

1) Q3(17): Ampere Maxwell Law, Radial Component of \underline{J}

The current density is given by:

$$J_1 = J_r = -\frac{A^{(0)}}{\mu_0} (R^1_{010} + R^1_{212} + R^1_{313}) \quad - (1)$$

$$J_2 = J_\theta = -\frac{A^{(0)}}{\mu_0} (R^2_{020} + R^2_{121} + R^2_{323}) \quad - (2)$$

$$J_3 = J_\phi = -\frac{A^{(0)}}{\mu_0} (R^3_{030} + R^3_{131} + R^3_{232}) \quad - (3)$$

We need:

$$R^1_{212} = r e^{-2\beta} \partial_1 \beta \quad - (4) \quad (C7.15)$$

$$R^1_{313} = r e^{-2\beta} \partial_1 \beta \sin^2 \theta \quad - (5) \quad "$$

$$R^2_{323} = (1 - e^{-2\beta}) \sin^2 \theta \quad - (6) \quad "$$

So: $R^1_{212} = -\frac{x}{2} \checkmark$, $R^1_{313} = -\frac{x}{2} \sin^2 \theta \checkmark$ - (7)

$$R^2_{323} = (1 - e^{-2\beta}) \sin^2 \theta = x \sin^2 \theta \quad - (8)$$

$$R^1_{212} = -\frac{x}{2}, \quad R^1_{313} = -\frac{x}{2} \sin^2 \theta, \quad R^2_{323} = x \sin^2 \theta,$$

$$g^{00} = -\frac{1}{1-x}, \quad g^{11} = 1-x, \quad g^{22} = \frac{1}{r^2}, \quad g^{33} = \frac{1}{r^2 \sin^2 \theta}$$

- (9)

So $R^1_{212} = g^{11} g^{22} R^1_{212} = \frac{x}{2r^2} (x-1) \quad - (10)$

$$R^1_{313} = g^{11} g^{33} R^1_{313} = \frac{x}{2r^2} (x-1) \quad - (11)$$

The third element of J_r is given by:

$$2) \quad R_{0^{10}}^1 = g'' g'' R_{010}^1 = g'' g'' g'' R_{1010} \\ = g'' g'' g'' g'' R_{101}^0,$$

$$\text{i.e.} \quad R_{0^{10}}^1 = (g'')^2 R_{101}^0 \quad - (9)$$

$$= - \frac{(1-x)^2}{2r^2} \left(\frac{x}{1-x} \right) \left(\frac{x}{1-x} - 1 \right)$$

$$= - \frac{x(1-x)}{2r^2} \left(\frac{x}{1-x} - 1 \right)$$

$$= - \frac{x}{2r^2} (x - (1-x)) = - \frac{x}{2r^2} (2x-1)$$

$$R_{0^{10}}^1 = - \frac{x}{2r^2} (2x-1), \quad R_{2^{12}}^1 = R_{3^{13}}^1 = \frac{x(x-1)}{2r^2}$$

- (10)

$$\text{So:} \quad R_{0^{10}}^1 + R_{2^{12}}^1 + R_{3^{13}}^1$$

$$= - \frac{x(2x-1)}{2r^2} + \frac{x(x-1)}{r^2}$$

$$= \frac{1}{r^2} \left(x(x-1) - \frac{1}{2} x(2x-1) \right)$$

$$= \frac{1}{r^2} \left(x^2 - x - x^2 + \frac{x}{2} \right) = - \frac{x}{2r^2}$$

$$\bar{J}_r = \frac{A^{(0)}}{\mu_0} \cdot \frac{x}{2r^2} \quad - (11)$$

$$x = \frac{2mG}{r^2}$$