THE DESCRIPTION OF PRECESSING ELLIPTICAL ORBITS WITH THE HOOKE / NEWTON INVERSE SQUARE LAW.

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ABSTRACT

It is demonstrated straightforwardly that a precessing elliptical orbit can be described with a Hooke / Newton inverse square force law provided that the plane polar coordinate system is rotating. The rotation generates a Christoffel connection. This is the simplest way to describe the observed planetary orbit and is preferred by Ockham's Razor. Some graphical representations of the theory are given.

Keywords: ECE theory, rotating coordinate system, planetary orbits.

UFT 214

1. INTRODUCTION

Planetary orbits were first rationalized by Kepler in terms of three laws which were based on the observations of Brahe. Kepler found that the orbit of the planet Mars was an ellipse, which was later found to be precessing. For a static elliptical orbit the Kepler laws can be explained in terms of an inverse square force law as is well known. The precessing elliptical orbit has been considered hitherto as a non Newtonian problem, in that precession cannot be explained with an inverse square law in a frame of reference described by the plane polar coordinates {1 - 12}. Einstein later claimed that his theory of general relativity can explain the precession of the perihelion. This claim was rejected by Schwarzschild almost as soon as it was made, and has been rejected since then. In recent papers of this series the Einsteinian general relativity (EGR) has been refuted logically in several ways, some of them very simple as in UFT202 (www.aias.us). So it is considered that EGR is an obsolete theory that was always obscure.

In Section 2 it is shown by elementary methods that the precessing ellipse can be described by an inverse square law in a rotating coordinate system (r, β) in which is defined by the product x θ . Here θ is the angular coordinate of the plane polar system (r, θ) and x is the precession constant. The rotation of the coordinate system is shown to be described by a Christoffel connection that is antisymmetric in its lower two indices. When the standard plane polar system (r , θ) is used the force law needed to describe a precessing elliptical orbit is the sum of an inverse square and inverse cube law. Therefore the precessing elliptical orbit of a planet can be described entirely without Einsteinian general relativity (EGR). Not only do we reach this conclusion, but it is also known that EGR produces an incorrect force law, a sum of an inverse square and inverse fourth power term in the radial coordinate r. These are very good reasons for abandoning EGR.

2. FORCE LAWS FOR A PRECESSING ELLIPTICAL ORBIT.

The precessing elliptical orbit is defined by:

$$r = \frac{\alpha}{1 + \epsilon \cos \beta} \qquad -(1)$$

where

$$\beta = x\theta . \qquad -(2)$$

Here x is the precession constant, 2α is the right magnitude and ϵ is the ellipticity.

Adopt the rotating plane polar coordinate system (r, β). This is the optimal choice of coordinates by Ockham's Razor, in that it rationalizes the precessing elliptical orbit in the simplest way. Consider the lagrangian:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\beta}^2 \right) - U(r) \qquad -(3)$$

where U is the potential energy. The Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} = 0, \qquad -(4)$$

and

$$\frac{\partial \mathcal{L}}{\partial \beta} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\beta}} = 0. \qquad -(5)$$

The total angular momentum is conserved so:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\beta}} = m r^2 \dot{\beta} = \text{constant} \qquad -(6)$$

and:

$$\frac{dL}{dt} = 0. \qquad -(7)$$

From Eqs. (3) and (4):

$$m\left(\ddot{r} - r\dot{\beta}^2\right) = -\frac{\partial U}{\partial r} = F(r) \quad -(8)$$

where F is the force. Denote:

$$u = \frac{1}{r} \quad -(9)$$

then:

$$\frac{du}{d\beta} = -\frac{1}{r^2}\frac{dr}{d\beta} = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\beta}. \quad -(10)$$

From Eq. (6):

$$\dot{\beta} = \frac{L}{mr^2} \quad -(11)$$

so:

$$\frac{du}{d\beta} = -\frac{m}{L}\dot{r}. \quad -(12)$$

Therefore:

$$\frac{d^2u}{d\beta^2} = \frac{d}{d\beta}\left(-\frac{m}{L}\dot{r}\right) = \frac{dt}{d\beta}\frac{d}{dt}\left(-\frac{m\dot{r}}{L}\right) \quad -(13)$$

$$= -\frac{m\ddot{r}}{L\dot{\beta}} = -\frac{m^2}{L^2}r^2\ddot{r}$$

and:

$$\ddot{r} = -\frac{L^2}{m^2}u^2\frac{d^2u}{d\beta^2}, \quad r\dot{\beta}^2 = \frac{L^2}{m^2}u^3. \quad -(14)$$

Therefore:

$$\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{m}{L^2}r^2F(r) \quad -(15)$$

where:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos\beta\right). \quad -(16)$$

Therefore:

$$\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\cos\beta \quad -(17)$$

and

$$\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} = -\frac{m}{L^2} r^2 F(r) \quad -(18)$$

and

$$F(r) = -\frac{L^2}{m\alpha^2 r^2}. \quad -(19)$$

If:

$$\alpha = \frac{L^2}{mk} \quad -(20)$$

then

$$F(r) = -\frac{k}{r^2} \quad -(21)$$

where

$$k = mMG. \quad -(22)$$

The force is a Hooke / Newton inverse square law. Therefore in the rotating frame a precessing elliptical orbit can be described without EGR at all.

This result means that the hamiltonian in the rotating frame is

$$H = E = \frac{1}{2}mv^2 + U(r) \quad -(23)$$

where the total linear velocity is:

$$v = \left(\dot{r}^2 + r^2 \dot{\beta}^2 \right)^{1/2} \quad -(24)$$

Therefore:

$$\dot{r} = \frac{dr}{dt} = \left(\frac{2}{m} \left(E - U \right) - \frac{L^2}{m^2 r^2} \right)^{1/2} \quad -(25)$$

Using:

$$\frac{d\beta}{dr} = \frac{d\beta}{dt} \frac{dt}{dr} \quad -(26)$$

it is found that:

$$\beta(r) = \int \left(r^2 \left(2m \left(E + \frac{k}{r} - \frac{L^2}{2mr^2} \right) \right) \right)^{-1/2} dr \quad -(27)$$

whose solution is:

$$\cos\beta = \left(\frac{L^2}{2mk} - 1 \right) \left(1 + \frac{2EL^2}{mk^2} \right)^{-1/2} \quad -(28)$$

Eqs. (1) and (28) are the same provided that:

$$\alpha = \frac{L^2}{mk}, \quad \epsilon = \left(1 + \frac{2EL^2}{mk^2} \right)^{1/2} \quad -(29)$$

Therefore the hamiltonian (23) describes a precessing elliptical orbit without EGR, QED.

If Eq. (15) is used with the static coordinate system (r, θ) then:

$$\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} = - \frac{mr^2}{L^2} F(r) \quad -(30)$$

where:

$$\frac{1}{r} = \frac{1}{\alpha} \left(1 + \epsilon \cos(x\theta) \right) \quad -(31)$$

So:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon x^2}{\alpha}\cos(x\theta) \quad -(32)$$

and

$$\frac{1}{\alpha}\left(1+\epsilon\left(1-x^2\right)\cos(x\theta)\right) = -\frac{mr^2}{L^2}F(r) \quad -(33)$$

giving a force law:

$$F(r) = -\frac{k}{r^2}\left(x^2+\left(1-x^2\right)\frac{\alpha}{r}\right) \quad -(34)$$

which is a sum of inverse square and inverse cubed terms. The EGR theory produces an incorrect sum of terms as shown in UFT196 on www.aias.us.

The precise meaning of the rotating coordinate system is deduced by considering the basic equations:

$$X = r\cos\beta, \; Y = r\sin\beta. \quad -(35)$$

Consider:

$$\theta \rightarrow \theta + 2\pi \quad -(36)$$

then:

$$\beta \rightarrow \beta + 2\pi x \quad -(37)$$

and:

$$X_1 = r\cos(\beta+2\pi x) = r\left(\cos\beta\cos(2\pi x) - \sin\beta\sin(2\pi x)\right)$$

$$Y_1 = r\sin(\beta+2\pi x) = r\left(\sin\beta\cos(2\pi x) + \cos\beta\sin(2\pi x)\right)$$

$$-(38)$$

in which:

$$r^2 = X^2 + Y^2 = X_1^2 + Y_1^2 \quad -(39)$$

It is seen that the coordinate system does not return to the same point after a rotation through 2π radians. The coordinate system and frame of reference rotates and there is a Christoffel connection present in the geometry. The unit vectors of the system are:

$$\underline{e}_r = \underline{i}\cos\beta + \underline{j}\sin\beta \quad -(40)$$

$$\underline{e}_\theta = -\underline{i}\sin\beta + \underline{j}\cos\beta \quad -(41)$$

so:

$$\underline{e}_\beta = \frac{d\underline{e}_r}{d\beta}, \quad \underline{e}_r = -\frac{d\underline{e}_\beta}{d\beta}. \quad -(42)$$

It follows that:

$$d\underline{e}_r = d\beta\,\underline{e}_\beta \quad -(43)$$

$$d\underline{e}_\beta = -d\beta\,\underline{e}_r \quad -(44)$$

and:

$$\dot{\underline{e}}_r = d\underline{e}_r / dt = \dot{\beta}\,\underline{e}_\beta \quad -(45)$$

$$\dot{\underline{e}}_\beta = d\underline{e}_\beta / dt = -\dot{\beta}\,\underline{e}_r. \quad -(46)$$

The linear velocity is:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{d}{dt}(r\underline{e}_r) = \dot{r}\underline{e}_r + r\dot{\underline{e}}_r = \dot{r}\underline{e}_r + r\dot{\beta}\underline{e}_\beta \quad -(47)$$

a result which leads to the lagrangian (3). From the equation of a precessing ellipse, eq. (1):

$$\frac{dr}{d\beta} = \frac{\epsilon}{\alpha} r^2 \sin\beta \quad -(48)$$

so:

$$\frac{dr}{dt} = \frac{dr}{d\beta}\frac{d\beta}{dt} = \left(\frac{L\epsilon}{md}\right)\sin\beta \quad -(49)$$

and

$$\dot{r} = \left(\frac{L\epsilon}{md}\right)\sin\beta, \quad \dot{\beta} = \frac{L}{mr^2}. \quad -(50)$$

As in UFT196 the acceleration is:

$$\underline{a} = \frac{d\underline{v}}{dr} = (\ddot{r} - r\dot{\beta}^2)\underline{e}_r + (r\ddot{\beta} + 2\dot{r}\dot{\beta})\underline{e}_\beta. \quad -(51)$$

Therefore:

$$\ddot{r} = \left(\frac{L\epsilon}{md}\right)\frac{d}{dt}(\sin\beta), \quad \ddot{\beta} = \frac{L}{m}\frac{d}{dt}\left(\frac{1}{r^2}\right). \quad -(52)$$

Now use:

$$\frac{df(r)}{d\beta} = \frac{df(r)}{dr}\frac{dr}{d\beta}, \quad -(53)$$

$$\frac{df(\beta)}{dr} = \frac{df(\beta)}{d\beta}\frac{d\beta}{dr}, \quad -(54)$$

to find that:

$$\frac{d}{dt}\left(\frac{1}{r^2}\right) = -\frac{2}{r^3}\frac{dr}{dt}, \quad \frac{d}{dt}(\sin\beta) = \cos\beta\frac{d\beta}{dt}. \quad -(55)$$

Therefore:

$$\ddot{r} = \left(\frac{\epsilon L^2}{m^2 d}\right)\frac{1}{r^2}\cos\beta, \quad \ddot{\beta} = -\left(\frac{2L^2\epsilon}{m^2 d}\right)\frac{\sin\beta}{r^3}. \quad -(56)$$

It follows that:

$$\ddot{r} - r\dot{\beta}^2 = \frac{\epsilon MG}{r^2}\cos\beta - \frac{L^2}{m^2 r^3} \quad -(57)$$

Similarly:

$$r\ddot{\beta} + 2\dot{r}\dot{\beta} = 0 \quad -(58)$$

so the force is:

$$\underline{F} = m\left(\frac{\epsilon MG}{r^2}\cos\beta - \frac{L^2}{m^2 r^3}\right)\underline{e}_r \quad -(59)$$

Finally use:

$$\epsilon\cos\beta = \frac{\alpha}{r} - 1 \quad -(60)$$

to find that:

$$\underline{F} = \left(-\frac{mMG}{r^2} + \frac{L^2}{mr^3} - \frac{L^2}{mr^3}\right)\underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r \quad -(61)$$

which is Eq. (19), QED. The centrifugal forces cancel as discussed in UFT196.

In the rotating frame the passive rotation of axes is described by a development of UFT199 as:

$$\begin{bmatrix} \underline{i}' \\ \underline{j}' \end{bmatrix} = \begin{bmatrix} \cos(x\theta) & -\sin(x\theta) \\ \sin(x\theta) & \cos(x\theta) \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix} \quad -(62)$$

Denote:

$$R_Z = \begin{bmatrix} \cos(x\theta) & -\sin(x\theta) \\ \sin(x\theta) & \cos(x\theta) \end{bmatrix} \quad -(63)$$

then:

$$\left.\frac{dR_z}{d\theta}\right|_{\theta=0} = x\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad -(64)$$

and the rotation generator is:

$$J_{ij} = -i x \epsilon_{ij} . \quad -(65)$$

The totally antisymmetric unit tensor is:

$$\epsilon_{ijk} = \epsilon_{ij}\epsilon_k \quad -(66)$$

so

$$\epsilon_{ij} = \epsilon^k_{\ ij}\,\epsilon_k \quad -(67)$$

and the antisymmetric Christoffel connection is:

$$\Gamma^k_{\ ij} = \frac{x}{r}\epsilon^k_{\ ij} \quad -(68)$$

using a development of the method in UFT212. The precessing elliptical orbit can be described by the Christoffel connection (68), and this is an elegant expression of the philosophy of relativity. It should replace the obsolete EGR by Ockham's Razor.

3. GRAPHICAL AND OTHER ANALYSIS OF THE RESULTS OF SECTION 2.

(Section by H. Eckardt, R. Delaforce and R. Cheshire)

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3 Graphical and other analysis of the results of section 2

First we show that the orbits of Einstein General Relativity are different from those of the observed kind. Define the radius function $r(\theta)$ of a precessing ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)}. \tag{69}$$

Rewriting of this eqation gives

$$\cos(x\theta) = \frac{\alpha}{\epsilon r} - \frac{1}{\epsilon} \tag{70}$$

which, with aid of

$$\sin(x\theta)^2 + \cos(x\theta)^2 = 1, \tag{71}$$

can be transformed into

$$\sin(x\theta)^2 = \frac{2\alpha}{\epsilon^2 r} - \frac{\alpha^2}{\epsilon^2 r^2} - \frac{1}{\epsilon^2} + 1. \tag{72}$$

This expression has to be eqated with the result of EGR for the $\sin(x\theta)^2$ term:

$$\epsilon^2 x^2 \left(\frac{2\alpha}{\epsilon^2 r} - \frac{\alpha^2}{\epsilon^2 r^2} - \frac{1}{\epsilon^2} + 1 \right) = \alpha^2 \left(\frac{r_0}{a^2 r} + \frac{r_0}{r^3} - \frac{1}{r^2} + \frac{1}{b^2} - \frac{1}{a^2} \right). \tag{73}$$

It is seen that the left hand side is a polynom of maximum degree $1/r^2$ while the right hand side is one of maximum degree $1/r^3$. Both sides can never be equal

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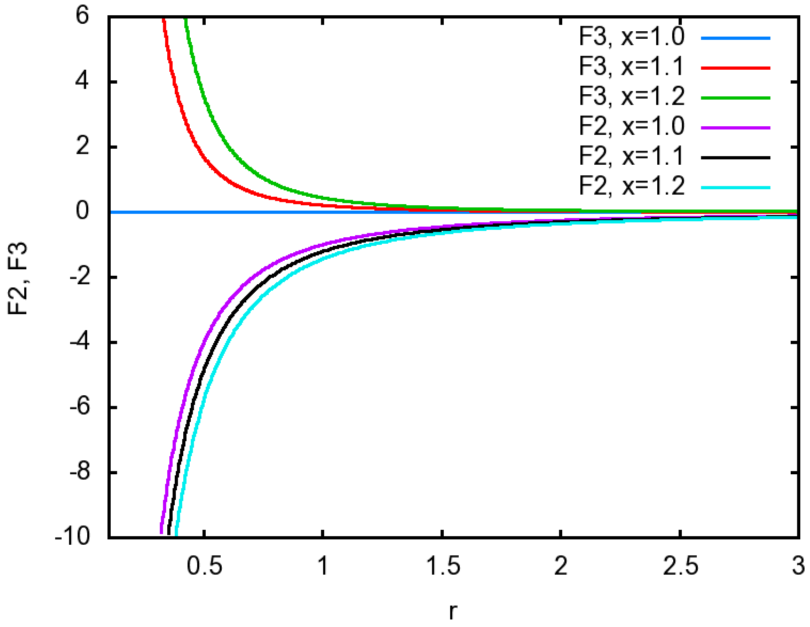

Figure 1: Force components $F2$ and $F3$ for different values of $x \geq 1$, with parameters $k = 1, \alpha = 1$.

for a certain range of r. However the variable r must be able to vary because it describes an ellipse - reduction ad absurdum.

Next we show some examples of force laws for dependencies of $1/r^2$, $1/r^3$ and their combinations. We define

$$F2 = -\frac{k\,x^2}{r^2}, \tag{74}$$

$$F3 = -\frac{\alpha\,k\,\left(1 - x^2\right)}{r^3}. \tag{75}$$

Both forces are shown in Fig. 1 for several values of $x \geq 1$. Observe that $F3$ is positive. The sum of both is graphed in Fig. 2. The positive contribution effects a minimum in the total force, similar to potentials of atomic orbitals.

The equivalents of Figs. 1 and 2 for parameter values $x \leq 1$ are graphed in Figs. 3 and 4. Here all force components are negative, leading to a lowering of the total force.

Finally we show particular forms of normal and precessing ellipses. Fig. 5 presents ellipses for several ellipticities ϵ. In Fig. 6 the region of common crossing points has been enlarged. Precessing ellipses have been plotted in Figs. 7-10. The factor x describes the "multiplicity" of the elllipse in case of integer values. For non-integral values, the ellipses precess around the near integers.

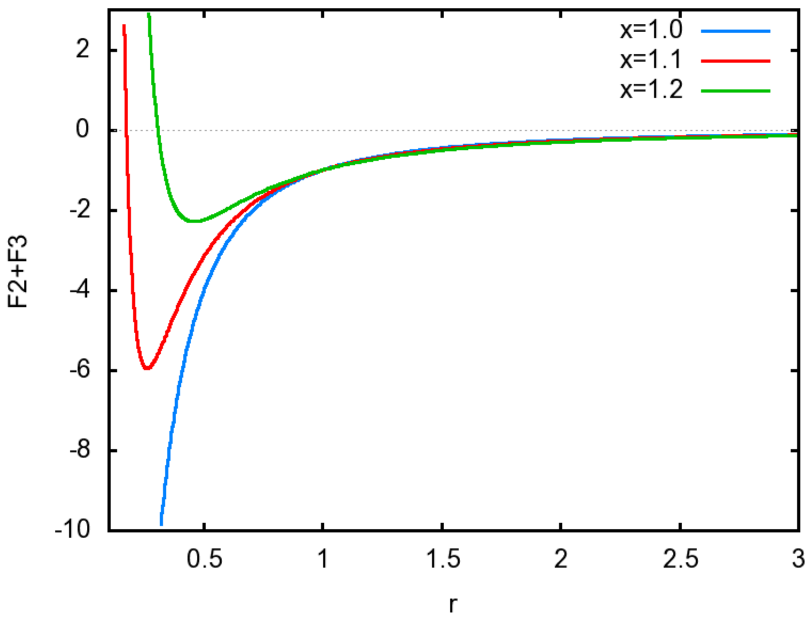

Figure 2: Sum of force components $F1 + F2$ of Fig.1.

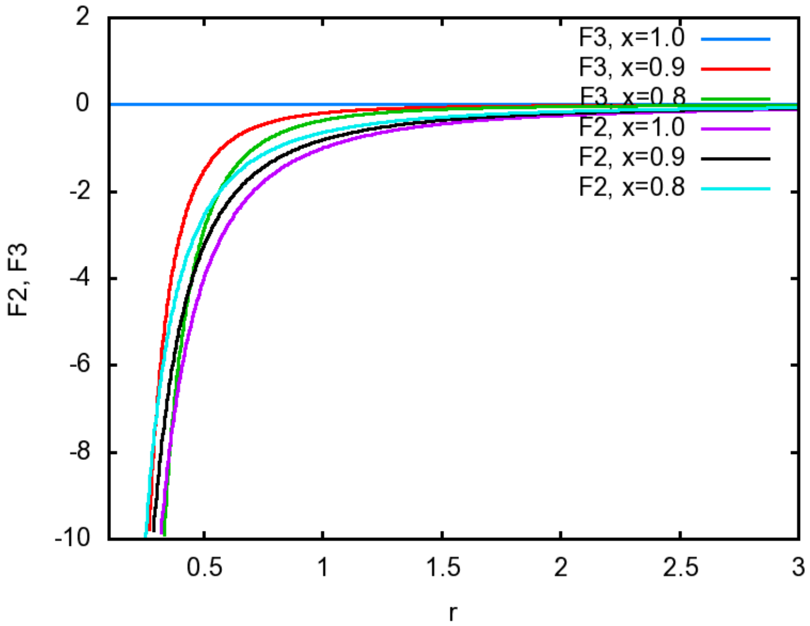

Figure 3: Force components $F2$ and $F3$ for different values of $x \leq 1$, with parameters $k = 1, \alpha = 1$.

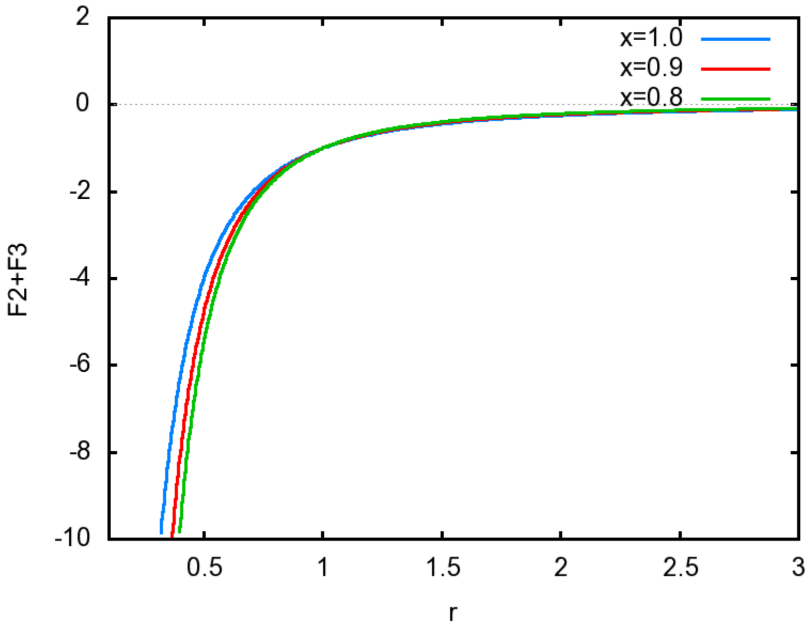

Figure 4: Sum of force components $F1 + F2$ of Fig.3.

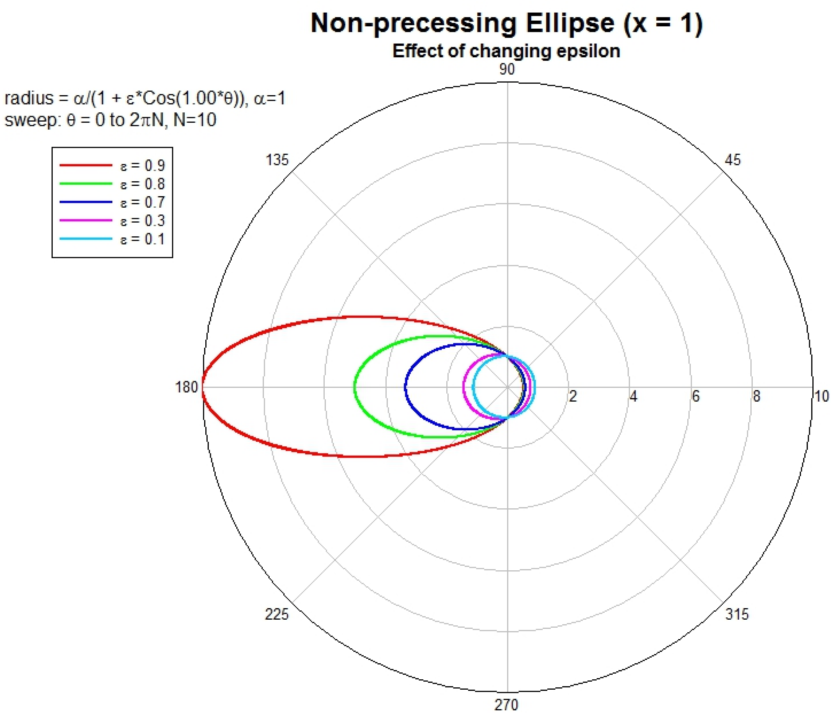

Figure 5: Ellipses for several values of ϵ.

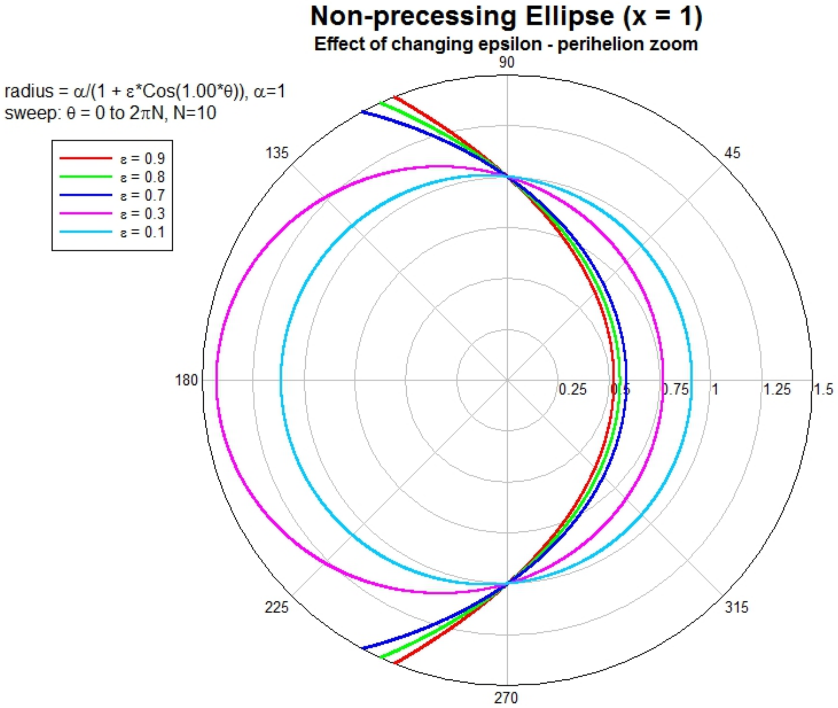

Figure 6: Enlarged view of Fig. 5.

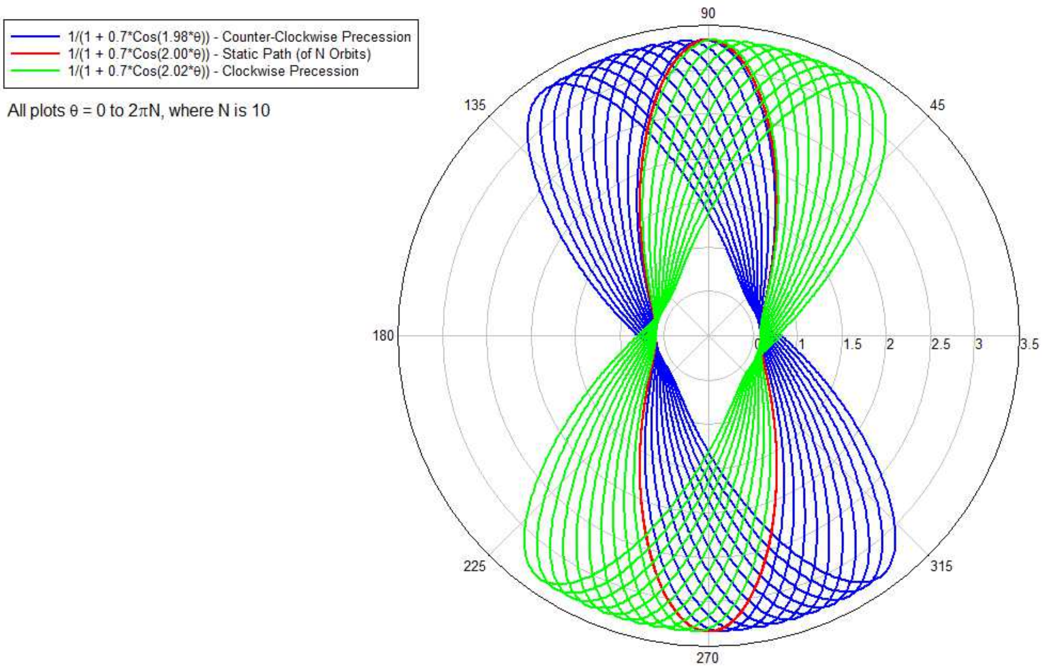

Figure 7: Ellipses with multiplicity 2.

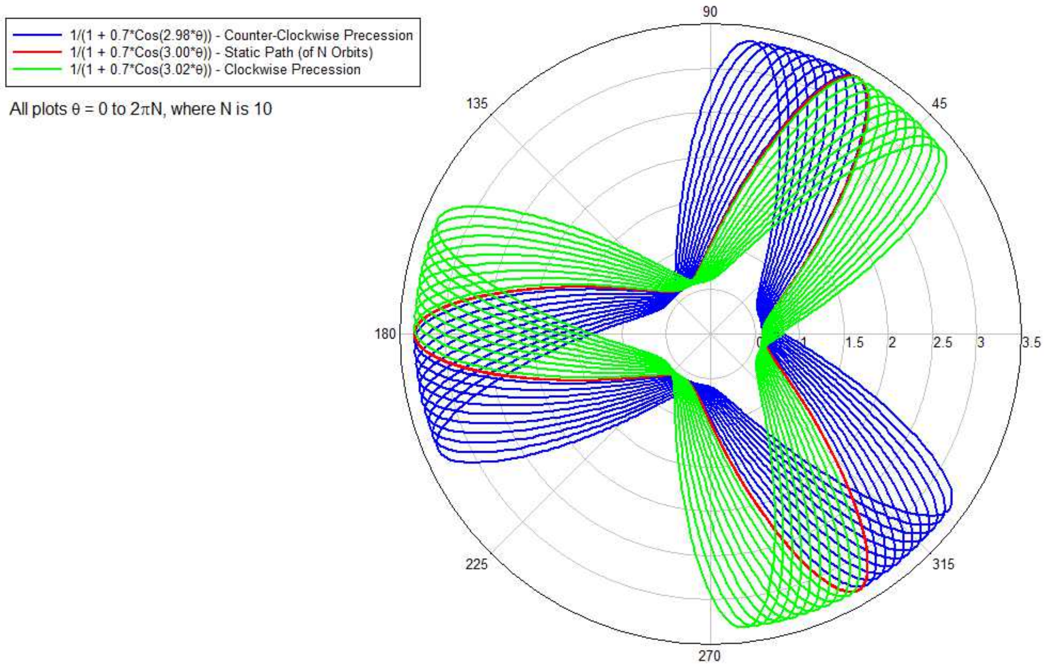

Figure 8: Ellipses with multiplicity 3.

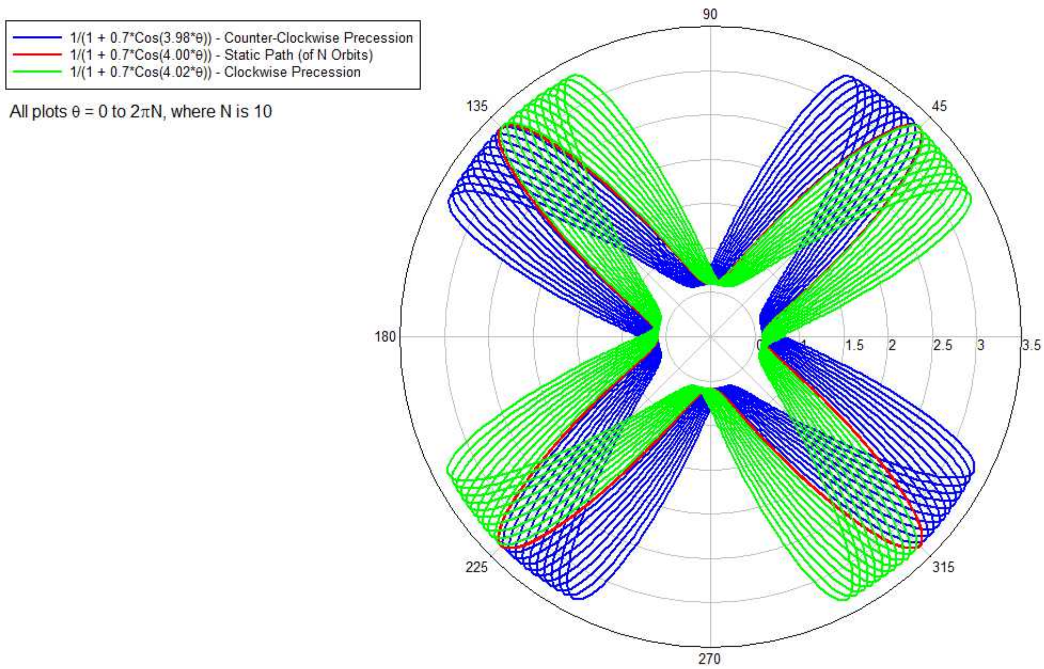

Figure 9: Ellipses with multiplicity 4.

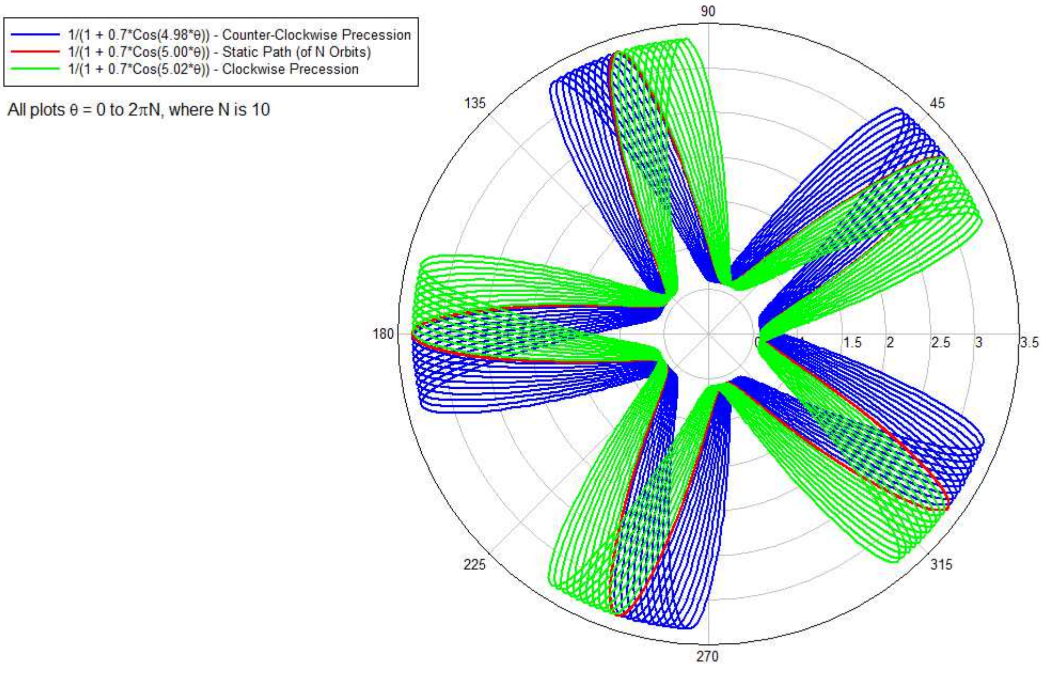

Figure 10: Ellipses with multiplicity 5.

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