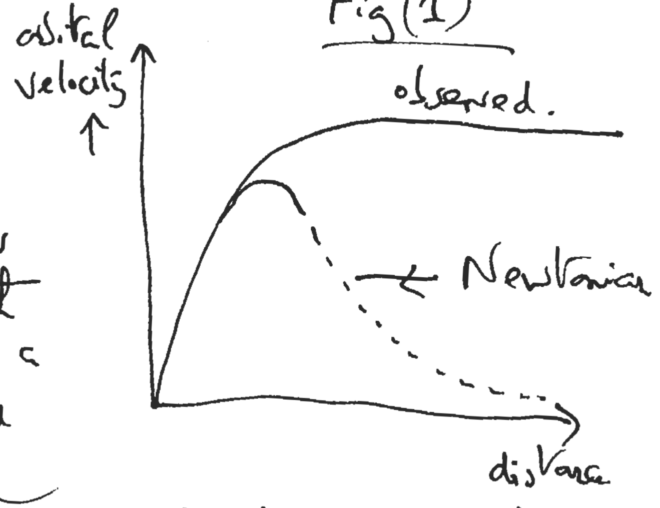
76(3): Cartan Torsion in Spiral Galaxies

Galactic rotation curves illustrate the velocity of rotation versus the distance from the center (Fig (1)).

Fig (1)



These cannot be explained by Newtonian dynamics. The curves are inferred using mass to light ratio, which is a comparison of the total mass of a galaxy or cluster compared with its luminosity. The sun is used as a base line ratio. Mass is calculated from the virial theorem or gravitational lensing. Luminosities are obtained from photometry and by correcting the observed brightness for distance dimming effects.

Rubin and Ford (1975) found that most stars in spiral galaxies orbit at the same velocity – and it was inferred that their mass densities are uniform. This is the origin of the dark matter hypothesis. This inference brings in numerous speculations which are not needed. In particular, the dark matter hypothesis is not needed, and can be replaced as follows.

A spiral galaxy has total angular momentum:

$$\underline{L} = m_i \sum_i \underline{r}_i \times \underline{v}_i \quad — (1)$$

where m_i, $\underline{r}_i$ and $\underline{v}_i$ are the mass, position and orbital velocity of star i. It is found experimentally (Fig. (1)) that $\underline{L}$ increases with distance $\underline{r}_i$, because m_i and $\underline{v}_i$ are constant. Thus:

$$\underline{L} = \underline{L}(\underline{r}) \quad — (2)$$

for each star, labelled i.

2)

The torque $\underline{Tq}$ may then be defined by:

$$\underline{Tq} = c \frac{d\underline{L}}{dr} \quad — (3)$$

Units Check

$$Tq = m s^{-1} \; J s \; m^{-1} = J$$
$$= \text{force} \times \text{dim} = kgm \; m \; s^{-2} \; m = J \;\; ✓✓$$

In general:

$$\underline{r} = X \underline{i} + Y \underline{j} + Z \underline{k} \quad — (4)$$

$$\underline{v} = v_x \underline{i} + v_y \underline{j} + v_z \underline{k} \quad — (5)$$

For simplicity consider:

$$\underline{r} = Y \underline{j}, \quad \underline{v} = v_x \underline{i}. \quad — (6)$$

For each star:

$$\underline{L} = m \, \underline{r} \times \underline{v} = m \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & Y & 0 \\ v_x & 0 & 0 \end{vmatrix}$$

$$= - m Y v_x \underline{k}$$

So:

$$\boxed{\underline{Tq} = c \frac{\partial \underline{L}}{\partial Y} = - m c v_x \underline{k}}$$

$$— (7)$$

This is a constant torque, because v_x is constant.

3). This constant torque derives from the Cartan torsion:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(8)$$

as in paper 55.

Therefore the observed motion of a spiral galaxy is due to a constant Cartan torsion.

Eq. (8) is fully relativistic and covariant, and is the first Cartan structure equation. In tensor notation eq. (8) is:

$$T^a{}_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu$$

$$-(9)$$

and the derivative ∂_μ is:

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t} , \underline{\nabla} \right). \quad -(10)$$

The c factor works its way into classical equation (3).

Conclusion

The motion of a spiral galaxy is due to Cartan torsion.