Sketch of Paper 86

In note 86(6) it was shown that the basic equation here is:

$$-\frac{\hbar^2}{2m}\left(1+\frac{\alpha}{4\pi}\right)^2\frac{d^2\rho}{dr^2} - V_{eff}^{(0)}\rho = E\rho \quad -(1)$$

and this is equivalent by hypothesis to:

$$-\frac{\hbar^2}{2m}\frac{d^2\underline{\rho}}{dr^2} - V_{eff}\,\underline{\rho} = E\underline{\rho} \quad -(2)$$

where:

$$V_{eff} = \frac{e^2}{4\pi\epsilon_0(r+r_{vac})} - \frac{l(l+1)\hbar^2}{2m(r+r_{vac})^2} \quad -(3)$$

Therefore to first order in α:

$$\boxed{-\frac{\hbar^2\alpha}{4\pi m}\frac{d^2\rho_0}{dr^2} = \left(V_{eff}^{(0)} - V_{eff}\right)\rho_0} \quad -(4)$$

where we have assumed:

$$\underline{\rho} = \rho_0. \quad -(5)$$

Here:

$$\rho_0 = rR(r) \quad -(6)$$

where $R(r)$ is <u>the H atom radial wavefunction</u>.

2)

Here also:

$$\bar{V}^{(0)}_{eff} = \frac{e^2}{4\pi\epsilon_0 r} - \frac{l(l+1)\hbar^2}{2mr^2}, \quad -(7)$$

So:

$$\bar{V}^{(0)}_{eff} - \bar{V}_{eff} = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r + r_{vac}}\right)$$

$$- \frac{l(l+1)\hbar^2}{2m}\left(\frac{1}{r^2} - \frac{1}{(r+r_{vac})^2}\right)$$

$$-(8)$$

Computer Algebra

This should be used to solve eq. (4) for the Lamb shift using eq (8). This method introduces the centrifugal repulsion, giving a Lennard - Jones type potential.

1) Lamb shift for 2s and ⟨2p⟩.

2) Perhaps higher orbital lamb shifts.

IN THIS METHOD THE ANGULAR DEPENDENCE IS INCORPORATED IN THE CENTRIFUGAL TERM, SO WE ONLY HAVE TO USE $l(l+1)$.