77(2): Light Bending in Ricci Flat Spacetimes

As Crothers has shown, Ricci flat spacetimes have the form:

$$ds^2 = \left(1 - \frac{\alpha}{C^{1/2}}\right)c^2 dt^2 - \left(1 - \frac{\alpha}{C^{1/2}}\right)^{-1} d\left(C^{1/2}\right) - C(r)\left(d\theta^2 + \sin^2\theta \, d\phi^2\right) \quad -(1)$$

where the radius of curvature is:

$$R_c = C^{1/2} = \left(|r - r_0|^n + \alpha^n\right)^{1/n} \quad -(2)$$

Here:

1) $C(r)$ is not determined by the field equations.
2) Any $C(r)$ can be used in eq. (1) without changing the spherical symmetry or violating the field equation.
3) $C(r)$ must be asymptotically Minkowski.
4) There is a difference between R_c and the geodesic proper radius R_p. The geodesic proper radius is:

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(r_0)}^{R_c(r)} \left(B(R_c(r))\right)^{1/2} dR_c(r) \quad -(3)$$

$$= \int_{r_0}^{r} \left(B(R_c(r))\right)^{1/2} \left(\frac{dR_c(r)}{dr}\right) dr \quad -(4)$$

) One cannot assume that $0 \leq R_c(r) < \infty$ if $0 \leq r < \infty$.

) The Ricci flat spacetimes are Schwarzschild, Kerr-Newman, Kerr, charged Kerr, and exterior of an incompressible spherical fluid. All describe the gravitational field in terms of a centre of mass.

Light bending occurs in a Ricci flat ...

2) Because the Christoffel symbols and Riemann tensor elements may be non-zero while:

$$R_{\mu\nu} = R = 0 \quad -(5)$$

the usual line element used to describe light bending is a special case of eq (1):

$$ds^2 = \left(1 - \frac{2mG}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2mG}{rc^2}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right) \quad -(6)$$

A photon of mass m is attracted by an object of mass M along a null geodesic. This has been discussed in volume 4 in detail. The photon is curved into an orbit around the object of mass M. This is a purely kinematic theory. In the classical electrodynamics of the ECE theory one can think of the phase velocity being changed from c to v, as in the theory of refraction. The line elements (1) and (6) both give:

$$\rho = 0, \quad \underline{J} = \underline{0} \quad -(7)$$

So the Ampère Maxwell law for a photon travelling at c is:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(8)$$

where:

$$\frac{1}{c^2} = \mu_0 \epsilon_0 \quad -(9)$$

3) where ϵ_0 and μ_0 are the vacuum permittivity and permeability. The vacuum is defined by eq. (5) and eq. (7). The wave velocity is eq. (8) is c. For light bent by an object of mass M the wave velocity is:

$$\frac{1}{v^2} = \mu \epsilon \quad - (10)$$

where ϵ and μ are the permittivity and permeability in region where the photon is curved along a null geodesic, in orbit around the object of mass M. So the Ampère Maxwell law becomes:

$$\underline{\nabla} \times \underline{B} - \frac{1}{v^2} \frac{\partial \underline{E}}{\partial t} = \underline{0} \quad - (11)$$

and as shown in note 97(1), its polarization is changed from circular to elliptical.

These considerations are also true for regions outside a sphere of incompressible spherical fluid, where:

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right) c^2 dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right) \quad - (12)$$

Inside the sphere:

$$\rho \neq 0, \; \underline{J} \neq \underline{0} \quad - (13)$$

4) But outside:

$$\rho = 0, \quad \underline{J} = \underline{0}. \quad -(14)$$

The problems with line element (6) are well known and discussed by Crothers in paper 93. In addition there is a passive mass, defined by:

$$m = \rho_0 \bar{V} \quad -(15)$$

and an active mass:

$$m = \frac{\alpha}{2} \quad -(16)$$

In eq. (12):

$$R_c = (|r - r_0|^n + \epsilon^n)^{1/3}, \quad -(17)$$

$$\alpha = \left(\frac{3}{\kappa\rho_0}\right)^{1/2} \sin^3|\chi_a - \chi_0| \quad -(18)$$

$$-(19)$$

$$\epsilon = \left(\frac{3}{\kappa\rho_0}\right)^{1/2}\left(\frac{3}{2}\sin^3|\chi_a - \chi_0| - \frac{9}{4}\cos|\chi_a - \chi_0|\left(|\chi_a - \chi_0| - \frac{1}{2}\sin 2|\chi_a - \chi_0|\right)\right)^{1/3}$$

The original Schwarzschild result is recovered for

$$n = 3, \quad r_0 = 0, \quad \chi_0 = 0, \quad r > 0 \text{ and } \chi_a > 0.$$

$$-(20)$$

The proper radius is determined by the line element of the interior and in general:

$$\alpha \neq M \quad -(21)$$