THE COULOMB AND AMPÈRE MAXWELL LAWS IN GENERALLY COVARIANT UNIFIED FIELD THEORY.

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ABSTRACT

The Coulomb and Ampère Maxwell laws are calculated exactly from Einstein Cartan Evans (ECE) unified field theory. The result are given for several stationary and dynamical line elements and metrics of the Einstein Hilbert field equation, and show that in general there are relativistic corrections of the same order as those responsible for the deflection of light by gravity and perihelion advance for example. In the special relativistic limit the Coulomb and Ampère Maxwell laws of classical electrodynamics are recovered self consistently. In the stationary Schwarzschild metric there is no charge density or current density. These are finite in the dynamic Friedman Lemaitre Robertson Walker metric. The laws of classical electrodynamics are investigated for the rigorously correct Crothers metric, and other stationary metrics.

Keywords : Einstein Cartan Evans (ECE) unified field theory, exact calculation of the generally covariant laws of electrodynamics, stationary and dynamical line elements.

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1. INTRODUCTION

In classical electrodynamics the Coulomb law and Ampère Maxwell laws are well known to be a precise laws of special relativity in Minkowski space-time {1}. However in a generally covariant unified field theory all the laws of classical electrodynamics become unified with those of gravitation and other fundamental fields {2-9}. In previous work {2-9} these laws have been developed using the spin connection, revealing the presence of resonance phenomena that can lead to new sources of energy. A dielectric formulation of the laws of classical electrodynamics has also been given. This showed that light deflected by gravity also changes polarization, as observed for example in light deflected by white dwarf stars {10}. More generally, there are many optical and electro-dynamical changes predicted by Einstein Cartan Evans (ECE) unified field theory {2-9}. In Section Two various well known line elements are used to compute the Coulomb Law and Ampère Maxwell laws, starting with the Bianchi identity of differential geometry. In Section Three a discussion is given of the shortcomings of Big Bang and black hole theory, based on the rigorously correct Crothers metric. The latter is also used in Section 3 to develop the laws of classical electrodynamics into laws of general relativity. Appendices give sufficient mathematical detail to follow the derivation step by step.

2. THE GENERALLY COVARIANT COULOMB AND AMPERE MAXWELL LAWS

The starting point of the derivation is the Bianchi identity {11} of Cartan geometry:

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b \quad -(1)$$

where T^a is the torsion form, $\omega^a{}_b$ is the spin connection, d ^ denotes the wedge product of differential geometry, $R^a{}_b$ is the curvature form and q^b is the tetrad form. Using the fundamental hypothesis {2-9}:

$$A^{a} = A^{(0)} q^{a}, \quad -(2)$$

$$F^{a} = A^{(0)} T^{a}, \quad -(3)$$

Eq. (1) becomes the ECE field equation:

$$d \wedge F^{a} = \mu_0 j^{a} = A^{(0)} \left(R^{a}{}_{b} \wedge q^{b} - \omega^{a}{}_{b} \wedge T^{b} \right). \quad -(4)$$

Here A^{a} is the potential form, F^{a} is the field form, and $cA^{(0)}$ is a primordial scalar in volts. The hypothesis (2) has been tested experimentally in an extensive manner (www.aias.us). The field equation (4) is generally covariant because the Bianchi identity is generally covariant. Under the general coordinate transformation the field equation becomes:

$$\left(d \wedge F^{a} \right)' = \left(\mu_0 j^{a} \right)' \quad -(5)$$

which is:

$$\left(d \wedge T^{a} + \omega^{a}{}_{b} \wedge T^{b} \right)' := \left(R^{a}{}_{b} \wedge q^{b} \right)'. \quad -(6)$$

It retains its form under the coordinate transform because it consists of tensorial quantities. This is the essence of general relativity.

Applying the Hodge dual transform to both sides of Eq. (4) (Appendix (1)) the inhomogeneous ECE field equation is obtained:

$$d \wedge \tilde{F}^{a} = \mu_0 J^{a} = A^{(0)} \left(\tilde{R}^{a}{}_{b} \wedge q^{b} - \omega^{a}{}_{b} \wedge \tilde{T}^{b} \right). \quad -(7)$$

Here the tilde denotes Hodge transformation {2-9, 11}. It is seen that the same Hodge transform is applied to two-forms on both sides of the equation. The generally covariant Coulomb and Ampère Maxwell laws are part of the inhomogeneous field equation (7). As shown in Appendix (2), the homogeneous and inhomogeneous field equations are the tensor equations:

$$\partial_\mu \tilde{F}^{a\mu\nu} = \mu_0 \tilde{j}^{a\nu} \quad -(8)$$

and

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} \quad -(9)$$

respectively. These tensor equations are generally covariant. They look like the Maxwell Heaviside field equations but contain more information. In the special case:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b \quad -(10)$$

the homogeneous field equation becomes:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad -(11)$$

and the inhomogeneous equation becomes:

$$\partial_\mu F^{a\mu\nu} = A^{(0)} \left(R^a{}_\mu{}^{\mu\nu} \right)_{grav.} \quad -(12)$$

It has been shown {2-9} that the special case (10) is pure rotation. A solution of Eq. (10) is:

$$R^a{}_b = -\frac{\kappa}{2} \epsilon^a{}_{bc} T^c, \quad \omega^a{}_b = -\frac{\kappa}{2} \epsilon^a{}_{bc} q^c, \quad -(13)$$

in which case the curvature is this well defined dual of the torsion and the spin connection is the well defined dual of the tetrad. These results are developed in all detail elsewhere {2-9}.

When the connection is the Christoffel connection, however, the gravitational torsion vanishes:

$$(T^a)_{grav} = 0, \qquad -(14)$$

$$(T^{\kappa}_{\mu\nu})_{grav} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} = 0 \qquad -(15)$$

and the curvature form becomes the Riemann tensor:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}. \qquad -(16)$$

In this case the inhomogeneous equation becomes:

$$\partial_\mu F^{a\mu\nu} = A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu} \qquad -(17)$$

and as shown in Appendix (3) can be written as two vector equations:

$$(\nabla \cdot E)^{0} = -\phi^{(0)}\left(R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}\right) -(18)$$

and

$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t} + \mu_0 J \qquad -(19)$$

where

$$J_r = J^1_1 = -\frac{A^{(0)}}{\mu_0}\left(R^{1}{}_{0}{}^{10} + R^{1}{}_{2}{}^{12} + R^{1}{}_{3}{}^{13}\right), -(20)$$

$$J_\theta = J^2_2 = -\frac{A^{(0)}}{\mu_0}\left(R^{2}{}_{0}{}^{20} + R^{2}{}_{1}{}^{21} + R^{2}{}_{3}{}^{23}\right), -(21)$$

$$J_\phi = J^3_3 = -\frac{A^{(0)}}{\mu_0}\left(R^{3}{}_{0}{}^{30} + R^{3}{}_{1}{}^{31} + R^{3}{}_{2}{}^{32}\right). -(22)$$

Eq. (18) is the generally covariant Coulomb Law, and Eq. (19) is the generally covariant

Ampère Maxwell law. As shown in Appendix (4) the index a for the Coulomb law must be zero on both sides because it is the time-like index indicating scalar quantities on both sides, and the a indices in eqs. (20) to (22) are obtained in a well defined manner from Cartan geometry.

The generally covariant Coulomb and Ampère Maxwell laws are given by evaluating the Riemann elements on the right hand side of Eq. (17) for well known stationary and dynamic line elements and metric elements and the rigorously correct Crothers metric { 12 }. The method is summarized in Appendix (5) and uses computer algebra. It consists of choosing line elements {11}, evaluating the Christoffel symbols and Riemann tensor elements, and finally raising indices with the relevant metric elements. The final results are given as follows.

For the Minkowski line element of special relativity:

$$ds^2 = -c^2 dt^2 + dX^2 + dY^2 + dZ^2, \quad -(23)$$

$$g_{00} = -1, \; g_{11} = 1, \; g_{22} = 1, \; g_{33} = 1 \quad -(24)$$

there is no charge density and no current density, because the space-time has no curvature. So all Christoffel and Riemann elements are zero in the Minkowski space-time. This shows that Maxwell Heaviside field theory has to use charge and current densities phenomenologically, and this is neither generally covariant (objective) nor rigorously correct nor self consistent. In the stationary Schwarzschild metric as usually used:

$$ds^2 = -\left(1 - \frac{2mG}{rc^2}\right)c^2 dt^2 + \left(1 - \frac{2mG}{rc^2}\right)^{-1} r^2 + r^2 d\Omega^2 \quad -(25)$$

$$\left.\begin{aligned} g_{00} &= -\left(1 - \frac{2mG}{rc^2}\right), \; g_{11} = \left(1 - \frac{2mG}{rc^2}\right)^{-1}, \\ g_{22} &= r^2, \quad g_{33} = r^2 \sin^2\theta \end{aligned}\right\} \quad -(26)$$

there is no charge density and no current density from Eq. (17) because there is no canonical energy momentum density used in deriving this Schwarzschild line element. Here M is mass, G the Newton constant, c the speed of light (S.I.units are used in Eq. (25)) and the spherical polar coordinate system (r , θ , ϕ) is used. Therefore in both of these line elements the Coulomb and Ampère Maxwell laws are:

$$\underline{\nabla} \cdot \underline{E} = 0, \qquad -(27)$$

$$\underline{\nabla} \times \underline{B} = \frac{1}{c^2} \frac{\partial E}{\partial t}. \qquad -(28)$$

The Friedman Lemaitre Robertson Walker dynamical line element { 13 } is:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right), \qquad -(29)$$

$$g_{00} = -1, \quad g_{11} = \frac{a^2(t)}{1 - kr^2}, \quad g_{22} = a^2(t) r^2, \quad g_{33} = a^2(t) r^2 \sin^2\theta \qquad -(30)$$

where a is governed by the Friedman equations. This metric is the result of homogeneity and isotropy, as is well known { 14 }, and the Einstein Hilbert field equations are used to define the line element through the Friedman equations. Well known types of cosmologies are defined by this line element { 14 }. The line element (29) produces the Coulomb law:

and the current density components:

These depend on the type of universe, or cosmology, being considered { 11 }. The Coulomb law (   ) depends directly on the Newton constant G and the mass density   , together with:

in the rest frame, where m is mass and V is volume. In the laboratory, Eq. (   ) is the well tested Coulomb law of electrodynamics, one of the most precise laws of physics {   }. Eq. (   ) is generally covariant and upon general coordinate transformation produces new physical effects. The generally covariant Ampère Maxwell law also produces new physical effects which can be looked for experimentally. Some are already known, notably the change in polarization of light deflected by gravitation {   }. Here, the scalar potential   has the units of volts, G is the Newton constant with units of meters per kilogram, r is the radial vector of the spherical polar coordinate system (   ),   is the electric charge density and   is the vacuum permittivity in S.I. units.

APPENDIX 1 : HODGE DUAL TRANSFORMATION

The general Hodge dual of a tensor is defined { 11 } as:

$$\tilde{V}_{\mu_1 \ldots \mu_{n-p}} = \frac{1}{p!} \epsilon^{\nu_1 \ldots \nu_p}_{\mu_1 \ldots \mu_{n-p}} V_{\nu_1 \ldots \nu_p} \quad -(A1)$$

where:

$$\epsilon_{\mu_1 \mu_2 \ldots \mu_n} = |g|^{1/2} \bar{\epsilon}_{\mu_1 \mu_2 \ldots \mu_n} \quad -(A2)$$

is the totally anti-symmetric tensor, defined as the square root of the modulus of the determinant of the metric multiplied by the Levi-Civita symbol:

$$\bar{\epsilon}_{\mu_1 \mu_2 \ldots \mu_n} = \begin{cases} 1 & \text{for even permutation} \\ -1 & \text{for odd permutation} \\ 0 & \text{otherwise} \end{cases} . \quad -(A3)$$

Using the metric compatibility condition { 11 }:

$$D_\mu g_{\nu\rho} = 0 \quad -(A4)$$

it is seen that:

$$D_\mu |g|^{1/2} = \partial_\mu |g|^{1/2} = 0 \quad -(A5)$$

because the determinant of the metric is made up of individual elements of the metric tensor. The covariant derivative of each element vanishes by Eq. (A4), so we obtain Eq. (A5). The pre-multiplier $|g|^{1/2}$ is a scalar, and we use the fact that the covariant derivative of a scalar is the same as its four-derivative { 11 }:

$$D_\mu V = \partial_\mu V. \quad -(A6)$$

The homogeneous field equation (4) in tensor notation is:

$$\partial_\mu F^a_{\nu\rho} + \partial_\rho F^a_{\mu\nu} + \partial_\nu F^a_{\rho\mu}$$

$$= -A^{(0)}\left(R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} + \omega^a_{\mu b} T^b_{\nu\rho} + \omega^a_{\rho b} T^b_{\mu\nu} + \omega^a_{\nu b} T^b_{\rho\mu}\right) \quad \text{— (A7)}$$

and this is equivalent {2-8,12} to:

$$\partial_\mu \tilde{F}^{a\mu\nu} = \mu_0 \tilde{j}^{\nu a} := -A^{(0)}\left(\tilde{R}^{a}{}_{\mu}{}^{\mu\nu} + \omega^a_{\mu b} \tilde{T}^{b\mu\nu}\right) \quad \text{— (A8)}$$

The Hodge dual of a two-form in four-dimensional space-time is another two-form. For example:

$$\tilde{F}^{a\mu\nu} = \frac{1}{2}|g|^{1/2} \bar{\epsilon}^{\mu\nu\rho\sigma} F^a_{\rho\sigma}, \quad \text{— (A9)}$$

$$\tilde{T}^{a\mu\nu} = \frac{1}{2}|g|^{1/2} \bar{\epsilon}^{\mu\nu\rho\sigma} T^a_{\rho\sigma}, \quad \text{— (A10)}$$

$$\tilde{R}^{a}{}_{b}{}^{\mu\nu} = \frac{1}{2}|g|^{1/2} \bar{\epsilon}^{\mu\nu\rho\sigma} R^a{}_{b\rho\sigma}. \quad \text{— (A11)}$$

— (A12)

The Bianchi identity:

$$d \wedge T^a_{\mu\nu} + \omega^a{}_b \wedge T^b_{\mu\nu} := -q \wedge R^a{}_{b\mu\nu}$$

is an identity between two-forms. So it remains true for:

$$d \wedge \tilde{F}^a_{\mu\nu} = -A^{(0)}\left(\tilde{R}^a{}_{b\mu\nu} + \omega^a{}_b \wedge \tilde{T}^a_{\mu\nu}\right) \quad \text{— (A13)}$$

because $\tilde{F}_{\mu\nu}$, $\tilde{R}_{\mu\nu}$, and $\tilde{T}_{\mu\nu}$ are two-forms, antisymmetric in their last two indices. In other words if we write down the sum:

$$\partial_\mu \tilde{F}^a_{\nu\rho} + \partial_\rho \tilde{F}^a_{\mu\nu} + \partial_\nu \tilde{F}^a_{\rho\mu} := d \wedge \tilde{F}^a \quad -(A14)$$

$$-(A15)$$

it is identically equal to the sum:

$$-A^{(0)}\left(\tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} + \omega^a_{\mu b}\tilde{T}^b_{\nu\rho} + \omega^a_{\rho b}\tilde{T}^b_{\mu\nu} + \omega^a_{\nu b}\tilde{T}^b_{\rho\mu}\right)$$
$$:= -A^{(0)}\left(q^b \wedge \tilde{R}^a{}_b + \omega^a{}_b \wedge \tilde{T}^b\right)$$

So the inhomogeneous field equation is:

$$d \wedge \tilde{F}^a = \mu_0 J^a = -A^{(0)}\left(q^b \wedge \tilde{R}^a{}_b + \omega^a{}_b \wedge \tilde{T}^b\right) \quad -(A16)$$

which is equivalent to:

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} \quad -(A17)$$

as given in the text.

APPENDIX 2 : EQUIVALENCE OF INDICES IN THE FIELD EQUATIONS

The homogeneous and inhomogeneous field equations can be written in equivalent ways, and the equivalence is proven in this Appendix. The first method of writing the homogeneous field equation is the sum:

$$\partial_\mu F^a_{\nu\rho} + \partial_\rho F^a_{\mu\nu} + \partial_\nu F^a_{\rho\mu} = \mu_0 \left(j^a_{\mu\nu\rho} + j^a_{\rho\mu\nu} + j^a_{\nu\rho\mu} \right) \quad \text{-- (B1)}$$

where the charge current density three-forms are defined by:

$$j^a_{\mu\nu\rho} + j^a_{\rho\mu\nu} + j^a_{\nu\rho\mu} := -\frac{A^{(0)}}{\mu_0} \Big(R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} + \omega^a_{\mu b} T^b_{\nu\rho} + \omega^a_{\rho b} T^b_{\mu\nu} + \omega^a_{\nu b} T^b_{\rho\mu} \Big). \quad \text{-- (B2)}$$

Consider individual tensor elements such as those defined by

$$\partial_0 \tilde{F}^{a01} + \partial_2 \tilde{F}^{a21} + \partial_3 \tilde{F}^{a31} = \frac{1}{2} |g| \bar{\epsilon}^{\mu 1\rho\sigma} \partial_\mu F^a_{\rho\sigma}$$
$$= \frac{1}{2} |g|^{1/2} \left(\bar{\epsilon}^{01\rho\sigma} \partial_0 F^a_{\rho\sigma} + \bar{\epsilon}^{21\rho\sigma} \partial_2 F^a_{\rho\sigma} + \bar{\epsilon}^{31\rho\sigma} \partial_3 F^a_{\rho\sigma} \right)$$
$$= |g|^{1/2} \left(\partial_0 F^a_{23} + \partial_2 F^a_{30} + \partial_3 F^a_{02} \right) \quad \text{-- (B3)}$$

which is a special case of the general result:

$$\partial_\mu \tilde{F}^{a\mu\nu} = |g|^{1/2} \left(\partial_\mu F^a_{\nu\rho} + \partial_\rho F^a_{\mu\nu} + \partial_\nu F^a_{\rho\mu} \right). \quad \text{-- (B4)}$$

Now consider the following current term for $\sigma = 1$ to obtain:

$$\tilde{j}^{a\sigma} = \frac{1}{6} |g|^{1/2} \bar{\epsilon}^{\mu\nu\rho\sigma} j^a_{\mu\nu\rho}, \quad (\sigma = 1) \quad \text{-- (B5)}$$

$$\tilde{j}^{a1} = \frac{1}{3} |g|^{1/2} \left(j^a_{023} + j^a_{302} + j^a_{230} \right). \quad \text{-- (B6)}$$

Similarly, the other two current terms

$$\tilde{j}^{a\sigma} = \frac{1}{6}|g|^{1/2}\bar{\epsilon}^{\rho\mu\nu\sigma}\dot{j}^{a}_{\rho\mu\nu} \quad -(B7)$$

and
$$\tilde{j}^{a\sigma} = \frac{1}{6}|g|^{1/2}\bar{\epsilon}^{\nu\rho\mu\sigma}\dot{j}^{a}_{\nu\rho\mu} \quad -(B8)$$

give Eq. (B6) two more times. So the right hand side of Eq. (B1) for $\nu = 1$ is:

$$\tilde{j}^{a1} = |g|^{1/2}\left(\dot{j}^{a}_{023} + \dot{j}^{a}_{302} + \dot{j}^{a}_{230}\right). \quad -(B9)$$

Finally use Eq. (A5) to find that:

$$\partial_\mu\left(|g|^{1/2}F^{a}_{\nu\rho}\right) = |g|^{1/2}\partial_\mu F^{a}_{\nu\rho} \quad -(B10)$$

and so derive Eq. (8) from Eq. (B1), Q.E.D. Note that the pre-multiplier $|g|^{1/2}$ cancels out on either side of Eq. (8).

Similarly it can be shown that the following expression of the inhomogeneous field equation:

$$\partial_\mu\tilde{F}^{a}_{\nu\rho} + \partial_\rho\tilde{F}^{a}_{\mu\nu} + \partial_\nu\tilde{F}^{a}_{\rho\mu} \quad -(B11)$$
$$= -A^{(0)}\left(\tilde{R}^{a}{}_{\mu\nu\rho} + \tilde{R}^{a}{}_{\rho\mu\nu} + \tilde{R}^{a}{}_{\nu\rho\mu} + \omega^{a}{}_{\mu b}\tilde{T}_{\nu\rho} + \omega^{a}{}_{\rho b}\tilde{T}_{\mu\nu} + \omega^{a}{}_{\nu b}\tilde{T}_{\rho\mu}\right)$$

is equivalent to:

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} \quad -(B12)$$

as used in the text.

As a familiar example of Appendices 1 and 2 consider the Maxwell Heaviside (MH) equations in free space. The homogeneous MH equation in differential form notation is

$$d \wedge F = 0 \qquad -(B13)$$

which is either:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad -(B14)$$

or

$$\partial_\mu F_{\nu\rho} + \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} = 0 \qquad -(B15)$$

in tensor notation. The inhomogeneous MH equation in differential form notation is:

$$d \wedge \tilde{F} = 0 \qquad -(B16)$$

which is either:

$$\partial_\mu F^{\mu\nu} = 0 \qquad -(B17)$$

or

$$\partial_\mu \tilde{F}_{\nu\rho} + \partial_\rho \tilde{F}_{\mu\nu} + \partial_\nu \tilde{F}_{\rho\mu} = 0 \qquad -(B18)$$

in tensor notation. The individual Hodge dual tensors are defined by:

$$\tilde{F}^{\nu\rho} = \frac{1}{2} \epsilon^{\nu\rho\mu\sigma} F_{\mu\sigma} \quad \text{etc.} \qquad -(B19)$$

and indices are lowered as follows:

$$\tilde{F}_{\nu\rho} = g_{\nu\rho}\, g_{\rho\kappa}\, \tilde{F}^{\rho\kappa} \quad \text{etc.} \qquad -(B20)$$

where $g_{\mu\nu}$ is the Minkowski metric in this case. The equivalent ECE equations in free space have the same properties exactly except of the addition of the index a to every tensor in the equations. Finally the homogeneous ECE equation in form notation is:

$$d \wedge F^a = \mu_0 j^a \quad - (B21)$$

which is

$$\partial_\mu \tilde{F}^{\mu\nu a} = \mu_0 \tilde{j}^{\nu a} \quad - (B22)$$

in tensor notation. The inhomogeneous ECE equation in form notation is:

$$d \wedge \tilde{F}^a = \mu_0 J^a \quad - (B23)$$

which is

$$\partial_\mu F^{\mu\nu a} = \mu_0 J^{\nu a} \quad - (B24)$$

in tensor notation. The individual Hodge duals are:

$$\tilde{F}^{\nu\rho a} = \frac{1}{2} |g|^{1/2} \epsilon^{\nu\rho\mu\sigma} F^a_{\mu\sigma} \quad \text{etc.} \quad - (B25)$$

and indices are lowered with the metric of the base manifold:

$$\tilde{F}^a_{\nu\rho} = g_{\nu\rho} \, g_{\rho\kappa} \, \tilde{F}^{\nu\rho a} \quad \text{etc.} \quad - (B26)$$

APPENDIX 3 : REDUCTION TO VECTOR NOTATION

In this appendix the tensorial form of the inhomogeneous ECE equation is reduced to the vector form, giving the Coulomb and Ampère Maxwell laws in generally covariant unified field theory. Begin with the inhomogeneous field equation:

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} = -\frac{A^{(0)}}{\mu_0}\left(R^{a}{}_{\mu}{}^{\mu\nu} + \omega^{a}{}_{\mu b} T^{b\mu\nu}\right). \quad -(C1)$$

In the Einstein Hilbert limit:

$$T^{b\mu 0} = 0 \quad -(C2)$$

so the equation becomes:

$$\partial_\mu F^{a\mu\nu} = -\frac{A^{(0)}}{\mu_0} R^{a}{}_{\mu}{}^{\mu\nu}. \quad -(C3)$$

The indices in the Riemann tensor elements are raised using the metric of the base manifold as follows:

$$R^{a}{}_{\mu}{}^{\sigma\rho} = g^{\sigma\nu} g^{\rho\kappa} R^{a}{}_{\mu\nu\kappa}. \quad -(C4)$$

The Coulomb Law is obtained for:

$$\nu = 0 \quad -(C5)$$

and is:

$$\partial_\mu F^{a\mu 0} = -\frac{A^{(0)}}{\mu_0}\left(R^{a}{}_{1}{}^{10} + R^{a}{}_{2}{}^{20} + R^{a}{}_{3}{}^{30}\right) \quad -(C6)$$

where summation over repeated μ indices has been carried out. The vector form of eq. (C6) is:

$$(\underline{\nabla} \cdot \underline{E})^a = -\phi \left(R^a{}_1{}^{10} + R^a{}_2{}^{20} + R^a{}_3{}^{30} \right). \quad -(C7)$$

The only possible value of a (see also Appendix Four) for the Coulomb Law is:

$$a = 0 \quad -(C8)$$

so we obtain the generally covariant Coulomb Law:

$$\underline{\nabla} \cdot \underline{E} = (\underline{\nabla} \cdot \underline{E})^0 = -\phi \left(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} \right). \quad -(C9)$$

Both sides are scalar valued quantities, so the time-like, or scalar, index a = 0 is used. Here ϕ is the scalar potential, having the units of volts. The units of $\underline{E}$ are volt / m and those of the R elements are inverse meters squared, so units are consistent.

The generally covariant Ampère Maxwell law is obtained with:

$$\nu = 1, 2, 3. \quad -(C10)$$

When:

$$\nu = 1 \quad -(C11)$$

Eq. (C3) becomes:

$$\partial_0 F^{a01} + \partial_2 F^{a21} + \partial_3 F^{a31} = -\frac{A^{(0)}}{\mu_0} R^a{}_\mu{}^{\mu 1}. \quad -(C12)$$

The vector form of this equation is:

$$(\underline{\nabla} \times \underline{B})^a_1 = \frac{1}{c^2}\frac{\partial E^a_1}{\partial t} + \frac{\mu_0}{c} J^a_1 . \quad -(C13)$$

Here, the 1 subscript denotes a component in a particular coordinate system. For example in the spherical polar system:

$$1 = r \quad -(C14)$$

or in the Cartesian system:

$$1 = X . \quad -(C15)$$

So Eq. (C13) is the r or X component of the Ampère Maxwell Law. If we adopt the spherical polar system for the Riemann elements (see Appendix 5) the value of a in Eq. (C13) must also be 1. If the complex circular basis {2 - 9} is chosen then:

$$a = (1), (2), (3) . \quad -(C16)$$

However, if the complex circular basis is chosen, then the relevant Riemann elements are:

$$R^{(1)\ \mu 1}_{\ \ \ \mu}, \ R^{(2)\ \mu 2}_{\ \ \ \mu}, \ R^{(3)\ \mu 3}_{\ \ \ \mu} \quad -(C17)$$

in which one index is complex circular, and the other three are spherical polar. It is possible to use either system, or any other system of coordinates for a. Therefore the generally covariant Ampère Maxwell Law is:

$$\underline{\nabla} \times \underline{B} = \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} + \frac{\mu_0}{c}\underline{J} \quad -(C18)$$

where the charge current density is defined as:

$$\underline{J} = J^1_1 \underline{e}_r + J^2_2 \underline{e}_\theta + J^3_3 \underline{e}_\phi \quad -(C19)$$

with the scalar valued components:

$$J^1_1 = -\frac{A^{(0)}}{\mu_0} \left(R^1{}_0{}^{01} + R^1{}_2{}^{21} + R^1{}_3{}^{31} \right), \quad -(C20)$$

$$J^2_2 = -\frac{A^{(0)}}{\mu_0} \left(R^2{}_0{}^{02} + R^2{}_1{}^{12} + R^2{}_3{}^{32} \right), \quad -(C21)$$

$$J^3_3 = -\frac{A^{(0)}}{\mu_0} \left(R^3{}_0{}^{03} + R^3{}_1{}^{13} + R^3{}_2{}^{23} \right). \quad -(C22)$$

A particular metric may finally be used to calculate these Riemann components exactly, and example is given in detail in Appendix 5.

The main result is that in the presence of space-time curvature, the electro-dynamical properties of light are changed, in addition to the well known effects of Einstein Hilbert theory there are polarization changes in light deflected by gravitation. These are due to the charge current density $\underline{J}$, which does not exist in the free space limit of Maxwell Heaviside theory. So these are predictions of ECE theory that are known already to be corroborated qualitatively {2-9}, because of observations of polarization changes in light deflected by a white dwarf for example.

APPENDIX 4 : THE MEANING OF THE a INDEX

It is first noted that the ECE field equations originate in the Bianchi identity:

$$D \wedge F^a := R^a{}_b \wedge A^b \qquad -(D1)$$

where the a and b indices denote those of a tangent space-time at point P in a base manifold in differential geometry. Thus:

$$D \wedge F^a = D_\mu F^a{}_{\nu\sigma} + D_\sigma F^a{}_{\mu\nu} + D_\nu F^a{}_{\sigma\mu} \qquad -(D2)$$

where the Greek indices of the base manifold have been restored. In generally covariant unified field theory the electromagnetic field tensor is therefore a vector-valued two-form, i.e. an anti-symmetric tensor for each a. The field two-form is defined as:

$$F^a{}_{\mu\nu} = q^a{}_\kappa F^\kappa{}_{\mu\nu} \qquad -(D3)$$

where $F^\kappa{}_{\mu\nu}$ is a tensor in the base manifold with three indices. It is seen that:

$$D \wedge F^\kappa := R^\kappa{}_b \wedge A^b \qquad -(D4)$$

using the tetrad postulate:

$$D_\mu q^a{}_\kappa = 0. \qquad -(D5)$$

Therefore Eq. (D1) can be written in the base manifold:

$$D_\mu F^\kappa{}_{\nu\sigma} + D_\sigma F^\kappa{}_{\mu\nu} + D_\nu F^\kappa{}_{\sigma\mu} := A^{(0)}\left(R^\kappa{}_{\mu\nu\sigma} + R^\kappa{}_{\sigma\mu\nu} + R^\kappa{}_{\nu\sigma\mu}\right). \qquad -(D6)$$

In general relativity and unified field theory the base manifold is four dimensional space-time in which curvature and torsion are both present in general. So the electromagnetic field in this base manifold is a rank three tensor, not a rank two tensor as in special relativity and

Minkowski space-time. In the latter type of space-time there is no curvature and no torsion, so Minkowski space-time is flat space-time.

For example, consider the electric field components:

$$F^{\kappa}{}_{\nu\sigma} = F^{\kappa}{}_{10}, \; F^{\kappa}{}_{20}, \; F^{\kappa}{}_{30}. \qquad -(D7)$$

In the complex circular basis:

$$\kappa = (1), (2), (3) \qquad -(D8)$$

and we recover the three vector components $E_x^{(1)}$, $E_y^{(1)}$, and $E_Z^{(1)}$. The first two denote complex conjugate plane waves:

$$\underline{E}^{(1)} = \underline{E}^{(2)*} = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) e^{i(\omega t - \kappa Z)}. \qquad -(D9)$$

So the meaning of κ Superimposed on $\nu\sigma$ is that one coordinate system is superimposed on another. When one coordinate system is imposed on the same coordinate system the only possibilities are:

$$F^{\kappa}{}_{\nu\sigma} = F^{1}{}_{10}, F^{2}{}_{20}, F^{3}{}_{30} = E^{1}{}_{1}, \; E^{2}{}_{2}, \; E^{3}{}_{3} \qquad -(D10)$$

as used in Appendix 4.

APPENDIX 5, CHRISTOFFEL SYMBOLS AND RIEMANN ELEMENTS

The non-vanishing Christoffel symbols and Riemann elements of each line element used in this paper were computed using a program written by Horst Eckardt based on Maxima { 15 }, after first hand checking the program for correctness. For the spherically symmetric line element:

$$ds^2 = -e^{2\alpha} dt^2 c^2 + e^{2\beta} dr^2 + r^2 d\Omega^2 \quad -(E1)$$

$$g_{00} = -e^{2\alpha}, \; g_{11} = e^{2\beta}, \; g_{22} = r^2, \; g_{33} = r^2 \sin^2\theta, \quad -(E2)$$

it was checked by hand calculation and by computer that the Christoffel symbols and Riemann elements are as given by Carroll { 11 } as follows:

$$\Gamma^0_{00} = \partial_0 \alpha, \; \Gamma^0_{01} = \partial_1 \alpha, \; \Gamma^0_{11} = e^{2(\beta-\alpha)} \partial_0 \beta,$$
$$\Gamma^1_{00} = e^{2(\alpha-\beta)} \partial_1 \alpha, \; \Gamma^1_{01} = \partial_0 \beta, \; \Gamma^1_{11} = \partial_1 \beta,$$
$$\Gamma^2_{12} = 1/r, \; \Gamma^1_{22} = -r e^{-2\beta}, \; \Gamma^3_{13} = 1/r,$$
$$\Gamma^1_{33} = -r e^{-2\beta} \sin^2\theta, \; \Gamma^2_{33} = -\sin\theta\cos\theta, \; \Gamma^3_{23} = \cos\theta / \sin\theta, \quad -(E3)$$

$$R^0{}_{101} = e^{2(\beta-\alpha)} \left(\partial_0^2 \beta + (\partial_0 \beta)^2 - \partial_0 \alpha \, \partial_0 \beta \right) + \partial_1 \alpha \, \partial_1 \beta - \partial_1 (\partial_1 \alpha) - (\partial_1 \alpha)^2$$
$$R^0{}_{202} = -r e^{-2\beta} \partial_1 \alpha,$$
$$R^0{}_{303} = R^0{}_{202} \sin^2\theta,$$
$$R^1{}_{212} = r e^{-2\beta} \partial_1 \beta,$$
$$R^1{}_{313} = r e^{-2\beta} \partial_1 \beta \sin^2\theta = R^1{}_{212} \sin^2\theta,$$
$$R^2{}_{323} = (1 - e^{-2\beta}) \sin^2\theta, \quad -(E4)$$
$$(0, 1, 2, 3) := (t, r, \theta, \phi)$$

The inverse metric elements are related to the metric elements as follows:

$$
\begin{aligned}
g^{00} &= g_{00}^{-1} = -e^{-2\alpha}, \\
g^{11} &= g_{11}^{-1} = e^{-2\beta}, \\
g^{22} &= g_{22}^{-1} = 1/r^2 \\
g^{33} &= g_{33}^{-1} = 1/(r^2 \sin^2\theta)
\end{aligned}
\qquad -(E5)
$$

in the spherical polar coordinate system (r , θ , ϕ). The Christoffel symbol in Riemann geometry is:

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right) \qquad -(E6)$$

where summation is implied over repeated indices in the covariant - contravariant system.

The non-vanishing Riemann elements are calculated from the Christoffel symbols using the definition of the Riemann tensor:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda} \Gamma^{\lambda}_{\mu\sigma} \qquad -(E7)$$

where summation is again implied over repeated indices.

In order to calculate the Coulomb law and Ampère Maxwell laws, indices must be raised with the metrics:

$$
\begin{aligned}
R^{0}{}_{1}{}^{10} &= -R^{0}{}_{1}{}^{01} = -g^{11} g^{00} R^{0}{}_{101} = R^{0}{}_{101}, \\
R^{0}{}_{2}{}^{20} &= -R^{0}{}_{2}{}^{20} = -g^{22} g^{00} R^{0}{}_{202}, \\
R^{0}{}_{3}{}^{30} &= -R^{0}{}_{3}{}^{03} = -g^{33} g^{00} R^{0}{}_{303},
\end{aligned}
\qquad -(E8)
$$

and this procedure was adhered to for each line element.