, The Spinning and Curving of Spacetime, The Electromagnetic and Gravitational Fields in the Evans Unified Field Theory, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
20. M. W. Evans, Derivation of O(3) Electrodynamics from the Evans Unified Field Theory, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
21. M. W. Evans, Calculation of the Anomalous Magnetic Moment of the Electron, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
22. M. W. Evans, Generally Covariant Electro-weak Theory, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
23. M. W. Evans, Evans field Theory of Neutrino Oscillations, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
24. M. W. Evans, The Interaction of Gravitation and Electromagnetism, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
25. M. W. Evans, Electromagnetic Energy from Gravitation, *Found. Phys. Lett.*, submitted, preprint on www.aias.us.
26. M. W. Evans, *Generally Covariant Unified Field Theory, the Geometrization of Physics*, (in press, 2005).
27. L. Felker, *The Evans Equations* (freshman text, in prep).
28. M. W. Evans and L. B. Crowell, *Classical and Quantum Electrodynamics and the* $B^{(3)}$ *Field* (World Scientific, Singapore, 2001).
29. M. W. Evans (ed.), *Modern Non-Linear Optics*, a special topical issue in three parts of I Prigogine and S. A. Rice, (series eds.), *Advances in Chemical Physics* (Wiley-Interscience, New York, 2001, 2nd ed., hardback and e book) vols. 119(1), 119(2) and 119(3).
30. M. W. Evans and S. Kielich (eds.), *Modern Non-Linear Optics*, a special topical issue in three parts of I. Prigogine and S. A. Rice (series eds.), *Advances in Chemical Physics* (Wiley-Interscience, New York, 1992, 1993, 1997, hardback and softback) vols. 85(1), 85(2) and 85(3).
31. M. W. Evans, J.-P. Vigier et alii, *The Enigmatic Photon* (van der Merwe Series, Kluwer, Dordecht, 1994 to 2002, hardback and softback) in five volumes.
32. M. W. Evans and A. A. Hasanein, *The Photomagneton in Quantum Field Theory*, (World Scientific, Singapore, 1994).
33. M. W. Evans, *The Photon's Magnetic Field: Optical NMR Spectroscopy* (World Scientific, Singapore, 1992).
34. M. W. Evans, Physica B, **182**, 227 and 237 (first scientific papers on the $B^{(3)}$ field).
35. M. W. Evans, Collected Scientific Papers website, Artspeed, California, in prep., a collection of over six hundred and fifty scientific papers.
36. over one million two hundred thousand page views of www.aias.us and aias.rfsafe.com, Jan-Sept 2004.