1)

90(2): Properties of the Spin Connection

It has been shown in note 90(1) that a spin connection of the type:

$$\omega_r = \omega_0^2 r - 4\beta \log_e r - \frac{4}{r} \quad -(1)$$

gives the damped resonance equation:

$$\frac{\partial^2 \phi}{\partial r^2} + 2\beta \frac{\partial \phi}{\partial r} + \omega_0^2 \phi = -\rho/\epsilon_0 \quad -(2)$$

If we write eq. (2) as:

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = A \cos \omega t \quad -(3)$$

its solution is well known to be:

$$x(t) = x_c(t) + x_p(t) \quad -(4)$$

where the particular integral is:

$$x_p(t) = \frac{A}{\left((\omega_0^2 - \omega^2)^2 + 4\omega^2 \beta^2\right)^{1/2}} \cos(\omega t - \delta) \quad -(5)$$

Now replace as follows:

$$x \to \phi, \quad t \to r, \quad \omega \to \kappa, \quad \omega_0 \to \kappa_0 \quad -(6)$$

to find that:

$$\phi_p(r) = \frac{A}{((k_0^2 - k^2)^2 + 4k^2\beta^2)^{1/2}} \cos(kr - \delta) \quad -(7)$$

If we consider the case:

$$\beta \to 0 \quad -(8)$$

then: $\omega_r \to \omega_0^2 r - \frac{4}{r}$ $-(9)$

and $\phi_p(r) \to \dfrac{A}{(k_0^2 - k^2)} \cos kr.$ $-(10)$

So when: $k_0 = k$ $-(11)$

$$\boxed{\phi_p(r) \to \infty} \quad -(12)$$

This is a simple way of demonstrating that eq. (1) leads to resonance.