Can energy be extracted from a double pendulum?

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Abstract

According to statements by Milkovic and various inventors of rotoverter systems, it should be possible to extract energy from rotating systems with at least two revolving or oscillating units, which corresponds to a mechanism for extracting spacetime energy. We base our analysis and modeling on the approach of an anonymous author who calculated the dynamics of a double pendulum according to classical mechanics, and proposed that with a certain load characteristic, it gains energy. We have proven that the author did not use the underlying equations of motion correctly when applying an external load. If they are put into canonical form, i.e., used in the intended way, there is no energy gain. We have investigated this for different types of load momenta. Such a system can serve as a source of energy only if one adopts non-conventional physical mechanisms. For this, a "spacetime resonance" was used in accordance with ECE theory. This then results in chaotic behavior, which, on average, leads to a significant gain in energy, with constant useful power being withdrawn from the system.

Keywords: double pendulum; Lagrangian mechanics; spacetime energy.

1 Introduction

Research into energy from spacetime (which has also been called vacuum energy, zero-point energy, and space energy) includes approaches that attempt to obtain energy using mechanical systems. The first known system of this type was the Bessler wheel. More recently, the Milkovic pendulum [1] and the Würth gearbox [2] have become known. However, there is no clear evidence of an energy gain with these devices. Qualitative descriptions from engineers and

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inventors who deal primarily with purely static configurations suggest that an energy gain is plausible.

However, mechanical devices are dynamic systems, because internal forces, and positions of the constituent parts change over time. Therefore, a complete description of the functionality is only possible using dynamical considerations and models.

The dynamics of mechanical systems can be described very elegantly with the methods of classical mechanics. When a model is structured simply enough, i.e. when it is a system of mass points, one uses the Lagrangian mechanics, which goes back to Leonhard Euler (1707-1783) and Joseph-Louis Lagrange (1736-1813), whose equations of motion are called Euler-Lagrange equations. Later, William Rowan Hamilton (1805-1865) brought these into a form that can be solved numerically, and today efficiently on a computer.

When it comes to systems of extended solids, either rigid or deformable bodies, the newer finite element method is used. It has been used very successfully in engineering since the advent of modern computers, although it is very computationally intensive.

As part of an attempt to explain a possible excess energy in the Milkovic pendulum, an anonymous author carried out a Lagrangian calculation [3]. To the best of our knowledge, this is the most in-depth analysis of this system. We checked the calculation, but found a serious modeling error by the author. That makes his results, which actually show an energy surplus, quite questionable. As a second system, which is based on the principle of the double pendulum, we examined the planetary gear according to Würth [2]. These results will be reported in a subsequent paper. This will probably be the first time that the Würth gearbox has been investigated in this modeling depth. In the following, we first briefly describe the application class of the double pendulum and the Lagrange method before we go into the results of the double pendulum.

2 Calculation method for the double pendulum

A double pendulum consists of two pendulums that are attached to one another, with each one having a pendulum body with a mass. When impacted in a vertical position, these masses perform unpredictable oscillations, which is a well-known example of a chaotic system. Lagrangian mechanics requires coordinates that correspond to the number of degrees of freedom. Here they are the angular deflections φ_1 and φ_2 from the vertical, see Fig. 1. To calculate the Lagrangian

$$\mathscr{L} = T - U \tag{1}$$

one needs the kinetic energy T and the potential energy U of the two masses. The easiest way to get the kinetic energy is from the Cartesian coordinates. They are (see Fig. 1):

$$x_1 = l_1 \sin(\varphi_1), \tag{2}$$

$$y_1 = -l_1 \cos(\varphi_1), \tag{3}$$

and

$$x_2 = x_1 + l_2 \sin(\varphi_2), \tag{4}$$
$$y_2 = y_1 - l_2 \cos(\varphi_2). \tag{5}$$

This gives the kinetic energy of the masses m_1 and m_2:

$$T_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2), \tag{6}$$
$$T_2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2), \tag{7}$$
$$T = T_1 + T_2. \tag{8}$$

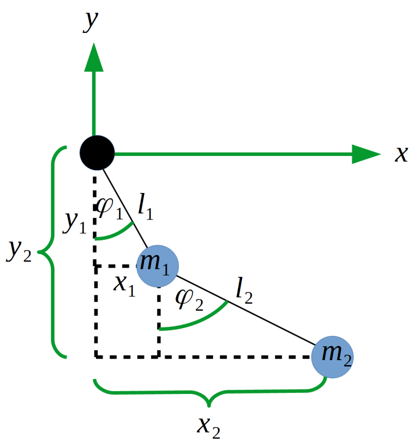

Figure 1: Coordinates of the double pendulum.

The dot denotes the time derivatives. The potential energy follows from the force of gravity in the y direction:

$$U = m_1 g\, y_1 + m_2 g\, y_2 \tag{9}$$

with g being the gravitational acceleration. The Lagrange function (1) is thus completely determined. The equations of motion follow from the Euler-Lagrange equations, where q_i stands for the coordinates φ_1 and φ_2:

$$\frac{d}{dt} \frac{\partial \mathscr{L}}{\partial \dot{q}_i} - \frac{\partial \mathscr{L}}{\partial q_i} = 0. \tag{10}$$

Conservation of energy applies to these equations, because they are derived from this principle. In addition, one can introduce dissipation functions D_i and generalized forces Q_i. A generalized force in our case is a torque. Then the equations take the form:

$$\frac{d}{dt} \frac{\partial \mathscr{L}}{\partial \dot{q}_i} - \frac{\partial \mathscr{L}}{\partial q_i} + \frac{\partial D_i}{\partial \dot{q}_i} = Q_i \tag{11}$$

and there is no longer any conservation of energy. We need this case here, since the system is supposed to provide additional energy. The dissipation functions can be replaced by

$$Q_{Ri} = -\frac{\partial D_i}{\partial \dot{q}_i} \tag{12}$$

and be traced back to generalized forces, with Q_{Ri} denoting any residual forces that are not covered by the dissipation functions.

The evaluation of equations (10) can lead to very complicated equations of motion. In the case of the double pendulum, they are just about manageable. In other cases, such as the planetary gear, they are so complicated that it is no longer possible to calculate them by hand. We use the computer algebra system Maxima for this. The Euler-Lagrange equations are two coupled differential equations for the variables $\ddot{\varphi}_1$ and $\ddot{\varphi}_2$. They are linear in these two variables and have to be solved for them so that the numerical solution (time integration) can be carried out. The following equations then result:

$$\begin{aligned}\ddot{\varphi}_1 = \Big[& (l_1\, l_2\, m_2 \sin(\varphi_2) \cos(\varphi_2 - \varphi_1) - (l_1\, l_2\, m_2 + l_1\, m_1\, l_2) \sin(\varphi_1))\, g \\ & + \left(l_1^2\, l_2\, m_2\, \dot{\varphi}_1^2 \cos(\varphi_2 - \varphi_1) + l_1\, l_2^2\, m_2\, \dot{\varphi}_2^2\right) \sin(\varphi_2 - \varphi_1) \\ & - l_1\, Q_2 \cos(\varphi_2 - \varphi_1) + l_2\, Q_1 \Big] \\ & \cdot \frac{1}{l_1^2\, l_2\, m_2 \sin(\varphi_2 - \varphi_1)^2 + l_1^2\, l_2\, m_1},\end{aligned} \tag{13}$$

$$\begin{aligned}\ddot{\varphi}_2 = \Big[& \left(\left(l_1\, l_2\, m_2^2 + l_1\, m_1\, l_2\, m_2\right) \sin(\varphi_1) \cos(\varphi_2 - \varphi_1) - \left(l_1\, l_2\, m_2^2 + l_1\, m_1\, l_2\, m_2\right) \sin(\varphi_2)\right) g \\ & + \left(\left(-l_1^2\, l_2\, m_2^2 - l_1^2\, m_1\, l_2\, m_2\right)\, \dot{\varphi}_1^2 - l_1\, l_2^2\, m_2^2\, \dot{\varphi}_2^2 \cos(\varphi_2 - \varphi_1)\right) \sin(\varphi_2 - \varphi_1) \\ & - Q_1\, l_2\, m_2 \cos(\varphi_2 - \varphi_1) + l_1\, Q_2\, m_2 + l_1\, m_1\, Q_2 \Big] \\ & \cdot \frac{1}{l_1\, l_2^2\, m_2^2 \sin(\varphi_2 - \varphi_1)^2 + l_1\, l_2^2\, m_1\, m_2}.\end{aligned} \tag{14}$$

3 Results

3.1 Verification and comparison with reference [3]

The aim of our calculations was initially to verify the results in reference document [3]. We first compared the equations of motion cited in this document with ours, and there was not a complete match. The anonymous author has not given the source of his equations and only speaks of “literature”. Since this literature could have come from a time when there was no computer algebra, it is possible that there was a calculation error. It wouldn’t be the first time such errors have been found in textbooks.

For comparison, we used the same parameters for the double pendulum as the anonymous author, see Table 1. Here $\omega_{1,2}$ is to be equated with the angular velocity $\dot{\varphi}_{1,2}$. The calculation was initially carried out without external forces and without gravitation. The initial angular velocity of the second pendulum

m_1	1 kg
m_2	0.1 kg
l_1	0.2 m
l_2	0.1 m
$\varphi_{1,\text{initial}}$	0
$\varphi_{2,\text{initial}}$	0
$\omega_{1,\text{initial}}$	0
$\omega_{2,\text{initial}}$	100·π rad/s

Table 1: Parameters and initial values for calculating the double pendulum.

is 50 Hz, so it is quite fast. In his Fig. 2, in addition to the time course for ω_1, the author also gives a "Pivot Torque" and a "Pivot Power", i.e., a torque and a power on the fixed axis, which he calculates as follows:

$$\tau_{\text{pivot torque}} = m_1 l_1^2 \ddot{\varphi}_1, \tag{15}$$

$$P_{\text{pivot power}} = m_1 l_1^2 \ddot{\varphi}_1 \omega_1. \tag{16}$$

This is where the accelerations on the axis appear. We have also evaluated these variables and shown them in Fig. 2. The angular velocity ω_2 results from the initial values and is exactly the same as in Fig. 2 of the reference document. Torque and power are a few percent lower, but otherwise the same. That may be an influence of the different equations of motion. Each of torque and power cancel out on average over time. The author speaks of reactive power, which has to be the case, since no power is taken out of the system.

Next, we look at the energy balance. Since we have neither external forces nor gravity, there are only kinetic energy contributions that have to be constant in sum for both masses. This is the case, as can be seen from Fig. 3. The sum corresponds to the initial rotation of the second mass at 50 Hz, which is a little over 49 joules. The same curve is found in Figure 3 of the reference document. So far, there is agreement.

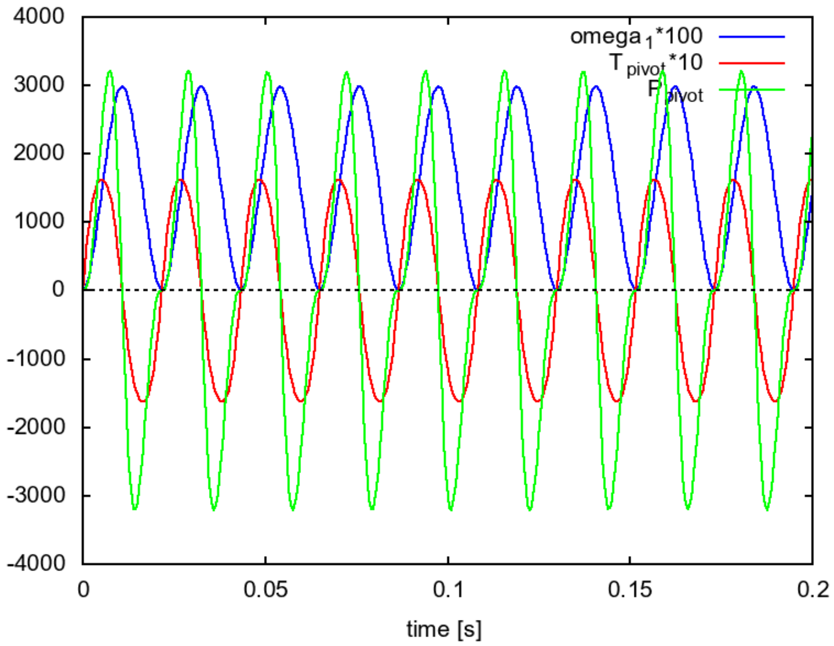

Figure 2: Angular velocity [rad/s], Pivot torque [Nm] und Pivot power [W].

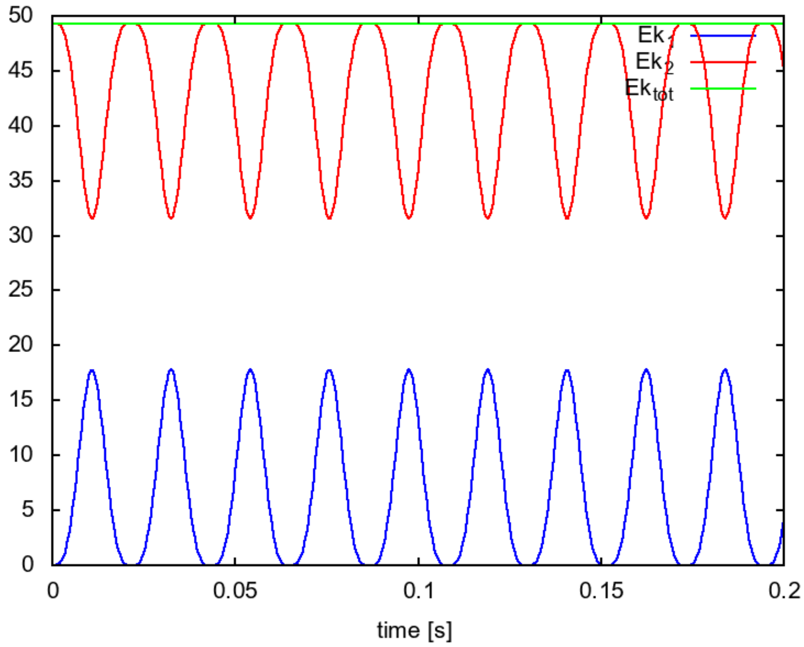

Figure 3: Kinetic energies [J] of both masses and total energy.

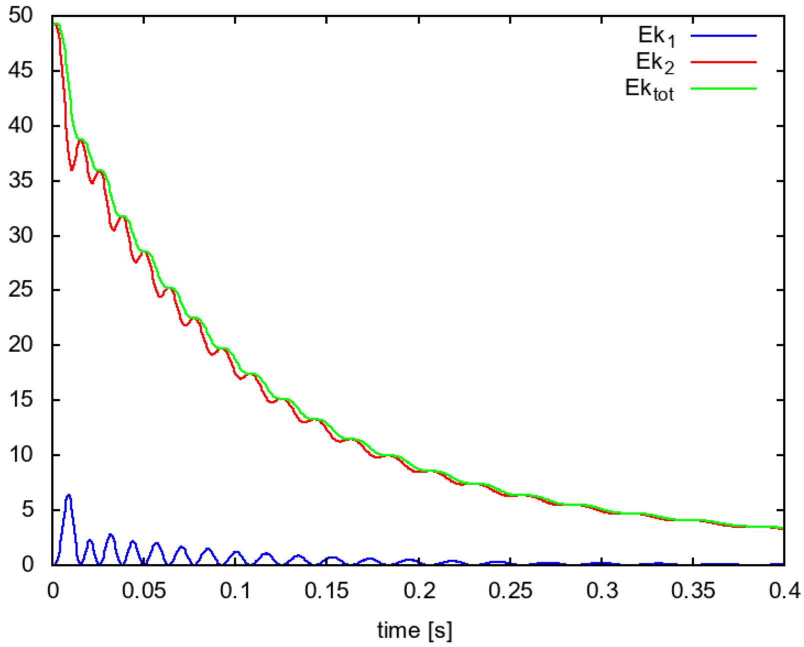

Figure 4: Kinetic energies [J] of both masses and total energy for external load, canonical form.

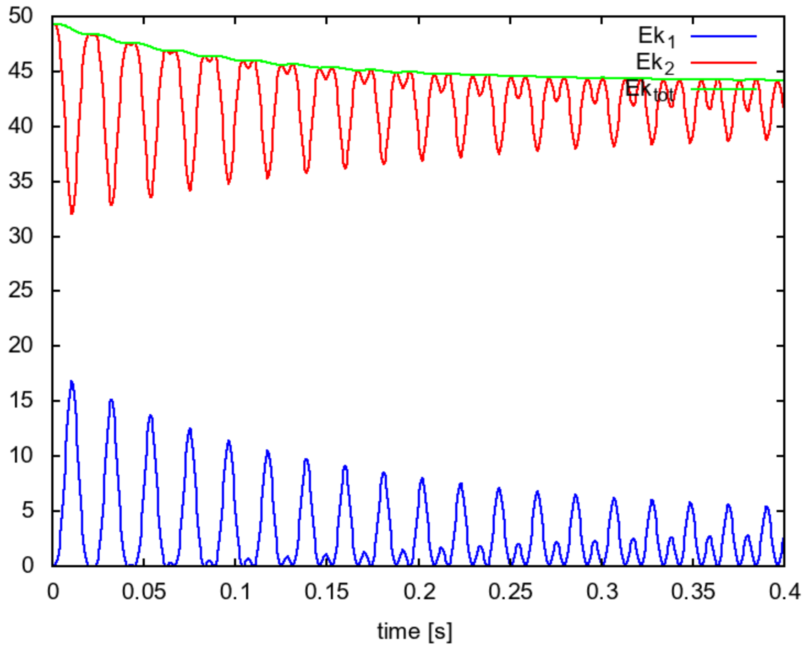

Figure 5: Kinetic energies [J] of both masses and total energy for external load according to [3].

We now apply an external load (a braking torque) to the fixed axis. This is modeled as a generalized force in the form

$$Q_1 = -\frac{\mu}{l_1}\omega_1, \tag{17}$$

where μ is a constant. We base this on reference document [3]. The braking torque is proportional to the angular velocity. The calculation with $\mu = 1$ shows that the total energy decreases exponentially, see Fig. 4. At the same time, the frequency slows down. The rotational energy of the outer, fast pendulum is transferred to the inner pendulum, where it is removed from the system by the load torque. The fact that the braking torque, which only acts on the axis of the first pendulum, also acts on the outer pendulum can be seen directly from the equation of motion (14). In addition to Q_2 (not used here), this equation also contains the braking torque Q_1.

The anonymous author of [3] received completely different results for the braking torque (14). He did not use the concept of generalized force as prescribed by the Lagrangian mechanics, but changed equation (13) for the acceleration of φ_1 a posteriori by making the replacement:

$$\ddot{\varphi}_1 \rightarrow \ddot{\varphi}_1 + \frac{\mu}{l_1}\omega_1. \tag{18}$$

Here the parameter μ has different physical units than in (17), but that is not decisive. More importantly, as a result of this arbitrary substitution, the braking torque only acts on the movement of the inner pendulum and the rotation of the outer pendulum is not affected. Our calculation with this approach results in the energy curve of Fig. 5, which is identical to that of Fig. 6 in [3]. The total energy only decreases slightly at the beginning and then remains constant. Only the form of oscillation changes; the oscillation frequencies are doubled due to the external braking torque. As we have explained, this is an arbitrary intervention in the "physics" of the double pendulum. Therefore, all results in [3] that are based on this must be regarded as unphysical, unfortunately.

3.2 Effect of different load torques

We now investigate how the load torque has to be changed so that there may be an increase in energy, if one uses the correct equations of motion. With the approach

$$Q_1 = -\frac{\mu}{l_1}|\omega_1| \tag{19}$$

(modulus of ω_1) and $\mu = 0.1$ the result of Fig. 6 comes out. A phase change occurs for both pendulums, whereby – after an initial decrease in the total energy – there is a gain. The question is whether this corresponds to an energy gain in the overall system or whether this increase is due to the supply of external energy. To this end, we examine the external torque or braking torque τ_{ext} and the input or output power P_{ext}. The following applies:

$$\tau_{\text{ext}} = Q_1 = -\frac{\mu}{l_1}|\omega_1|, \tag{20}$$

$$P_{\text{ext}} = \tau_{\text{ext}}\omega_1. \tag{21}$$

Both are shown in Fig. 7. The torque, based on the modulus in Eq. (20), is always negative. For the phases in which the angular velocity ω_1 is also negative, this leads to a torque of the same sign, i.e., the mass m_1 is driven in this direction and the angle even reverses direction. Accordingly, the performance (21) is positive, i.e., energy is supplied, as can be seen from Fig. 7. Therefore, this is a drive effect, and the system does not provide any energy gain.

One can avoid the drive phase by only using real braking phases for the braking torque:

$$\tau_{\text{ext}} = Q_1 = \begin{cases} -\frac{\mu}{l_1}\omega_1 & \text{for } \omega_1 > 0, \\ 0 & \text{else.} \end{cases} \tag{22}$$

Then the total energy adjusts to a final value after an initial braking phase, as shown in Fig. 8, for $\mu = 0.1$. The external energy flow can be determined via the integral

$$E_{\text{ext}} = \int P_{\text{ext}} dt. \tag{23}$$

The numerical evaluation (Fig. 9) shows that in fact energy initially flows away (negative values), but then the energy remains constant, i.e., the drain has "dried up". Therefore, there is no energy gain to be drawn from the system itself in this way.

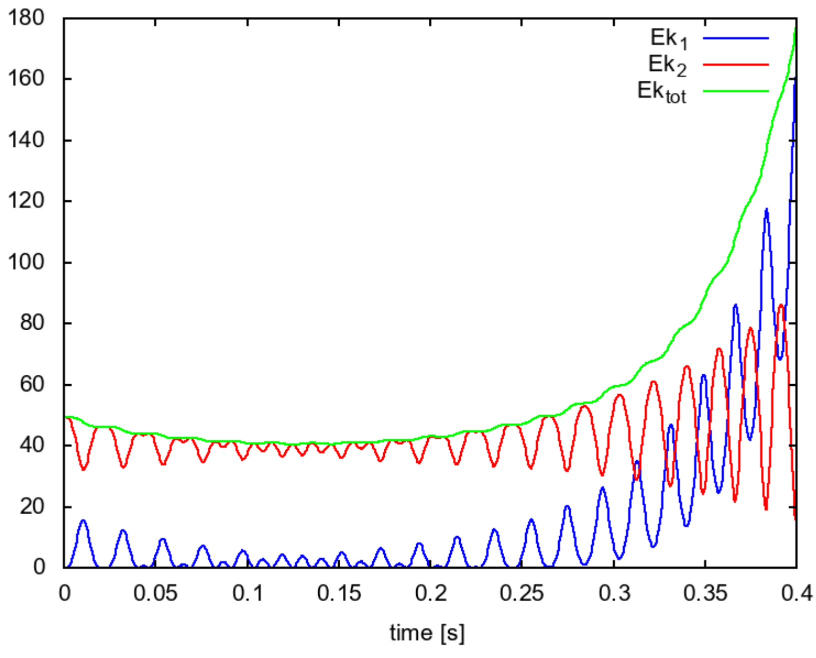

Figure 6: Kinetic energies of both masses and total energy for load (19).

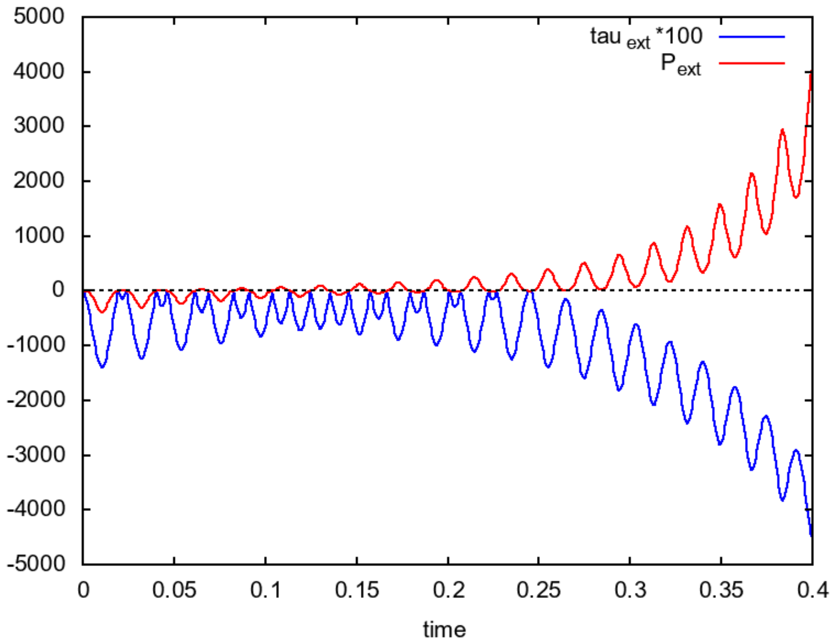

Figure 7: External load torque and power for load (19).

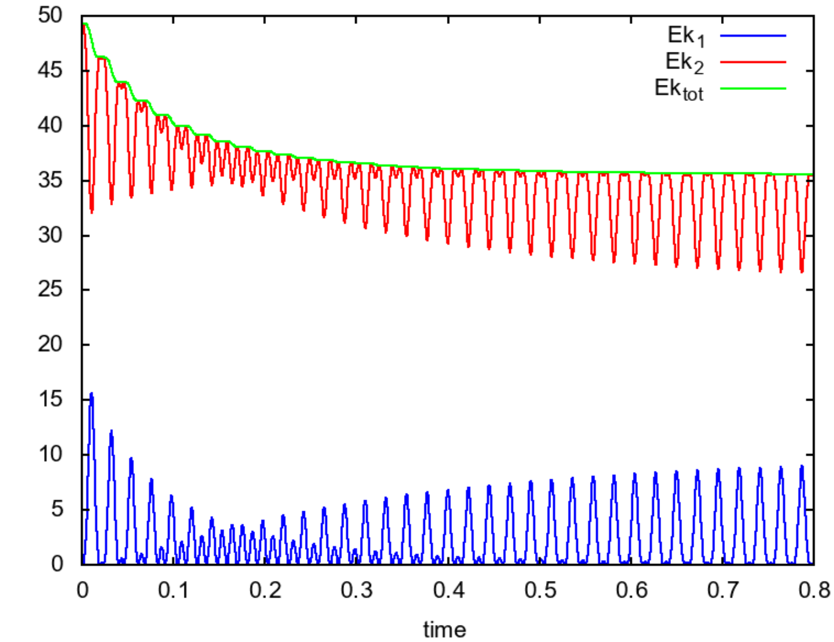

Figure 8: Kinetic energies of both masses and total energy for load (20).

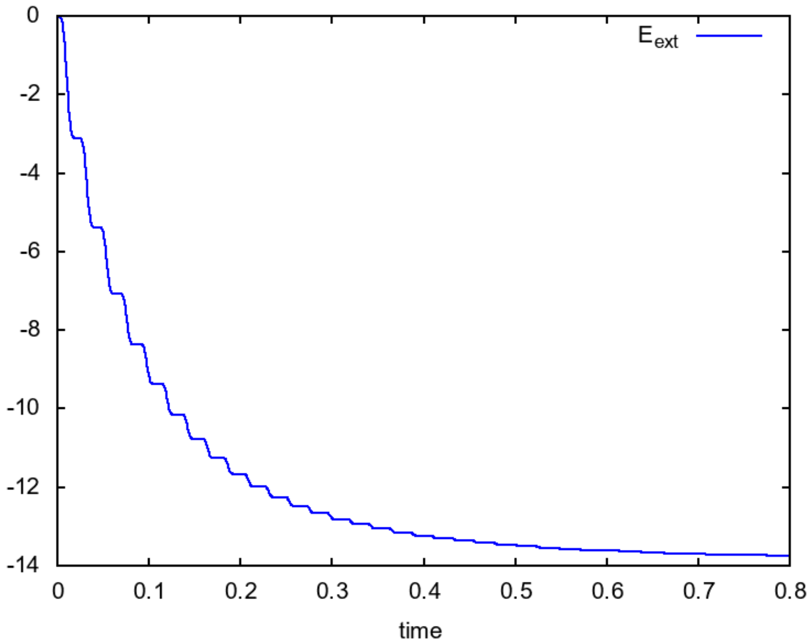

Figure 9: Externally released energy for load (20).

3.3 Effect of interaction with "spacetime energy"

To get the desired effect of energy gain, we have to consider mechanisms that cannot be found in conventional physics. We assume a resonance mechanism for this, which is predicted by ECE theory [4–6]. This mechanism was calculated for electromagnetic systems, but due to the complete equivalence between electromagnetic and mechanical systems, it also applies to dynamics. According to Eq. (20) in [6], the resonance equation applies to

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} + \omega_t \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial \omega_t}{\partial t} \mathbf{A} = \frac{1}{\epsilon_0} \mathbf{J}, \tag{24}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{J}$ is the electrical current density and ω_t is the spin connection (a frequency) of Cartan geometry. Applied to mechanics, this equation reads:

$$\frac{\partial^2 \mathbf{Q}}{\partial t^2} + \omega_t \frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \omega_t}{\partial t} \mathbf{Q} = G\, \mathbf{J}_m, \tag{25}$$

where $\mathbf{Q}$ is the mechanical equivalent of the vector potential, $\mathbf{J}_m$ is the mass flow density and G is the Einstein constant. $\mathbf{Q}$ has the units of a velocity, and one can consider it as "aether flow". If we restrict ourselves to the rotation component (the φ component Q_φ) of $\mathbf{Q}$ and assume a linear time dependence of ω_t, this equation can be written as:

$$\frac{d^2 Q_\varphi}{dt^2} + \alpha \frac{dQ_\varphi}{dt} + \omega_0^2 Q_\varphi = G\, J_\varphi, \tag{26}$$

and with a periodic excitation $J_\varphi = G\,J_0\cos(\omega t)$ as:

$$\frac{d^2Q_\varphi}{dt^2} + \alpha\frac{dQ_\varphi}{dt} + \omega_0^2 Q_\varphi = G\,J_0\cos(\omega t). \tag{27}$$

This is the equation of a damped resonance with a resonance frequency ω_0 and a damping constant α. The solution to this differential equation is:

$$Q_\varphi = G\,J_0\frac{\alpha\omega\sin(\omega t) + \left({\omega_0}^2 - \omega^2\right)\cos(\omega t)}{(\omega_0^2-\omega^2)^2+\alpha^2\omega^2}. \tag{28}$$

For $\alpha \approx 0$ the solution is simplified to

$$Q_\varphi = G\,J_0\frac{\cos(\omega t)}{\omega_0^2-\omega^2}, \tag{29}$$

which means a resonance increase of Q_φ of infinite, i.e., arbitrarily high strength. We now apply this to the double pendulum. We assume that the outer pendulum rotates relatively quickly, as assumed in the previous calculations. Then it makes sense to assume an energy transfer due to the rotational potential Q_φ. In the Lagrange formalism, this then appears as the external torque Q_2:

$$Q_2 = Q_\varphi = G\,J_0\frac{\alpha\omega_2\sin(\omega_2 t) + \left({\omega_0}^2 - {\omega_2}^2\right)\cos(\omega_2 t)}{(\omega_0^2-\omega_2^2)^2+\alpha^2\omega_2^2}. \tag{30}$$

We have set $\omega = \omega_2$, the angular velocity of the outer pendulum. So that an influence of Q_2 becomes visible, we have to place the initial value of ω_2 close to the resonance frequency ω_0. In addition, we assume a decrease in energy due to deceleration on the central axis of rotation, as previously modeled by Eq. (17):

$$Q_1 = -\frac{\mu}{l_1}\omega_1. \tag{31}$$

The new constants and initial values are listed in Table 2, resulting in the kinetic energy curve shown in Fig. 10. The resonance structure of Q_2 creates chaotic behavior in parts, which makes numerical stability of the result difficult. However, the solution shown could be reproduced qualitatively when the time integration step size Δt was varied. In Fig. 10, the kinetic energy calculated from the initial conditions is also shown. You can see that the resonance provides a significant amount of additional energy, except in an initial transient range.

μ	0.05
GJ_0	50 000
α	5.0
ω_0	$11\cdot\pi$
$\omega_{2,\text{initial}}$	$10\cdot\pi$

Table 2: Parameters and modified initial values for calculating the spacetime energy effect.

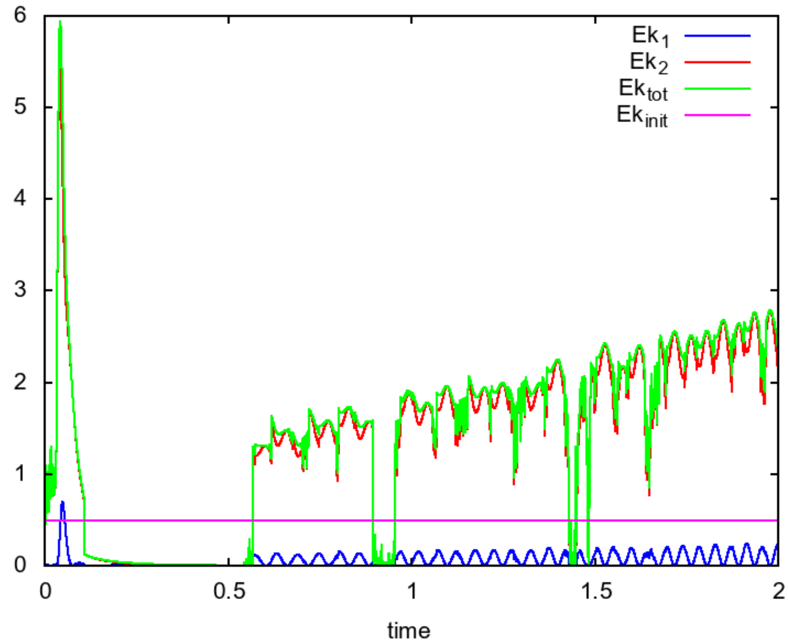

Figure 10: Kinetic energies of both masses, total energy and initial energy Ek_{init} with spatce energy coupling.

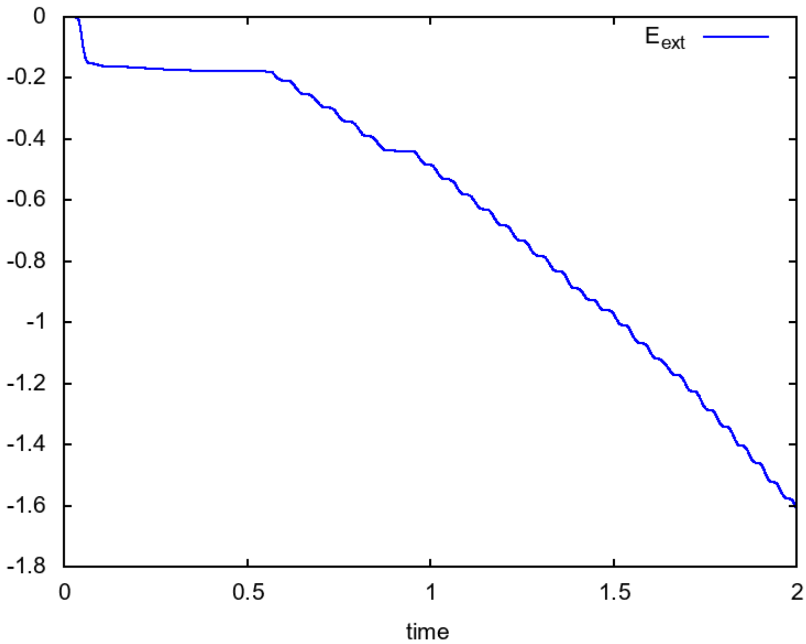

Figure 11: Externally emitted energy with space energy coupling.

Since we have taken the braking force into account, the energy increase takes place with the release of useful energy. This was calculated according to Eq. (23) (as for Fig. 9) and is shown in Fig. 11. After a settling phase, an approximately constant amount of energy per time unit is emitted, i.e., we can withdraw a constant continuous output from the system. We have thus found a possible mechanism for a double pendulum that is fed by spacetime energy, provided that the prediction made by ECE theory actually applies and can be demonstrated in the experiment.

4 Summary

The extraction of power from a system with two coupled, vibrating units, claimed by Milkovic and the anonymous author, could not be confirmed in this study. A classical approach to power extraction always leads to a decrease in rotational energy, i.e., a conservation of the total energy, which is to be expected from classical rotating systems. Such a system can only generate energy from itself if it is in exchange with an external energy reservoir, for example, the space energy of the non-empty vacuum. The model developed in this study has shown that rotary fields of the vacuum can provide such an effect. Nevertheless, any device that is claimed to interact with such an energy source still requires proper scientific verification, including reproducibility and repeatability of experiments.

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