Solution of the ECE Metric Equations for the Infinite Solenoid

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Abstract

Recently, the structure of spacetime was incorporated into the ECE equations of electromagnetism. In this paper, the field equations are shown to possess nonlinear terms that appear when the metric coefficients depend on the value of the magnetic field. This is investigated for the case of an infinite solenoid. With reasonable simplifications, the non-linear hyperbolic partial differential equation is shown to reduce to a distorted wave equation which offers solutions that show amplification effects. Heterodyning behaviour was observed, corresponding to the resonance for the first standing wave in the solenoid core. At higher frequencies, the magnetic field becomes asymptotically linear in time, indicating some form of resonant growth.

Keywords: ECE theory, Maxwell-Heaviside Equations, electromagnetism

Introduction

In previous publications [1,2,3], it was shown that the standard ECE electromagnetic theory for a single polarization is equivalent in a mathematical sense to traditional electromagnetic theory whenever the vector potential is a continuous function of time. We note that this equivalence is superficial because the ECE theory of electromagnetism is not restricted to the Minkowski metric, the basis of Maxwellian theory. We noted also that this continuous state is very stable and that once a system is there, it takes a vector potential that is not continuous in time to jar it from this stability.

This suggests that devices such as the energy savings devices developed in Mexico [4] are not described by the original ECE theory of electromagnetism because observational data suggests that the potentials in the device are continuous in time. Other devices, such as the Bedini

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machine [5] and toroidal power devices [6] may have explanation in the “first generation” ECE theory if they can actually achieve a state where the vector potential is not continuous. The practicality of achieving this theoretical state remains to be demonstrated.

A “second generation” ECE theory of electromagnetism has been introduced [7] where a connection of the field variables was made with the structure of the metric. It will be shown in this paper that this connection offers an explanation for over-unity energetics in “more or less” traditional electronic and electromechanical devices. A design strategy is presented based upon the analysis of an infinitely long solenoid where the metric properties depend upon the applied fields.

The Field Equations

The basic premise of this paper is that if anomalous behaviour is to be observable in a device, it should show up as an anomaly in the electric and/or magnetic fields. This allows the use of the field equations without the introduction of potentials and spin connections as described in ECE theory [7], except for boundary conditions that require potential sources.

Paper [7] provides the basis for introducing the metric into the ECE electromagnetic equations. In that paper, the metric has been assumed to be diagonal, with the components of the electric displacement $\boldsymbol{D}$ given by

$$D^1 = \varepsilon_0 g^{00} g^{11} E^1 \,,$$

$$D^2 = \varepsilon_0 g^{00} g^{22} E^2 \,, \qquad (1)$$

$$D^3 = \varepsilon_0 g^{00} g^{33} E^3 \,.$$

In vector form, this is

$$\boldsymbol{D} = \overline{\overline{\boldsymbol{G}_{\boldsymbol{E}}}} \cdot \boldsymbol{E} \qquad (2)$$

where $\boldsymbol{E}$ is the electric intensity, $\boldsymbol{D}$ is the electric displacement, and $\overline{\overline{\boldsymbol{G}_{\boldsymbol{E}}}}$ is related to the metric tensor through

$$G_E^{ii} = \varepsilon_0 g^{00} \, g^{ii}. \qquad (3)$$

For notational purposes, we write

$$\boldsymbol{D} = (D^1, D^2, D^3)^T \ , \tag{4}$$

$$\boldsymbol{E} = (E^1, E^2, E^3)^T \ . \tag{5}$$

Similarly for the magnetic intensity $\boldsymbol{H}$ and the magnetic induction $\boldsymbol{B}$ [7],

$$H^1 = \frac{1}{\mu_0} g^{22} g^{33} B^1 \ ,$$

$$H^2 = \frac{1}{\mu_0} g^{11} g^{33} B^2 \ , \tag{6}$$

$$H^3 = \frac{1}{\mu_0} g^{11} g^{22} B^3$$

can be rewritten as

$$\boldsymbol{H} = \overline{\overline{\boldsymbol{G}_{\boldsymbol{H}}}} \cdot \boldsymbol{B} \tag{7}$$

where $\overline{\overline{\boldsymbol{G}_{\boldsymbol{H}}}}$ is related to the metric through

$$G_H^{ii} = \frac{g^{jj} g^{kk}}{\mu_0} \tag{8}$$

and

$$\boldsymbol{H} = (H^1, H^2, H^3)^T \ , \tag{9}$$

$$\boldsymbol{B} = (B^1, B^2, B^3)^T \ . \tag{10}$$

For simplicity, we will ignore additional constant polarization and magnetization effects that are often added for completeness in traditional electromagnetism [8].

The homogeneous field equations of ECE theory are

$$\underline{\nabla} \cdot \boldsymbol{B} = 0 \ , \tag{11}$$

$$\underline{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \ . \tag{12}$$

The inhomogeneous equations of the ECE electromagnetic theory are given by

$$\underline{\nabla} \cdot \boldsymbol{D} = \rho \ , \tag{13}$$

$$\underline{\nabla} \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J} \ . \tag{14}$$

Using equation (2) and equation (7), equations (13) and (14) can be rewritten as

$$\underline{\nabla} \cdot \left(\overline{\overline{\boldsymbol{G}_{\boldsymbol{E}}}} \cdot \boldsymbol{E} \right) = \rho \ , \tag{15}$$

$$\underline{\nabla} \times \left(\overline{\overline{\boldsymbol{G}_{\boldsymbol{H}}}} \ \cdot \boldsymbol{B} \right) - \overline{\overline{\boldsymbol{G}_{\boldsymbol{E}}}} \cdot \frac{\partial \boldsymbol{E}}{\partial t} - \frac{\partial \overline{\overline{\boldsymbol{G}_{\boldsymbol{E}}}}}{\partial t} \cdot \boldsymbol{E} = \boldsymbol{J} \ . \tag{16}$$

In this paper, we shall assume that diagonal elements of the metric tensors vary in an amount proportional to the work done by the electromagnetic field, which when the electric field is negligible in comparison to the magnetic field, is

$$G_H^{ii} = \frac{1}{\mu_c}(1 + \mu_m \boldsymbol{B} \cdot \boldsymbol{B}), \tag{17}$$

$$G_E^{ii} = \varepsilon_c (1 + \varepsilon_m \boldsymbol{B} \cdot \boldsymbol{B}).$$

μ_c and ε_c are the permeability and permittivity of the material in the absence of electromagnetic fields, and the terms μ_m and ε_m are constants yet to be determined.

Infinite Solenoid

Let us now consider the case of an infinitely long solenoid. The solenoid has cross-sectional geometry as illustrated in Figure 1; the outer region consists of the windings of the solenoid of thickness h, and the inner region is the core with outer radius r_0.

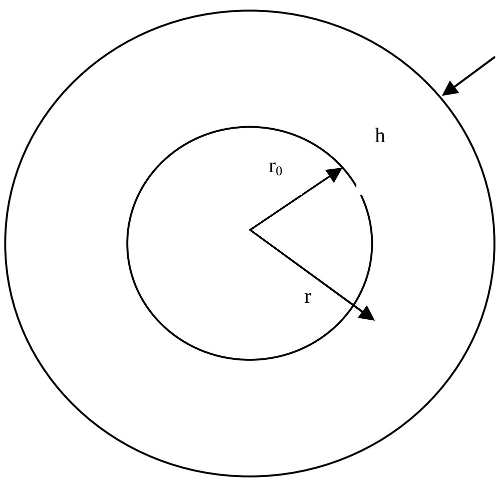


Figure 1. Solenoid Geometry

Because the core is infinite in length, and radially symmetrical, all functions depend upon radial position and time only. Thus we can write for a cylindrical polar coordinate system,

$$\boldsymbol{B} = \left(0, 0, B_z(r,t)\right)^T \ , \tag{18}$$

$$\boldsymbol{E} = (0, E_\theta(r,t), 0)^T \ . \tag{19}$$

Because of equation (18), Gauss' Law, equation (11), is automatically satisfied.

Equations (12), Faraday's Law, reduces to

$$\frac{1}{r}\frac{\partial(rE_\theta)}{\partial r} + \frac{\partial B_z}{\partial t} = 0 \ . \tag{20}$$

Equation (13) requires that

$$\rho = 0 \ .$$

Finally, equation (14) becomes

$$-\frac{\partial\left(G_H^{33}B_z\right)}{\partial r} - \frac{\partial\left(G_E^{22}E_\theta\right)}{\partial t} = J_\theta \ . \tag{21}$$

The metric properties for equation (21) are from equation (17), which are for this situation,

$$G_H^{33} = \frac{1}{\mu_c}(1 + \mu_m B_z^2) \ , \tag{22}$$

$$G_E^{22} = \varepsilon_c(1 + \varepsilon_m B_z^2) \ . \tag{23}$$

Incorporating the material properties of equations (22) and (23) into (21) gives

$$-\frac{\partial}{\partial r}\left(B_z(1 + \mu_m B_z^2)\right) - \mu_c\varepsilon_c\frac{\partial\left((1+\varepsilon_m B_z^2)E_\theta\right)}{\partial t} = \mu_c J_\theta \ . \tag{24}$$

Equation (24) can be simplified to

$$-\frac{\partial B_z}{\partial r} - \mu_m\frac{\partial}{\partial r}B_z^3 - \frac{1}{c^2}\frac{\partial E_\theta}{\partial t} - \frac{\varepsilon_m}{c^2}\frac{\partial\left(B_z^2 E_\theta\right)}{\partial t} = \mu_c J_\theta \tag{25}$$

where

$$\mu_c \varepsilon_c = \frac{1}{c^2} . \tag{26}$$

Equations (20) and (25) can be combined into a single equation. To do this, we multiply equation (25) by r and then take the derivative with respect to r.

$$-\frac{\partial}{\partial r}\left(r\frac{\partial B_z}{\partial r}\right) - \mu_m \frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}B_z^3\right) - \frac{1}{c^2}\frac{\partial^2(rE_\theta)}{\partial r \partial t} - \frac{\varepsilon_m}{c^2}\frac{\partial^2\left(B_z^2 rE_\theta\right)}{\partial r \partial t} = \mu_c \frac{\partial}{\partial r}(rJ_\theta) . \tag{27}$$

Using Faraday's Law, equation (20), in various forms, converts equation (27) to a partial differential equation in B_z. If we integrate equation (20),

$$rE_\theta = -\int r\frac{\partial B_z}{\partial t} dr + f(t) \tag{28}$$

where $f(t)$ is a constant of integration that can be taken to be zero for E_θ not to be singular at the centre of the core.

If we take the time derivative of equation (20) we get

$$\frac{\partial^2(rE_\theta)}{\partial r \partial t} = -r\frac{\partial^2 B_z}{\partial t^2} . \tag{29}$$

Expanding the last term of the left hand side of equation (27), we have

$$\frac{\partial^2\left(B_z^2 rE_\theta\right)}{\partial r \partial t} = \frac{\partial}{\partial t}\left(B_z^2 \frac{\partial}{\partial r}(rE_\theta) + rE_\theta \frac{\partial B_z^2}{\partial r}\right) .$$

This expands to

$$\frac{\partial^2\left(B_z^2 rE_\theta\right)}{\partial r \partial t} = B_z^2 \frac{\partial^2}{\partial t \partial r}(rE_\theta) + \frac{\partial}{\partial t}(B_z^2)\frac{\partial}{\partial \mathrm{r}}(rE_\theta) + \frac{\partial}{\partial t}\left(rE_\theta \frac{\partial B_z^2}{\partial r}\right) .$$

If we substitute equation (29) into the first term, equation (20) into the middle term, and equation (28) into the last term we have

$$\frac{\partial^2\left(B_z^2 rE_\theta\right)}{\partial r \partial t} = B_z^2\left(-r\frac{\partial^2 B_z}{\partial t^2}\right) + \frac{\partial}{\partial t}(B_z^2)\left(-r\frac{\partial B_z}{\partial t}\right) + \frac{\partial}{\partial t}\left(-\frac{\partial B_z^2}{\partial r}\int r\frac{\partial B_z}{\partial t} dr\right) .$$

The first two terms of this expression combine

$$B_z^2\left(-r\frac{\partial^2 B_z}{\partial t^2}\right) + \frac{\partial}{\partial t}(B_z^2)\left(-r\frac{\partial B_z}{\partial t}\right) = -r\frac{\partial}{\partial t}\left(B_z^2 \frac{\partial B_z}{\partial t}\right) = -\frac{r}{3}\frac{\partial^2 B_z^3}{\partial t^2}$$

leaving

$$\frac{\partial^2(B_z^2 r E_\theta)}{\partial r\partial t} = -\frac{r}{3}\frac{\partial^2 B_z^3}{\partial t^2} - \frac{\partial}{\partial t}\left(\frac{\partial B_z^2}{\partial r}\int r\frac{\partial B_z}{\partial t}dr\right) .$$

If we now substitute this and equation (20) into equation (27), we get

$$\left(-\frac{\partial}{\partial r}\left(r\frac{\partial B_z}{\partial r}\right) + \frac{r}{c^2}\frac{\partial^2 B_z}{\partial t^2}\right) - \mu_m\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}B_z^3\right) + \frac{r\varepsilon_m}{3c^2}\frac{\partial^2 B_z^3}{\partial t^2} + \frac{\varepsilon_m}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial B_z^2}{\partial r}\int r\frac{\partial B_z}{\partial t}dr\right) = \mu_c\frac{\partial}{\partial r}(rJ_\theta) . \tag{30}$$

Dividing this equation by *r* and reorganizing puts this into a more traditional form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(B_z+\mu_m {B_z}^3)}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2\left(B_z+\frac{\varepsilon_m}{3}{B_z}^3\right)}{\partial t^2} - \frac{\varepsilon_m}{rc^2}\frac{\partial}{\partial t}\left(\frac{\partial B_z^2}{\partial r}\int r\frac{\partial B_z}{\partial t}dr\right) = -\frac{\mu_c}{r}\frac{\partial}{\partial r}(rJ_\theta). \tag{31}$$

In this nonlinear wave equation, if the last term is significantly smaller than its predecessors, equation (31) simplifies to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(B_z+\mu_m {B_z}^3)}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2\left(B_z+\frac{\varepsilon_m}{3}{B_z}^3\right)}{\partial t^2} = -\frac{\mu_c}{r}\frac{\partial}{\partial r}(rJ_\theta) . \tag{32}$$

The traditional wave equation emerges from equation (32) if the magnitudes of both μ_m and ϵ_m are significantly less than one (for nominal magnitudes of B_z).

Boundary and Loading Conditions

The electromagnetic field equations (11) through (14) are mathematically similar to the field equations of traditional electromagnetic theory. Boundary conditions are found in any good textbook on the subject [8].

The boundary and loading conditions as applied to this problem are,

- H_z is continuous at the interface between the winding and core. Some finite element software based on calculus of variations techniques use "flow conditions" at an interface, which in this case would be that the volume integrals over the interface of $\underline{\nabla}\cdot\boldsymbol{B} = 0$ (which is automatically satisfied), and $\underline{\nabla}\times\left(\overline{\overline{\boldsymbol{G_H}}}\cdot\boldsymbol{B}\right) = 0$ at the interface between the coil and the core, and the coil and the outside environment.
- $\frac{\partial B_z}{\partial r} = 0$ at the centre of the core

Loading of the coil is provided through J_θ. If one assumes that the current density through the windings is constant, then

$$J_\theta = \frac{n\,I(t)}{h} \tag{33}$$

where n is the number of turns per unit length in the coil, h is the thickness of the coil, and $I(t)$ is the current flow in the wires of the coil, assumed uniform.

Numerical Solution

Equation (31) and (32) were solved using a commercial finite element solver [9] subject to the loading and boundary conditions discussed in the previous section. The factors $\frac{1}{c^2}$ and $\frac{\varepsilon_m}{c^2}$ grossly imbalance the relative size of the terms in the equation, making the solutions prone to numerical error. This is alleviated using the following transformation:

$$q = \frac{\omega r}{c}\,, \tag{34}$$

$$\tau = \omega t\ . \tag{35}$$

Equations (31) and (32) then become upon substituting (34) and (35)

$$\frac{1}{q}\frac{\partial}{\partial q}\left(q\,\frac{\partial\left(B_z+\mu_m {B_z}^3\right)}{\partial q}\right) - \frac{\partial^2\left(B_z+\frac{\varepsilon_m}{3}{B_z}^3\right)}{\partial \tau^2} - \frac{\varepsilon_m}{q}\frac{\partial}{\partial \tau}\left(\frac{\partial B_z^2}{\partial q}\int q\,\frac{\partial B_z}{\partial \tau}\,dq\right) = -\frac{\mu_c}{q}\left(\frac{\omega}{c}\right)\frac{\partial}{\partial q}\left(qJ_\theta\right) \tag{36}$$

and

$$\frac{1}{q}\frac{\partial}{\partial q}\left(q\,\frac{\partial\left(B_z+\mu_m {B_z}^3\right)}{\partial q}\right) - \frac{\partial^2\left(B_z+\frac{\varepsilon_m}{3}{B_z}^3\right)}{\partial \tau^2} = -\frac{\mu_c}{q}\left(\frac{\omega}{c}\right)\frac{\partial}{\partial q}\left(qJ_\theta\right)\ . \tag{37}$$

The last term of equation (36) was calculated separately and shown to be significantly less in magnitude than the other terms in the equation. Because of the limitations of the software, $\frac{\partial B_z}{\partial \tau}$ was taken to be spatially constant, so that

$$\int q\,\frac{\partial B_z}{\partial \tau}\,dq \approx \frac{\partial B_z}{\partial \tau}\frac{q^2}{2}$$

which then gives the last term approximately as

$$\frac{\varepsilon_m}{q}\frac{\partial}{\partial\tau}\left(\frac{\partial B_z^2}{\partial q}\int q\frac{\partial B_z}{\partial\tau}dq\right) \approx \frac{\varepsilon_m q}{2}\frac{\partial}{\partial\tau}\left(\frac{\partial B_z^2}{\partial q}\frac{\partial B_z}{\partial\tau}\right). \tag{38}$$

The constants for the problem were taken as "typical" expected values, although at this time, realistic estimates of ε_m and μ_m are not available. The following properties (in SI units) were used in the calculations:

$r_0 = 0.01\ m$

$\mu_c = 4\pi\ 10^{-4}$

$\varepsilon_c = 8.85\ 10^{-12}$

$\varepsilon_m = 0.001$

$\mu_m = 0.01$

$J_\theta = I_0 \frac{n}{h}\sin(\beta t)$

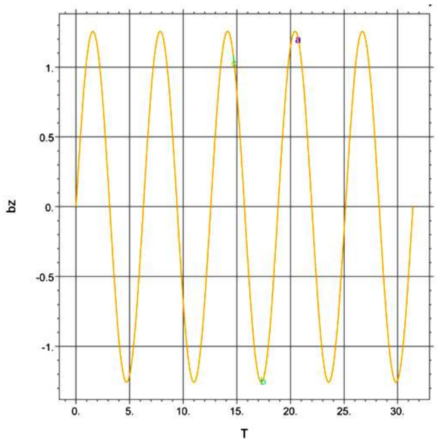
Where

$n = 1000$,

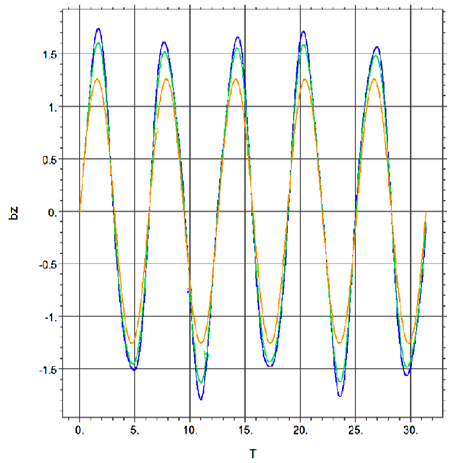
$h = 0.001\ m$,

$I_0 = 0.001\ Amperes$.

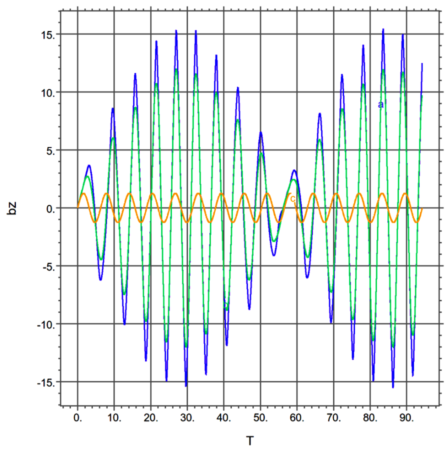
For small values of $\frac{\omega r}{c}$, the magnetic field is constant across the cross-section, and follows the driving function as is shown in Figure 1.



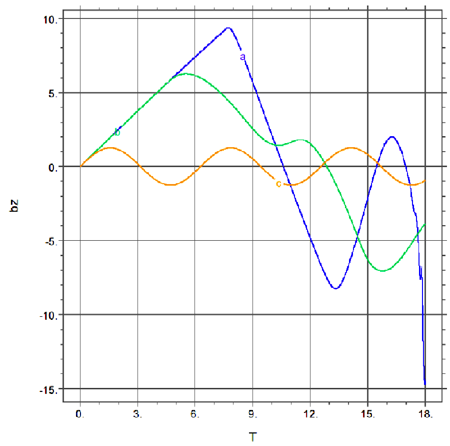
(a) $\frac{\omega r_0}{c} = 0.15$



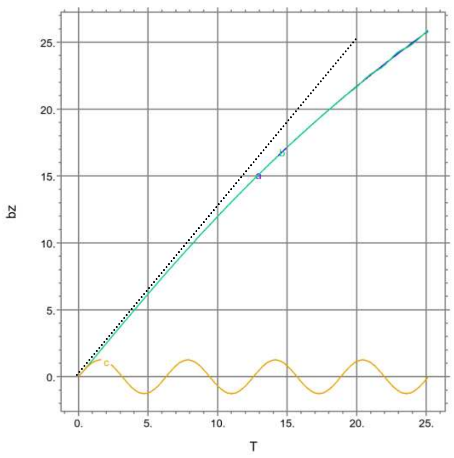
(b) $\frac{\omega r_0}{c} = 1.5$



(c) $\frac{\omega r_0}{c} = 2.3$



$(d)\,\frac{\omega r_0}{c} = 15$



(e) $\frac{\omega r_0}{c} = 150$

Figure 1. B_z with sinusoidal driving function (orange: $r = r_0$; green: $r = \frac{r_0}{2}$; blue: $r = 0$)

For values of $\frac{\omega r_0}{c}$ of about two, the system becomes increasingly unstable. This happens to be the first zero in the zeroth order Bessel function of the first kind, corresponding to a standing half-wave across the diameter of the core. This is the first resonant point with heterodyning appearing in the solution. Similar behaviour has been observed previously in studies of the ECE form of Coulomb's Law [10]. The details of the resonance cannot be calculated at this time, due to the unstable nature of the solution algorithm in this region.

For large values of $\frac{\omega r_0}{c}$ stability again returns. *Figure 1 (e)* suggests an asymptotic behaviour for large values of $\frac{\omega r_0}{c}$. When this occurs, $\frac{\partial B_z}{\partial q}$ is practically constant across the core. This suggests that the second degree temporal term in equation (32) dominates i.e.

$$\frac{\partial^2\left(B_z + \frac{\varepsilon_m}{3} {B_z}^3\right)}{\partial \tau^2} = 0\ . \tag{39}$$

The analytic solution of this equation is,

$$B_z = \frac{\sqrt[3]{2}}{\sqrt[3]{\sqrt{{\varepsilon_m}^3\left(9c_1^2\varepsilon_m(c_2+t)^2+4\right)} - 3c_1 t{\varepsilon_m}^2 - 3c_1 c_2{\varepsilon_m}^2}} - \frac{\sqrt[3]{\sqrt{\epsilon{\varepsilon_m}^3\left(9c_1^2\varepsilon_m(c_2+t)^2+4\right)} - 3c_1 t{\varepsilon_m}^2 - 3c_1 c_2{\varepsilon_m}^2}}{\sqrt[3]{2}\varepsilon_m} \tag{40}$$

where c_1 and c_2 are constants of integration.

If this is written as a power series in $q = 3c_1\tau$

then

$$B_z \approx a_1 + a_2 q + \cdots \tag{41}$$

At $\tau = 0$ the field B_z is zero, so that $a_1 = 0$.

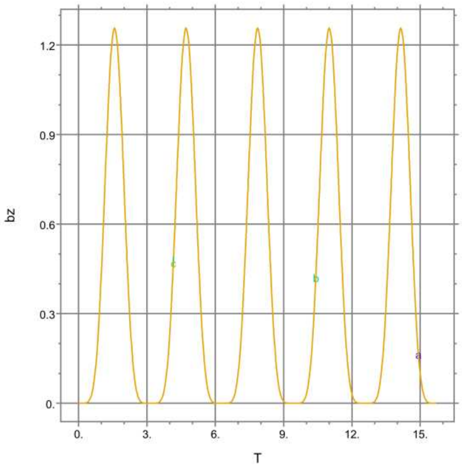
We also have that

$B_z \approx \mu_c I_0 \frac{n}{h} f(\tau)$ so that

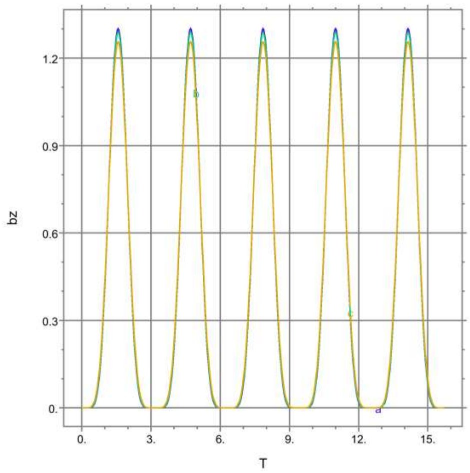
$$B_z \approx \mu_c I_0 \frac{n}{h}\ \tau \tag{42}$$

This is shown in Figure 1(e) using the reference data for the finite element calculation. The agreement observed here is fairly strong evidence for the validity of the solution at these higher frequencies.

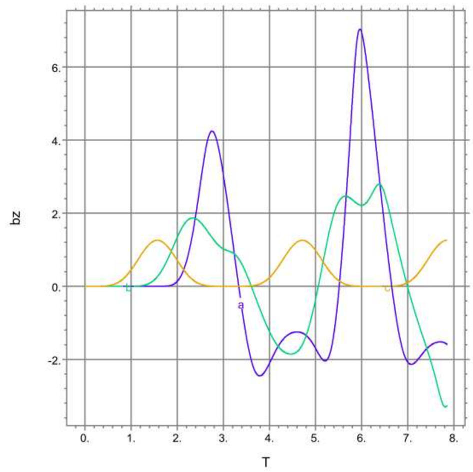
Solutions to equations (36) or (37) were attempted using driving functions with discontinuous time derivatives such as $|\sin(\omega t)|$ without success. Pulse-like driving functions such as $\sin(\omega t)^n$ were analyzed successfully and the results for $n = 6$, are presented in *Figure 2*. This system was more unstable than for the sinusoidal driving function. Even though the term $\frac{\partial B_z}{\partial q}$ was zero across the core for higher frequency solutions, the computation was not stable. Heterodyning behaviour is suspected again when $\frac{\omega r_0}{c}$ is about two.



(a) $\frac{\omega r_0}{c} = 0.015$



(b) $\frac{\omega r_0}{c} = 0.15$



(c) $\frac{\omega r_0}{c} = 1.5$

Figure 2. B_z with pulse-like driving function (orange: $r = r_0$; green: $r = \frac{r_0}{2}$; blue: $r = 0$)

Conclusions

Amplification of the magnetic field was observed for the infinitely long solenoid, when the metric coefficients were not of unit value, and depended upon the magnitude of the magnetic field. A resonant frequency is indicated, but its value could not be determined precisely because of numerical instabilities that occurred when solving the equations. Heterodyning behaviour occurs when $\frac{\omega r_0}{c} \approx 2$. This is the resonance for the first standing wave. Similar behaviour has been observed previously in studies of the ECE form of Coulomb's Law [10]. Further, at higher frequencies, the solution become asymptotically linear in time, perhaps indicating some form of resonant growth.

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