Working out the $2p_z$ Orbital

This is worked out directly from Atkins:

$$\psi_{2pz} = R_{21} Y_{10} \quad \text{—— (1)}$$

where

$$Y_{10} = \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} \cos\theta \quad \text{—— (2)}$$

$$R_{21} = \frac{1}{a^{3/2}}\left(\frac{1}{2\sqrt{6}}\right)\left(\frac{r}{a}\right) e^{-r/(2a)} \quad \text{—— (3)}$$

So

$$\psi_{2pz} = \frac{1}{4}\frac{1}{\sqrt{6}}\left(\frac{3}{\pi}\right)^{1/2}\frac{1}{a^{3/2}}\left(\frac{r}{a}\right) e^{-r/(2a)} \cos\theta$$

$$\boxed{\psi_{2pz} = \frac{1}{4}\left(\frac{1}{2\pi}\right)^{1/2}\frac{1}{a^{3/2}}\left(\frac{r}{a}\right) e^{-r/(2a)} \cos\theta}$$

$$\boxed{\psi_{2pz} = \boxed{\frac{1}{2\sqrt{2}}\cdot\frac{1}{2\pi^{1/2}}}\frac{1}{a^{3/2}}\left(\frac{r}{a}\right) e^{-r/(2a)} \cos\theta}$$

This is the same as Atkins, page 76.

$$\boxed{\psi_{2s} = \boxed{\frac{1}{2\sqrt{2}}\cdot\frac{1}{2\pi^{1/2}}}\left(2-\frac{r}{a}\right) e^{-r/2a}}$$