79(5): First Order Force on an Electron, Hamilton - Jacobi Equation and Lorentz Force Equation.

From eq. (11) of paper 78 the orbital velocity is:

$$V_x = \frac{ecB^{(0)}}{m\omega} \cos\omega t, \quad V_y = -\frac{ecB^{(0)}}{m\omega} \sin\omega t \quad -(1)$$

and therefore the force is:

$$\left.\begin{aligned} F_x &= m\frac{dV_x}{dt} = -ecB^{(0)} \sin\omega t, \\ F_y &= m\frac{dV_y}{dt} = -ecB^{(0)} \cos\omega t. \end{aligned}\right\} \quad -(2)$$

So

$$F = \left(F_x^2 + F_y^2\right)^{1/2} = ecB^{(0)} \quad -(3)$$

$$= eE^{(0)}$$

calculated in the non-relativistic limit. This is the same as the non-relativistic Lorentz force. This force is to first order in the electric and magnetic field strengths of the electromagnetic field. The force components may be expressed as:

$$F_x = -e\omega A^{(0)} \sin\omega t \quad -(4)$$

$$F_y = -e\omega A^{(0)} \cos\omega t. \quad -(5)$$

A more complete calculation of the force proceeds from the energy of interaction, which may be calculated as follows from the angular momentum:

$$E_n = \frac{1}{2}\omega J \quad -(6)$$

2) where
$$J = \frac{e^2 c^3 B^{(0)2}}{\gamma \omega^2} \quad -(7)$$

and
$$\gamma = mc \left(1 + \left(\frac{eB^{(0)}}{m\omega} \right)^2 \right)^{1/2} \quad -(8)$$

Therefore:
$$E_n = \frac{e^2 c^2}{2m\omega^2} \left(\frac{B^{(0)2}}{\left(1 + \frac{e^2 B^{(0)2}}{m^2 \omega^2} \right)^{1/2}} \right) \quad -(9)$$

$$\boxed{E_n = \frac{e^2 c^2}{2\omega} \left(\frac{B^{(0)2}}{\left(m^2 \omega^2 + e^2 B^{(0)2} \right)^{1/2}} \right)} \quad -(10)$$

In the limit:
$$eB^{(0)} \gg m\omega \quad -(11)$$

$$E_n \rightarrow \frac{e c^2 B^{(0)}}{2\omega} = \frac{c}{2\omega} e E^{(0)} \quad -(12)$$

$$\boxed{E_n = \frac{1}{2} e A^{(0)} c} \quad -(13)$$

In the limit
$$m\omega \gg eB^{(0)} \quad -(14)$$

$$\boxed{E_n = \frac{1}{2} \frac{e^2 A^{(0)2}}{m}} \quad -(15)$$

3) Eqn. (15) is the non-relativistic kinetic energy:

$$E_n = \frac{p^2}{2m} \qquad (16)$$

$$= \int F\,dr = \frac{1}{m}\int p\,dp$$

where F is the corresponding force:

$$F = \frac{dp}{dt} \qquad (17)$$

Conclusion

In the interaction of an electron with a circularly polarized electromagnetic field the Lorentz force $F = eE^{(0)}$ gives rise to a relativistic energy of interaction:

$$E_n = \frac{e^2 E^{(0)2}}{2\omega\left(m^2\omega^2 + \frac{e^2 E^{(0)2}}{c^2}\right)^{1/2}} \qquad (18)$$

In the non-relativistic limit:

$$E_n = \frac{1}{2}\frac{e^2 E^{(0)2}}{m\omega^2} \qquad (19)$$

$$F = eE^{(0)} \qquad (20)$$