1) 96(a): Charge and Current Densities for the Time or Original Static Vacuum Schwarzschild Solution.

In this case the charge and current densities are well defined and non-zero for $0 < r < \infty$. They are plotted in Fig 1c of paper 93. In this note they are given in terms of r from Crothers' solution.

Charge Density

$$J^0 = \frac{\phi}{AB}\left(\frac{r^4}{2(r^3+\alpha^3)^{8/3}} - \frac{r}{(r^3+\alpha^3)^{5/3}}\right) \quad -(1)$$

$$\rightarrow 0 \text{ as } r \rightarrow \infty.$$

Radial Current Density

$$J_r = \frac{A^{(0)} r}{2\mu_0 B^2}\left(\frac{3r^3 - 10\alpha^3}{(r+\alpha)^2(r^2-\alpha r+\alpha^2)^2(r^3+\alpha^3)^{2/3}}\right)$$

$$\rightarrow 0 \text{ as } r \rightarrow \infty. \quad -(2)$$

Angular Current Densities

$$J_\theta = J_\phi \sin^2\theta = \frac{A^{(0)}}{\mu_0}\left(\frac{2BC^{1/2} - \ddot{C}}{2BC^{5/2}}\right) \quad -(3)$$

For $B = 1$:

$$J_\theta = J_\phi \sin^2\theta = \frac{A^{(0)}}{\mu_0}\left(\frac{1}{C^2} - \frac{\ddot{C}}{2C^{5/2}}\right) \quad -(4)$$

where:

$$C = (r^3+\alpha^3)^{2/3} \quad -(5)$$

$$\ddot{C} = 4r(r^3+\alpha^3)^{-1/3} - 2r^4(r^3+\alpha^3)^{-4/3} \quad -(6)$$

2) So:

$$J_\theta = J_\phi \sin^2\theta = \frac{A^{(0)}}{\mu_0}\left(\frac{1}{(r^3+d^3)^{4/3}} + \frac{r^4}{(r^3+d^3)^{8/3}} - \frac{r}{(r^3+d^3)^{5/3}}\right)$$

$\rightarrow 0$ as $r \rightarrow \infty$

These results must be interpreted in a manner analogous with the Lehnert charge / current density, a Maxwell displacement current. As in "Advances in Chemical Physics", vol 119(2) p. 5 by Bo Lehnert:

"the divergence of the electric field may differ from zero, and a corresponding "space-charge current" may exist in vacuo. This concept should not become less conceivable than the earlier one regarding introduction of the displacement current, which implies that a non-vanishing curl of the magnetic field and a corresponding "current density ~~may~~ can exist in vacuo.