

# Table 1a: Metrics for the Coulomb and Ampère Maxwell Laws

| Metric            | Metrical Structure   |
|-------------------|--|
| Minkowski         | $ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2,$ $g_{00} = -1, g_{11} = 1, g_{22} = 1, g_{33} = 1$   |
| Schwarzschild     | $ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$ $g_{00} = -\left(1 - \frac{2GM}{rc^2}\right), g_{11} = \left(1 - \frac{2GM}{rc^2}\right)^{-1}, g_{22} = r^2, g_{33} = r^2 \sin^2 \theta$   |
| Gödel             | $ds^2 = \frac{1}{2\omega^2} \left( - (dt + \exp(x) dz)^2 + dx^2 + dy^2 + \frac{1}{2} \exp(2x) dz^2 \right)$ <p style="text-align: center;">Diagonals and off-diagonals</p>   |
| FLRW              | $ds^2 = -c^2 dt^2 + a(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$ $g_{00} = -1, g_{11} = \frac{a^2(t)}{1 - kr^2}, g_{22} = a^2 r^2, g_{33} = a^2 r^2 \sin^2 \theta$ <p>wie: <math>\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G (\rho + 3p),</math><br/> <math>\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \rho - \frac{k}{a^2}</math></p> |
| General Spherical | $ds^2 = -e^{2\alpha(r,t)} c^2 dt^2 + e^{2\beta(r,t)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$ $g_{00} = -e^{2\alpha}, g_{11} = e^{2\beta}, g_{22} = r^2, g_{33} = r^2 \sin^2 \theta$  |

Table 1a Continued

## Metrical Structure

| Metric                            | Metrical Structure  |
|-----------------------------------|---|
| Crothers General                  | $ds^2 = -A(c(r))^{1/2} dt^2 + B(c(r))^{1/2} r^2$ $+ c(r) (d\theta^2 + \sin^2 \theta d\phi^2),$ <p>where <math>c(r) = ( r - r_0 ^n + d^n)^{2/n}</math></p>   |
| Crothers / Original Schwarzschild | $n = 3, r_0 = 0, r > r_0$   |
| Crothers / Schwarzschild          | $n = 1, r_0 = d, r > r_0$   |
| Crothers Type one                 | $ds^2 = -c^2 dt^2 + dr^2,  r - r_0 ^2 (d\theta^2 + \sin^2 \theta d\phi^2)$  |
| Static de Sitter                  | $ds^2 = \left(1 - \frac{r^2}{d^2}\right) c^2 dt^2 + \left(1 - \frac{r^2}{d^2}\right)^{-1} dr^2 + r^2 d\Omega^2$ $g_{00} = -\left(1 - \frac{r^2}{d^2}\right), g_{11} = \left(1 - \frac{r^2}{d^2}\right)^{-1}, g_{22} = r^2,$ $g_{33} = r^2 \sin^2 \theta.$ |
| Kasner                            | $ds^2 = -c^2 dt^2 + \sum_{j=1}^{D-1} t^{2p_j} (dx_j)^2,$ $\sum_{j=1}^{D-1} p_j = 1, \quad \sum_{j=1}^{D-1} p_j^2 = 1, \quad D > 3$  |
| Perfect Fluid Sphere              | $ds^2 = -(1 + ar^2) c^2 dt^2 + \frac{(1 - 3ar^2)^{2/3}}{(1 + 3ar^2)^{2/3} - br^2} dr^2$ $+ r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$   |
| Friedmann Dust                    | $ds^2 = -c^2 dt^2 + \left(\cosh\left(\frac{3t}{a}\right) - 1\right)^{2/3} (x^2 + y^2 + z^2)$  |

Table 1b: Charge and Current Densities for the Metrics of Table 1a.

| Metric                  | Charge and Current Densities   |
|-------------------------|--|
| Wilkowski               | $J^0 = 0, J_r = 0, J_\theta = 0, J_\phi = 0$   |
| Schwarzschild           | $J^0 = 0, J_r = 0, J_\theta = 0, J_\phi = 0$   |
| Cottler / Schwarzschild | $J^0 = 0, J_r = 0, J_\theta = 0, J_\phi = 0$   |
| Gödel                   | $J^0 = -\phi \left( \frac{64\omega^8}{1024\omega^8 - 64\omega^4 + 1} \right),$ $J_r = -\frac{A^{(0)}}{\mu_0} \left( \frac{64\omega^8 - 4\omega^4}{32\omega^4 - 1} \right),$ $J_\theta = 0,$ $J_\phi = -\frac{A^{(0)}}{\mu_0} \left( \frac{(128\omega^8 - 8\omega^4) e^{-2x}}{1024\omega^8 - 64\omega^4 + 1} \right).$      |
| FLRW                    | $J^0 = -3\phi \ddot{a}/a = 4\pi\phi\sigma(\rho + 3p),$ $J_r = -\frac{A^{(0)}}{\mu_0} \left( \frac{2}{a^4} (k + \ddot{a}^2)(kr^2 - 1) + \frac{\ddot{a}}{a^3} (kr^2 - 1) \right),$ $J_\theta = J_\phi \sin^2\theta = \frac{A^{(0)}}{\mu_0} \left( \frac{2}{a^4 r^2} (k + \ddot{a}^2) + \frac{\ddot{a}}{a^3 r^2} \right).$    |
| Cottler General         | $J^0 = -\ddot{c}\phi / (4ABC^2),$ $J_r = -\frac{A^{(0)}}{\mu_0} \left( \frac{5C\ddot{c} - 4\dot{c}^2}{4B^2C^2} \right),$ $J_\theta = \frac{A^{(0)}}{\mu_0} \left( \frac{2BC^{1/2} - \ddot{c}}{2BC^{5/2}} \right)$ $= J_\phi \sin^2\theta,$ <p>where <math>\ddot{c} = \frac{d^2C}{dr^2}, \dot{c} = \frac{dC}{dr}</math></p> |

## Table 1b Continued.

Works with  
Defined  $c(r)$

Here  $c(r) = (|r - r_0|^n + d^n)^{2/n}$   
Finite charge and current densities. In the  
special case:

$$r_0 = 0, \quad d = 0$$

The following are obtained.

$$J^0 = -\frac{\phi}{2r^4 AB},$$

$$J_r = \frac{A^{(0)}}{\mu_0} \cdot \frac{3}{2r^4 B^2},$$

$$J_\theta = \sin^2 \theta \cdot J_\phi = \frac{A^{(0)}}{\mu_0} \left( \frac{r^2 B - |r|}{r^6 B} \right)$$

Static  
de Sitter

$$J^0 = -\frac{3\phi}{r^2 - d^2},$$

$$J_r = -\frac{A^{(0)}}{\mu_0} \left( \frac{3r^2 - 3d^2}{d^4} \right),$$

$$J_\theta = \sin^2 \theta \cdot J_\phi = \frac{A^{(0)}}{\mu_0} \cdot \frac{3}{d^2 r^2}$$

Wormhole  
Metric

$$ds^2 = -c^2 dt^2 + dr^2 + (e^2 + k^2)(d\theta^2 + \sin^2 \theta d\phi^2).$$

Here:

$$J^0 = 0,$$

$$J_r = -\frac{A^{(0)}}{\mu_0} \left( \frac{2k^2}{(e^2 + k^2)^2} \right),$$

$$J_\theta = J_\phi = 0.$$

Alcubierre,  
Kerr and  
Charged Kerr

Here  $R \wedge \nu \neq 0$  (?)  
so these metrics do not correctly obey  
the Ricci cyclic equation, usually known  
as the first Bianchi identity.

# Table 1b Continued

General Spherical Metric

$$ds^2 = -c^2 dt^2 e^{2\alpha} + e^{2\beta} dr^2 + d\theta^2 + \sin^2 \theta d\phi^2,$$

Here:

$$J^0 = \frac{1}{r} \left( e^{-2\beta-4\alpha} \left( \left( e^{2\beta} \frac{d^2\beta}{dt^2} \right) + e^{2\beta} \left( \frac{d\beta}{dt} \right)^2 - \frac{d\alpha}{dt} e^{2\beta} \frac{d\beta}{dt} + e^{2\alpha} \frac{d\alpha}{dr} \frac{d\beta}{dr} - e^{2\alpha} \frac{d^2\alpha}{dr^2} \right) r - 2e^{2\alpha} \frac{d\alpha}{dr} \right) / r,$$

$$J_r = \frac{A^{(0)}}{\mu_0} \left( e^{-4\beta-2\alpha} \left( \left( e^{2\beta} \frac{d^2\beta}{dt^2} + e^{2\beta} \left( \frac{d\beta}{dt} \right)^2 - \frac{d\alpha}{dt} e^{2\beta} \frac{d\beta}{dt} + e^{2\alpha} \frac{d\alpha}{dr} \frac{d\beta}{dr} - e^{2\alpha} \frac{d^2\alpha}{dr^2} - e^{2\alpha} \left( \frac{d\alpha}{dr} \right)^2 \right) r + 2e^{2\alpha} \frac{d\beta}{dr} \right) / r,$$

$$J_\theta = J_\phi \sin^2 \theta$$

$$= \frac{A^{(0)}}{\mu_0} \frac{1}{r^4} e^{-2\beta} \left( \left( \frac{d\beta}{dr} - \frac{d\alpha}{dr} \right) r + e^{2\beta} - 1 \right).$$

The charge and current densities are not zero in general.

Metric

Charge and Current Densities

Friedman  
Dust

$$J^0 = -\frac{\phi}{a^2} \left( 6 \tanh \left( \frac{3t-a}{a} \right)^2 - 9 \right),$$

$$J_r = \frac{A^{(0)}}{\mu_0} \cdot \frac{3}{a^2 \cosh \left( \frac{3t-a}{a} \right)^{2/3}},$$

$$= J_\theta = J_\phi$$

Perfect  
Spherical  
Fluid

$$J^0 = -\phi \left( (1-3ar^2)^{1/3} (3ar^2+1)^{1/3} \right. \\ \left. (7a^3br^6 + 7a^2br^4 - 4abr^2) \right. \\ \left. + (1-3ar^2)^{1/3} (-18a^4r^6 - 23a^3r^4 + 6a^2r^2 + 3a) \right) \\ / \left( (3ar^2+1)^{1/3} (9a^5r^{10} + 21a^4r^8 + 10a^3r^6 \right. \\ \left. - 6a^2r^4 - 3ar^2 + 1) \right),$$

$$J_r = \frac{A^{(0)}}{\mu_0} \left( (3ar^2+1)^{2/3} (3a^3b^2r^8 + 5a^2b^2r^6 \right. \\ \left. - 4ab^2r^4 - 2b^2r^2) + (3ar^2+1)^{1/3} \right. \\ \left. (-9a^4br^8 - 15a^3br^6 + 27a^2br^4 + 19abr^2 + 2b) \right. \\ \left. - 9a^4r^6 - 63a^3r^4 - 47a^2r^2 - 9a \right) \\ / \left( (1-3ar^2)^{1/3} (3ar^2+1)^{2/3} \right. \\ \left. (9a^4r^8 + 12a^3r^6 - 2a^2r^4 - 4ar^2 + 1) \right),$$

$$J_\theta = \cos^2 \theta \cdot J_\phi = \frac{A^{(0)}}{\mu_0} \left( (3ar^2+1)^{1/3} (7a^2br^6 \right. \\ \left. + (1-3ar^2)^{2/3} (3a^2r^4 + 2ar^2 - 1) + abr^4 - 2br^2) \right. \\ \left. - 18a^3r^6 - 5a^2r^4 + 6ar^2 + 1 \right) / \left( (1-3ar^2)^{2/3} \right. \\ \left. (3ar^2+1)^{1/3} (3a^2r^8 + 2ar^6 - r^4) \right)$$

## Table 1b Continued

Metric

Charge and Current Densities

Kasner

$$J^0 = \frac{\phi}{t^2} (P_3^2 - P_3 + P_2^2 - P_2 + P_1^2 - P_1),$$

$$J_r = \frac{A^{(0)}}{\mu_0} (P_1 P_3 + P_1 P_2 + P_1^2 - P_1) t^{-2P_1 - 2},$$

$$J_\theta = \frac{A^{(0)}}{\mu_0} (P_2 P_3 + P_2^2 + (P_1 - 1) P_2) t^{-2P_2 - 2},$$

$$J_\phi = \frac{A^{(0)}}{\mu_0} (P_3^2 + (P_2 + P_1 - 1) P_3) t^{-2P_3 - 2}.$$

Reissner -  
Nordström

$$J^0 = -\phi Q^2 / (r^2 Q^2 - 2r^3 m + r^4),$$

$$J_r = -\frac{A^{(0)}}{\mu_0} \cdot \frac{1}{r^6} (Q^4 + (r^2 - 2rm) Q^2),$$

$$J_\theta = \frac{A^{(0)}}{\mu_0} \frac{Q^2}{r^6}, \quad J_\phi = J_\theta / \sin^2 \theta.$$

Simple  
Kerr

Not a solution of the Einstein-Hilbert field equation, finite charge and current densities.

Crothers 1a

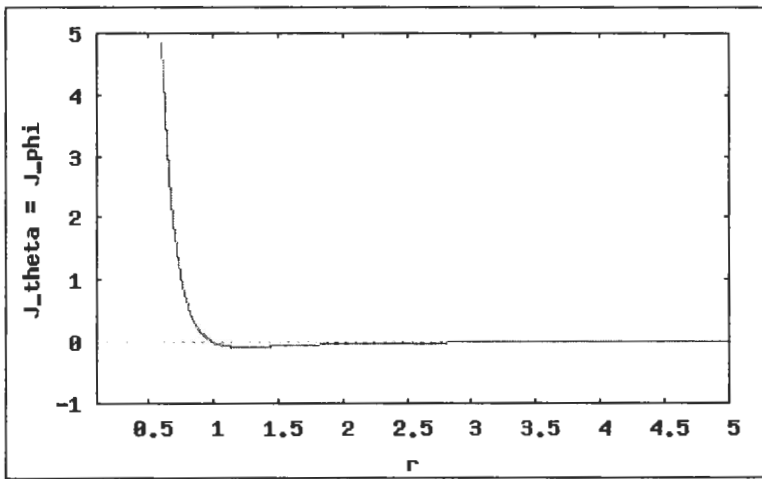
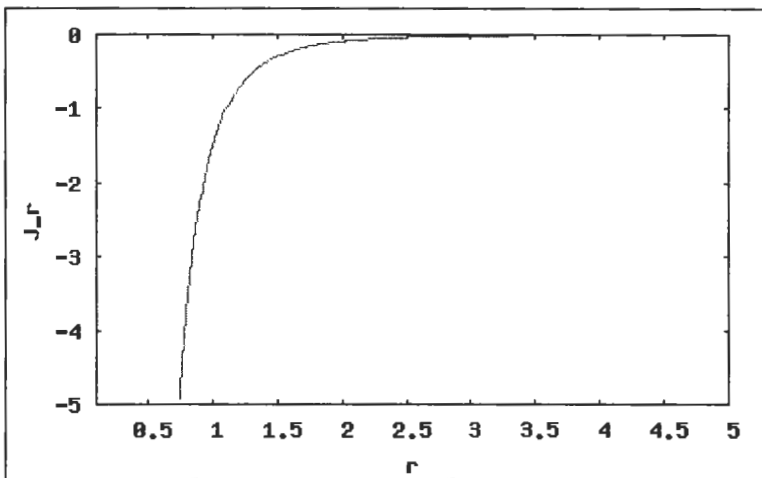
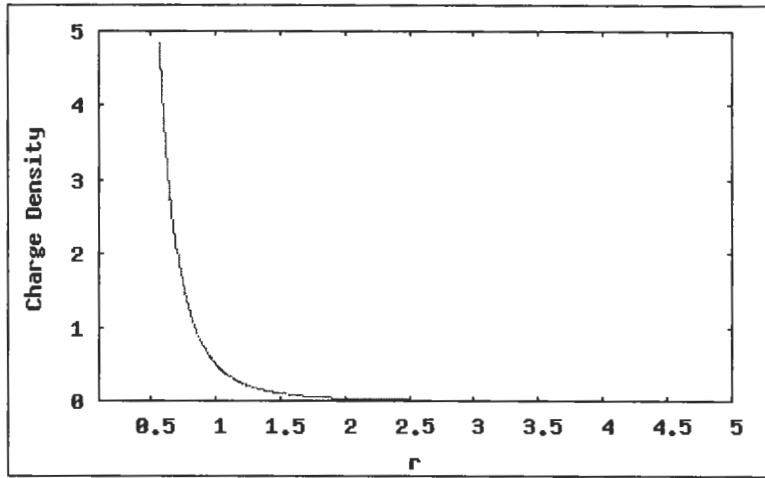


Fig. 1



## Crothers 1a

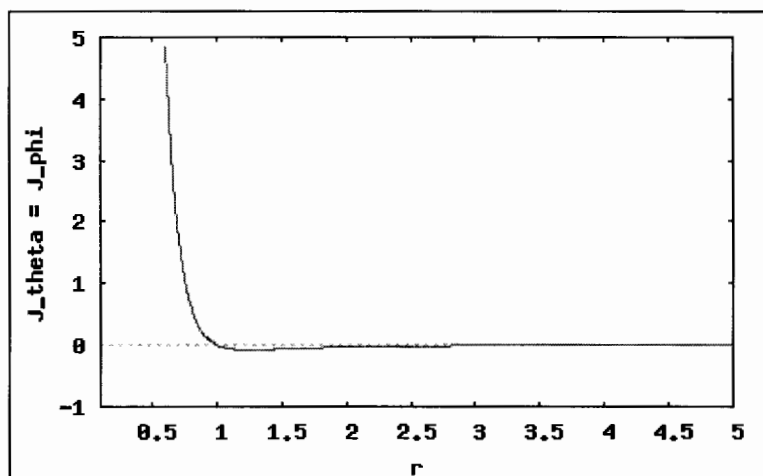
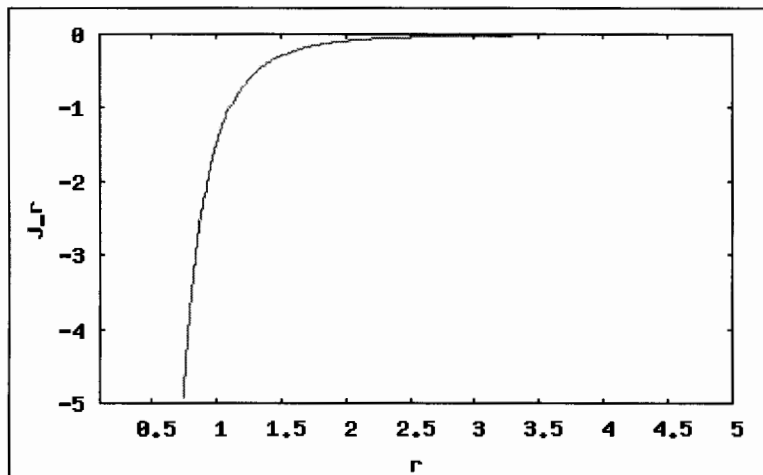
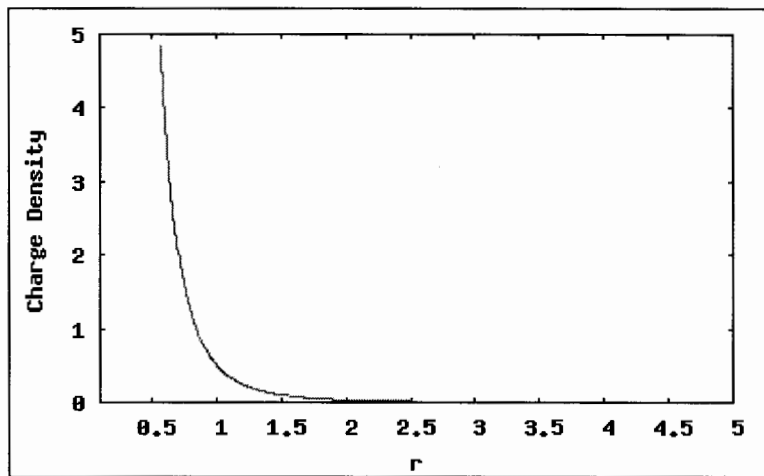
 $r_0=0$ ,  $\alpha=0$ ,  $n=1$ ,  $A=B=1$ 

Fig 1a

Crothers 1a

$r_0=1, \alpha=1, n=1, A=B=1$  (Crothers / Schwarzschild)

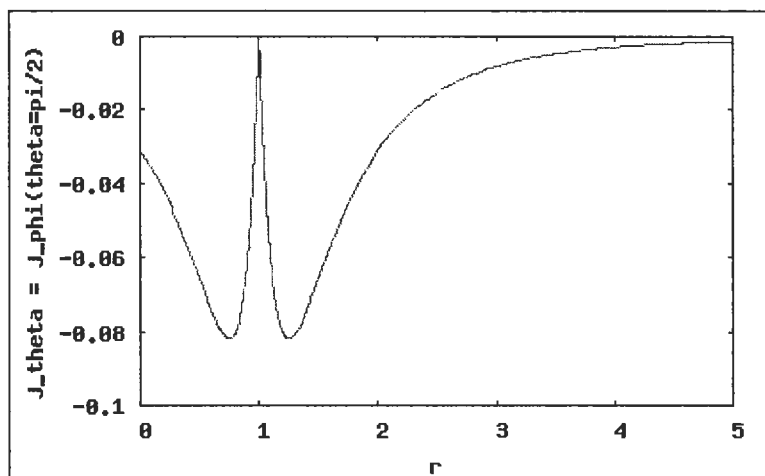
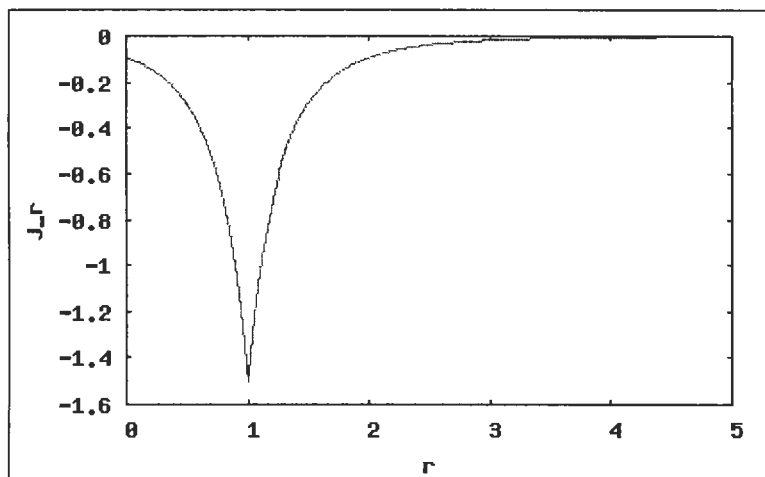
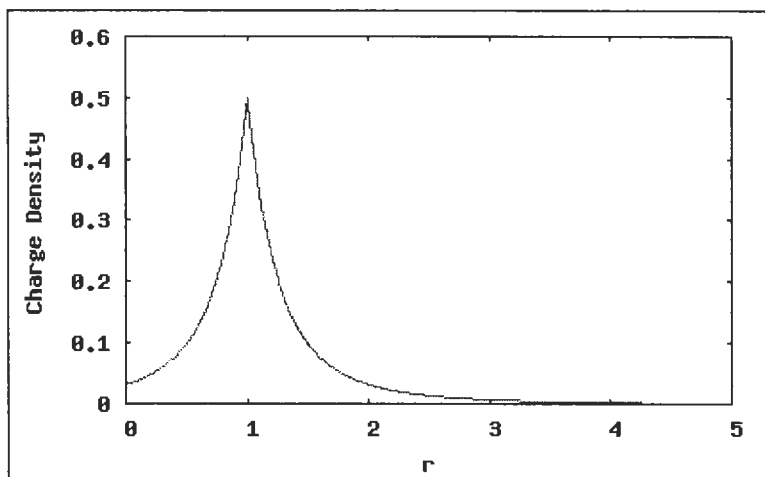


Fig 1b

Crothers 1a

$r_0=0$ ,  $\alpha=1$ ,  $n=3$ ,  $A=B=1$  (Original Schwarzschild)

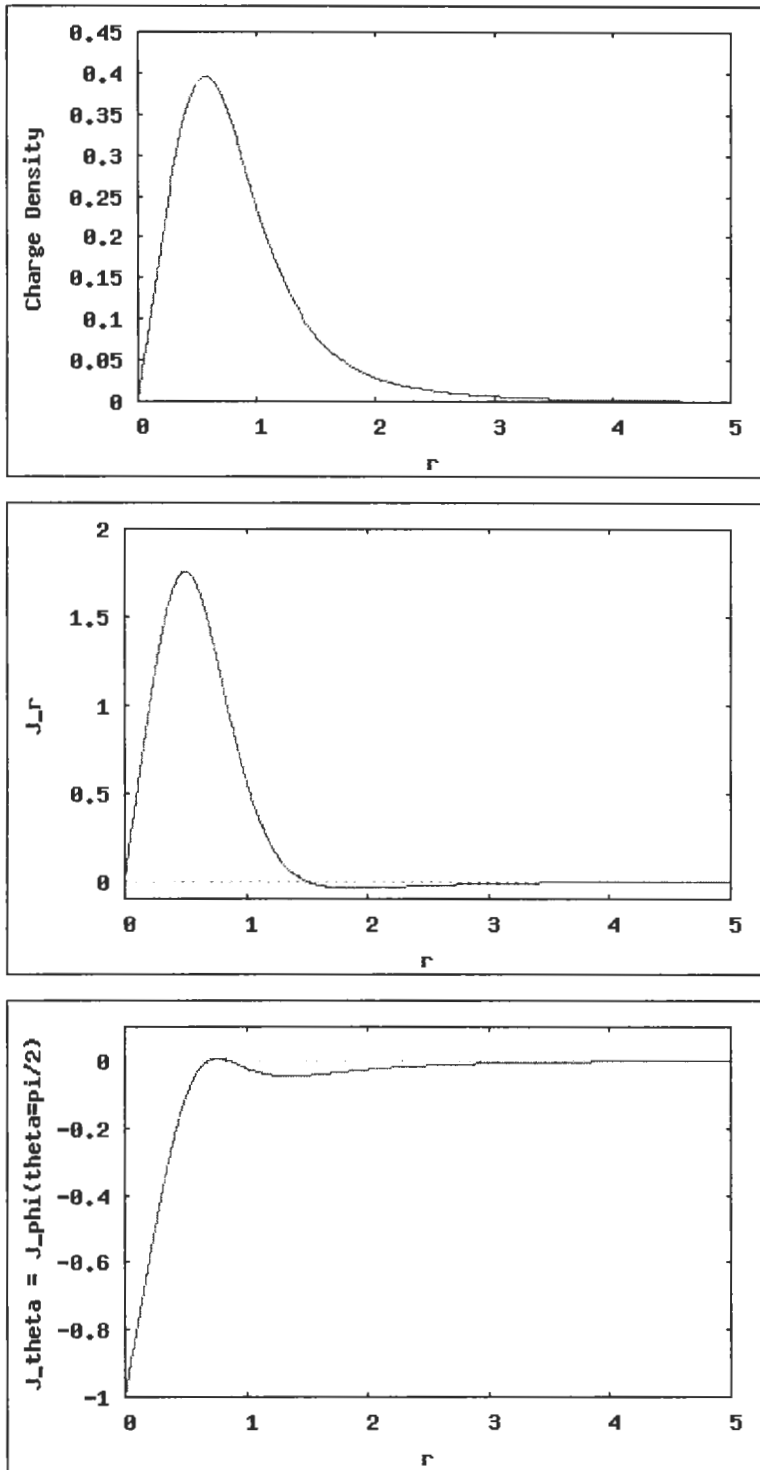


Fig 1c

Friedmann-Dust

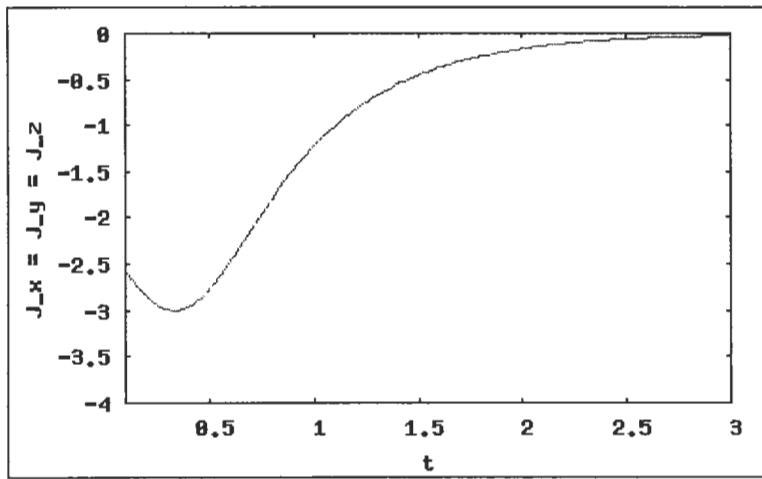
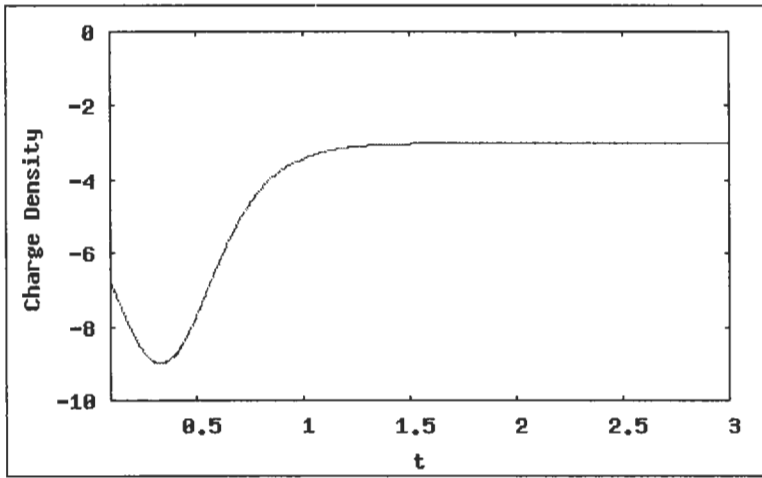


Fig. 2

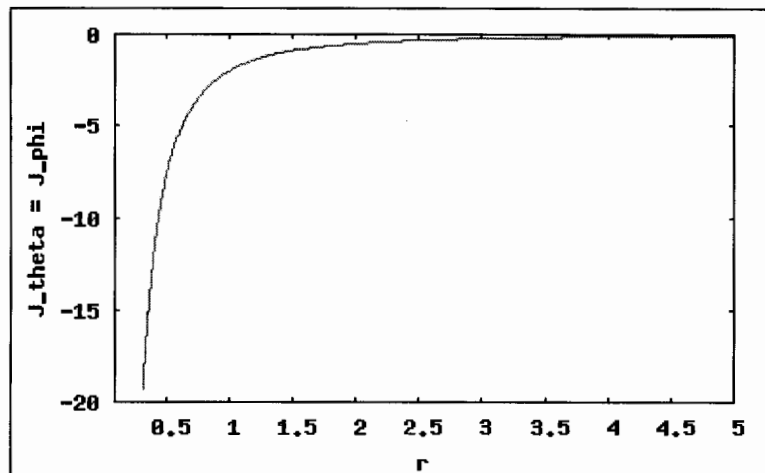
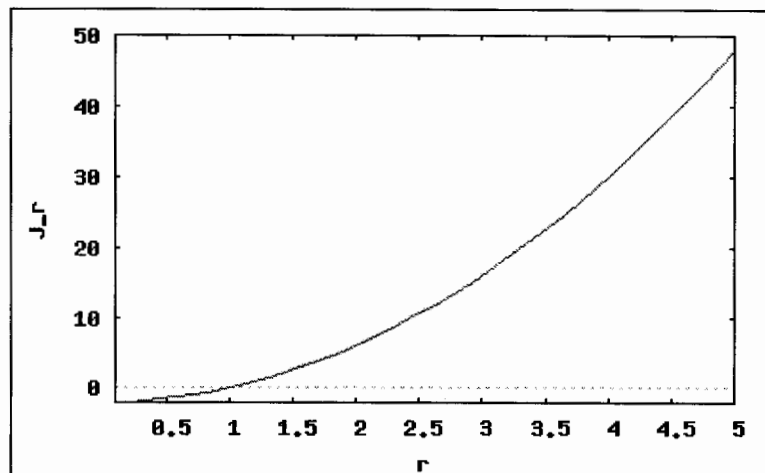
Robertson-Walker with  $a=1=\text{const}$  $\rho = 0$ 

Fig. 3

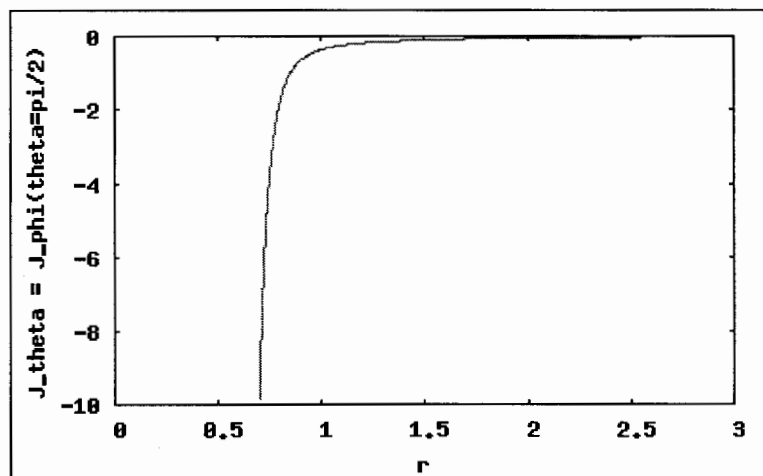
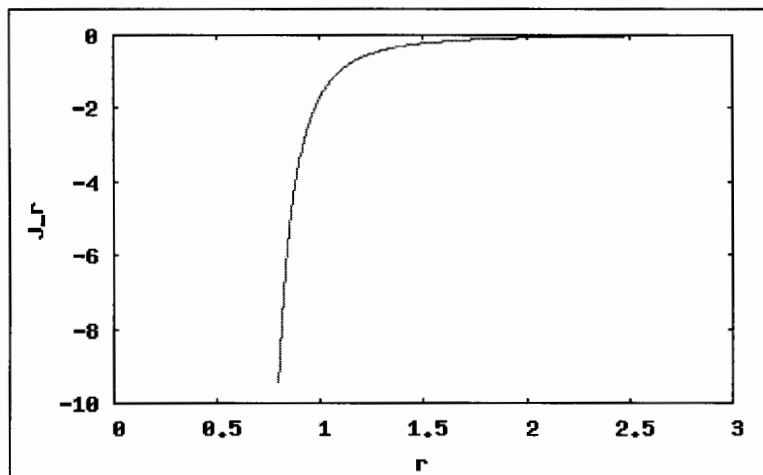
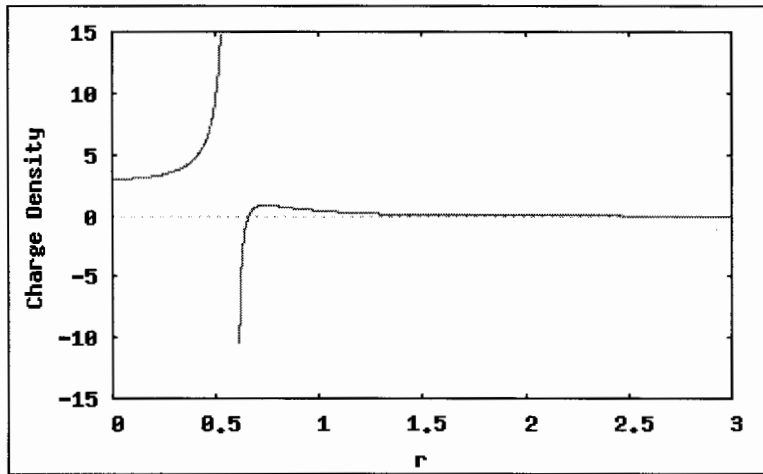
Perfect Fluid,  $a=1$ ,  $b=1$ 

Fig. 4

Static de-Sitter

alpha=1

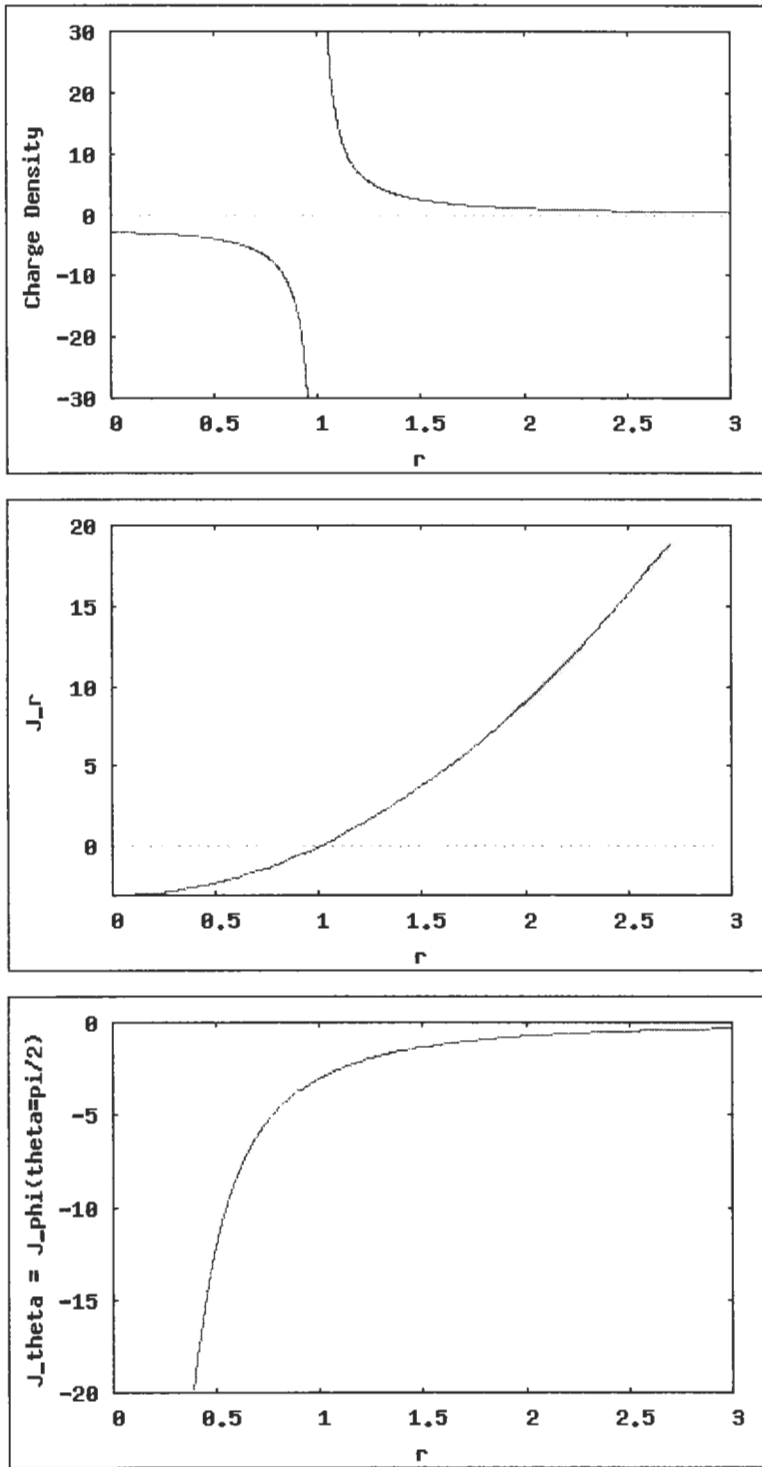


Fig 5

Reissner-Nordstrom

$Q=1, M=2$

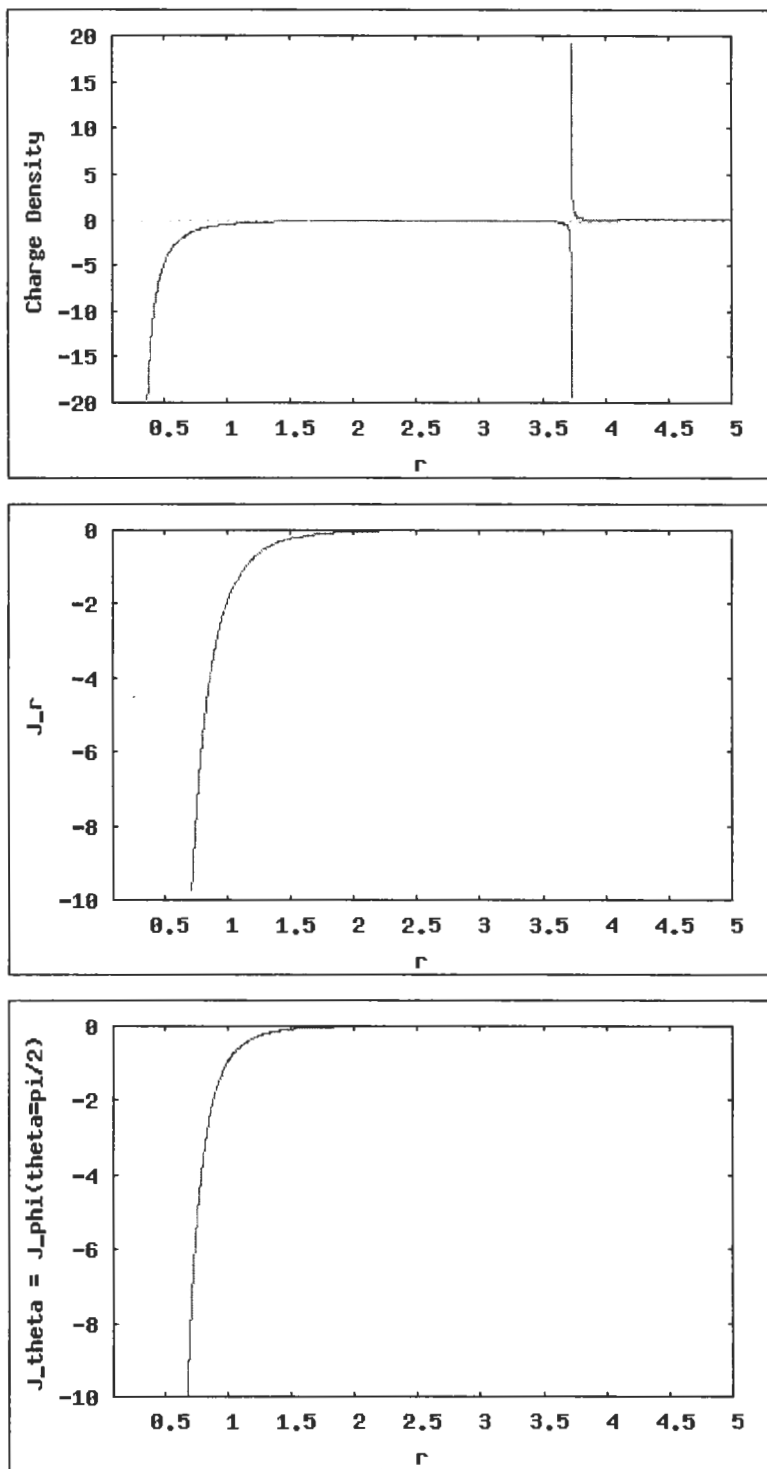


Fig. 6



Reissner-Nordstrom

Q=2, M=1

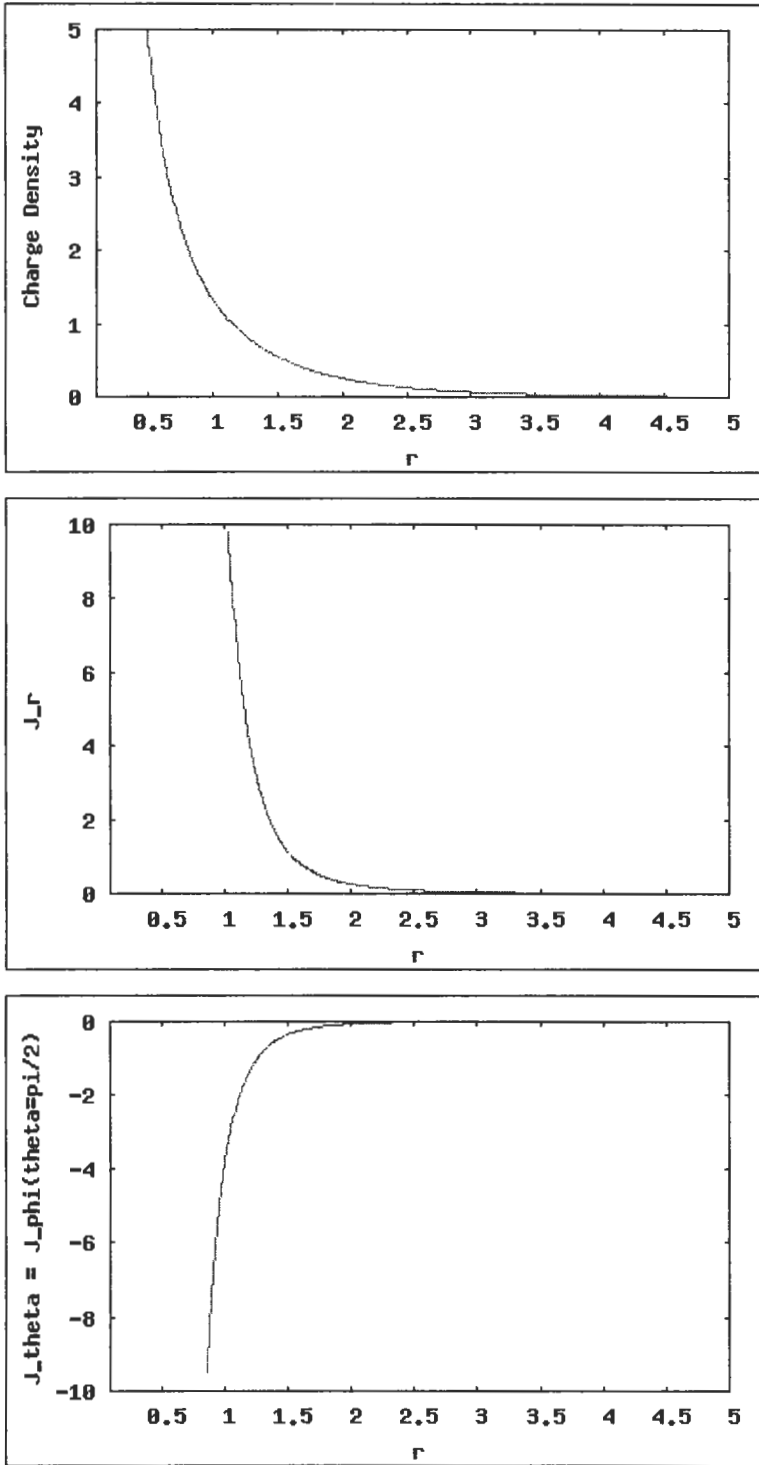


Fig 6a