1) 85(1): Setting Up the Equation of Ortho Positronium

Ortho-positronium is one electron and one positron in the same spin 1/2 state. This is theoretically forbidden by the Pauli exclusion principle, which states that the total wave-function must be antisymmetric with respect to the interchange of any pair of fermions, irrespective of their charge. The Slater orbital of ortho positronium must therefore be:

$$\psi(1,2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi^{\alpha}(1) & \psi^{\beta}(1) \\ \psi^{\alpha}(2) & \psi^{\beta}(2) \end{vmatrix} \quad -(1)$$

To distinguish between the electron and positron the Dirac equations of both must be considered in the electromagnetic field. For the electron:

$$p^{\mu} \rightarrow p^{\mu} + eA^{\mu} \quad -(2)$$

and for the positron:

$$p^{\mu} \rightarrow p^{\mu} - eA^{\mu}; \quad -(3)$$

where

$$p^{\mu} = i\hbar\partial^{\mu}, \quad -(4)$$

$$\partial^{\mu} = -\frac{i}{\hbar}p^{\mu}. \quad -(5)$$

The Dirac equation of the free electron and positron are:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0 \quad -(6)$$

which is obtained from the ECE wave equation:

$$\left(\square + kT \right) q^a_\mu = 0 \quad - (7)$$

as in previous work.

From eq. (5):

$$\square = \partial^\mu \partial_\mu = - \frac{1}{\hbar^2} p^\mu p_\mu \quad - (8)$$

Eq. (7) factorizes into:

$$\left(i \gamma^\mu \partial_\mu - \frac{mc}{\hbar} \right) q^a_\mu = 0 \quad - (9)$$

or

$$\left(i \gamma^\mu \partial_\mu - \frac{mc}{\hbar} \right) \psi = 0 \quad - (10)$$

For the electron:

$$p_\mu = i\hbar \partial_\mu \rightarrow i\hbar \partial_\mu + e A_\mu \quad - (11)$$

and for the positron:

$$p_\mu = i\hbar \partial_\mu \rightarrow i\hbar \partial_\mu - e A_\mu \quad - (12)$$

Dirac Equation for Electron

$$\left(\gamma^\mu \left(i \partial_\mu + \frac{e}{\hbar} A_\mu \right) - \frac{mc}{\hbar} \right) \psi = 0 \quad - (13)$$

Dirac Equation for Positron

$$\left(\gamma^\mu \left(i \partial_\mu - \frac{e}{\hbar} A_\mu \right) - \frac{mc}{\hbar} \right) \psi = 0 \quad - (14)$$

3)

The Dirac equation for orthopositronium in the absence of an electromagnetic field is therefore:

$$\left(i \gamma^{\mu} \partial_{\mu} - \frac{mc}{\hbar} \right) \psi_{-} = 0 \quad -(15)$$

There are two fermions described by an anti-symmetric wavefunction:

$$\psi_{-} = \frac{1}{\sqrt{2}} \left(\psi^{\alpha}(1) \psi^{\beta}(2) - \psi^{\beta}(1) \psi^{\alpha}(2) \right)$$

$$-(16).$$

When

$$\underline{r_1} = \underline{r_2} \quad -(17)$$

$$\psi_{-} = 0. \quad -(18)$$

Here the label 1 denotes the electron, and the label 2 denotes the positron.

From eq. (16) it is seen immediately that orthopositronium cannot exist theoretically, because α and β are the same in ortho-positronium, i.e. the half integral spins of electron and positron are the same. Finally, consideration of Coulombic interaction leads to:

$$\boxed{\left(\gamma^{\mu} \left(i \partial_{\mu} + \frac{e A_{\mu}}{\hbar} \right) - \frac{mc}{\hbar} \right) \psi_{-} = 0} \quad -(1$$

where eA_{μ} now represents Coulomb attraction between electron and positron.