1) 94(5): Vector Notation used in Papers 93 and 94.

In differential form notation the electromagnetic field in ECE theory is:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(1)$$

which in tensor notation is:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu. \quad -(2)$$

The electromagnetic potential is:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(3)$$

where q^a_μ is a rank two tensor defined by:

$$V^a = q^a_\mu V^\mu. \quad -(4)$$

Here V^a and V^μ are four vectors in different frames of reference in four dimensional space-time.

Consider a particular example of eq. (2):

$$F^1_{23} = \partial_2 A^1_3 - \partial_3 A^1_2 + \omega^1_{2b} A^b_3 - \omega^1_{3b} A^b_2. \quad -(5)$$

Either side of this equation there are rank three tensors whose components must correspond to each other on both sides. Thus:

$$F^1_{23} = (\partial_2 A_3 - \partial_3 A_2)^1 + (\omega_{2b} A^b_3 - \omega_{3b} A^b_2)^1 \quad -(6)$$

Inside the brackets on the right hand side are antisymmetric tensor components which correspond to the components of an axial vector (magnetic field) or polar vector (electric field). The magnetic vector components are defined by:

$$B^1_i = \frac{1}{2} \epsilon_{ijk} F^1_{jk}, \quad -(7)$$

thus

$$B^1_1 = \frac{1}{2} \left(\epsilon_{123} F_{23} + \epsilon_{132} F_{32} \right)^1$$

$$\boxed{B^1_1 = F^1_{23}} \quad -(8)$$

This is recognized as the X component:

$$B_X = B^1_1 \quad -(9)$$

of the magnetic field:

$$\underline{B} = B_X \underline{i} + B_Y \underline{j} + B_Z \underline{k}. \quad -(10)$$

Similarly:

$$B_Y = B^2_2 = F^2_{31}, \quad -(11)$$

$$B_Z = B^3_3 = F^3_{12}. \quad -(12)$$

These results were used in paper 93 and checked by computer. So eq. (6) becomes, for the magnetic field:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega}_b \times \underline{A}^b \quad -(13)$$

in the vector notation used by electrical engineers. In this notation:

3)

$$(\underline{\omega}_b \times \underline{A}^b)_X = (\omega_{3b} A^b_2 - \omega_{2b} A^b_3)^1 \quad -(14)$$

where the minus sign follows the notation of previous papers. This is a convention.

These results are obtained by considering the special case:

$$a = \mu \quad -(15)$$

in eq. (4). This means that the vector $\bar{V}^a$ and vector $\bar{V}^\mu$ are written in the same frame of reference. This means that q^a_μ is a diagonal rank two tensor:

$$\left.\begin{aligned} \bar{V}^0 &= q^0_0 \bar{V}^0, \quad \bar{V}^1 = q^1_1 \bar{V}^1, \\ \bar{V}^2 &= q^2_2 \bar{V}^2, \quad \bar{V}^3 = q^3_3 \bar{V}^3 \end{aligned}\right\} -(16)$$

and from eq (3), A^a_μ must be diagonal. So in eq. (14):

$$(\underline{\omega}_b \times \underline{A}^b)_X = (\omega_{32} A^2_2 - \omega_{23} A^3_3)^1$$

$$\boxed{(\underline{\omega}_b \times \underline{A}^b)_X = \omega^1{}_{32} A^2_2 - \omega^1{}_{23} A^3_3} \quad -(17)$$

Similarly:

$$(\underline{\omega}_b \times \underline{A}^b)_Y = \omega^2{}_{13} A^3_3 - \omega^2{}_{31} A^1_1 \quad -(18)$$

$$(\underline{\omega}_b \times \underline{A}^b)_Z = \omega^3{}_{21} A^1_1 - \omega^3{}_{12} A^2_2 \quad -(19)$$

Therefore the meaning of the b index is given by eqs. (17) to (19). The final result is:

$$\boxed{\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}} \quad -(20)$$

as used in previous papers. The spin connection has been reduced to a vector $\underline{\omega}$. The components of this vector are, in analogy to eqs. (9) to (12),

$$\boxed{\begin{aligned} \omega_X &= \omega^1{}_1 = \omega^1{}_{23} = -\omega^1{}_{32} \quad &-(21)\\ \omega_Y &= \omega^2{}_2 = \omega^2{}_{31} = -\omega^2{}_{13} \quad &-(22)\\ \omega_Z &= \omega^3{}_3 = \omega^3{}_{12} = -\omega^3{}_{21} \quad &-(23)\end{aligned}}$$

So:

$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} + \omega_Z \underline{k}. \quad -(24)$$

Finally, if we adopt the complex circular basis:

$$\underline{B}^{(3)*} = \underline{\nabla} \times \underline{A}^{(3)*} - i\underline{\omega}^{(1)} \times \underline{A}^{(2)} \quad -(25)$$

and if:

$$\underline{\omega}^{(1)} = g\underline{A}^{(1)} \quad -(26)$$

we obtain the $\underline{B}^{(3)}$ spin field:

$$\boxed{\underline{B}^{(3)*} = -ig\underline{A}^{(1)} \times \underline{A}^{(2)}} \quad -(27)$$