71(2): Review of Gauge Theory

The gauge principle evolved from Noether's Theorem, which states that if an action is invariant under a given group of transformations (symmetry) there exist one or more conserved quantities (constants of motion) associated with these transformations. For example time translation leads to conservation of energy, space translation to conservation of momentum and rotation to angular momentum conservation laws respectively. The gauge principle extended this idea, so if a given Lagrangian is invariant under a certain symmetry, it would be possible to define the field, or the form of the interaction between particles. The gauge principle was introduced by Salam and Ward: "Our basic postulate is that it should be possible to generate strong, weak and e/m interaction terms (with all their correct symmetry properties and also with clues regarding their relative strengths) by making local gauge transformations on the kinetic energy terms in the free Lagrangian for all particles."

This principle could only be made to work by introducing two new ideas: spontaneous symmetry breaking for very heavy weak gauge bosons, and asymptotic freedom for the strong field (e.g. quark theory). SSB requires in turn the Higgs mechanism and Higgs boson, and asymptotic freedom is part of short range perturbation theory.

2) The problem at present is that the Higgs boson has not been observed at the predicted energy level. Another serious problem of gauge theory is that it cannot deal with a finite photon mass because the Proca equation is not gauge invariant. It is known from NASA Cassini that there must be finite photon mass within an accuracy of 1 : 100,000. There are many problems with the U(1) theory of electrodynamics as the AIAS work shows very clearly. An example of the confusion generated by gauge theory is the half century debate over the Aharonov Bohm effects. Attempts at saving the gauge principle are unconvincing. It is a weak principle compared with Einstein's principle of general covariance. The gauge principle is also abstract and not easy to understand by non-specialists. It has generated as many problems as it has solved.

<u>Meaning of Gauge Invariance.</u>

Usually this is introduced by the well known U(1) definitions:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}, \quad \underline{B} = \underline{\nabla} \times \underline{A} \quad -(1)$$

3)

These equations do not change if the gauge transform is made:

$$\phi \rightarrow \phi' = \phi - \frac{\partial \chi}{\partial t}, \quad \underline{A} \rightarrow \underline{A}' = \underline{A} + \underline{\nabla} \chi \quad -(2)$$

This well known result of classical physics is translated to quantum mechanics using the operator equivalence:

$$\underline{p} \rightarrow -i\hbar \underline{\nabla} \quad -(3)$$

and the hamiltonian:

$$H = \frac{1}{2m}\left(\underline{p} - e\underline{A}\right)^2 + e\phi \quad -(4)$$

This gives the Schrödinger equation for a particle in an electromagnetic field:

$$\left(\frac{1}{2m}\left(-i\hbar\underline{\nabla} - e\underline{A}\right)^2 + e\phi\right)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t} \quad -(5)$$

This can be written as

$$\frac{1}{2m}\left(-i\underline{D}\right)^2\psi = iD_0\psi \quad -(6)$$

where:

$$\underline{\nabla} \rightarrow \underline{D} = \underline{\nabla} - i\frac{e}{\hbar}\underline{A}, \quad -(7)$$

$$\frac{\partial}{\partial t} \rightarrow D_0 = \frac{\partial}{\partial t} + i\frac{e}{\hbar}\phi. \quad -(8)$$

If the gauge transform is now made:

$$(\phi, \underline{A}) \rightarrow (\phi', \underline{A}') \quad -(9)$$

4) then $\frac{1}{2m}\left(-i\underline{D}'\right)^2\psi' = iD_0'\psi'$ — (10)

is not the same equation, i.e is not gauge invariant. In order to save the gauge principle the wave-function must be changed to:

$$\psi' = e^{\frac{ie}{\hbar}\chi}\psi$$
$$= \exp\left(i\frac{e}{\hbar}\chi\right)\psi \quad — (11)$$

Thus:

$$\underline{D}'\psi' = \left(\underline{\nabla} - i\frac{e}{\hbar}\left(\underline{A} + \underline{\nabla}\chi\right)\right)e^{ie\chi/\hbar}\psi$$

$$= \exp\left(i\frac{e}{\hbar}\chi\right)\left(\underline{\nabla}\psi\right) + i\frac{e}{\hbar}\left(\underline{\nabla}\chi\right)\exp\left(i\frac{e}{\hbar}\chi\right)\psi$$

$$- i\frac{e}{\hbar}\underline{A}\exp\left(i\frac{e}{\hbar}\chi\right)\psi - i\frac{e}{\hbar}\left(\underline{\nabla}\chi\right)\exp\left(i\frac{e}{\hbar}\chi\right)\psi$$

$$= \exp\left(i\frac{e}{\hbar}\chi\right)\underline{D}\psi \quad — (12)$$

and $$D_0'\psi' = \exp\left(i\frac{e}{\hbar}\chi\right)D_0\psi. \quad — (13)$$

The Schrödinger equation becomes:

$$\frac{1}{2m}\left(-i\underline{D}'\right)^2\psi' = \frac{1}{2m}\left(-i\underline{D}'\right)\left(-i\underline{D}'\psi'\right)$$

$$= \frac{1}{2m}\left(-i\underline{D}'\right)\left(-i\exp\left(i\frac{e}{\hbar}\chi\right)\underline{D}\psi\right)$$

$$= \exp\left(i\frac{e}{\hbar}\chi\right)\frac{1}{2m}\left(-i\underline{D}\right)^2\psi$$

$$= \exp\left(i \frac{e}{\hbar} \chi\right)\left(i D_0\right)\psi = i D_0' \psi' \quad -(14)$$

Therefore

$$|\psi|^2 = |\psi'|^2 \quad -(15)$$

This is an illustration of the gauge principle:

$$\underline{\nabla} \rightarrow \underline{D}, \quad \frac{\partial}{\partial t} \rightarrow D_0 . \quad -(16)$$

It is noted that the derivatives are replaced by a type of covariant derivative, but in special relativity. ECE is more general than the gauge principle because it uses covariant derivatives in general relativity for all fields. There is also no need for SSB or asymptotic freedom, because these are introduced only to save the gauge principle. Modern physics has proceeded to use the gauge principle to infer fields and dynamics. In ECE this is replaced by ~~the~~ principle of general covariance.

Gauge Invariance in Quantum Electrodynamics.

Using the usual shorthand notation of this subject, the free fermion lagrangian is written as:

$$\mathcal{L}_\phi = \bar{\psi}\left(i \not{\partial} - m\right)\psi \quad -(17)$$

The gauge transformation in this case is:

$$\psi \to \psi' = \exp(-i\alpha(x))\psi \quad -(18)$$

This is a phase transformation of the spinor ψ. The Lagrangian becomes:

$$\mathcal{L}_\psi \to \mathcal{L}_\psi' = \mathcal{L}_\psi + \bar{\psi}\gamma_\mu\psi(\partial^\mu\alpha) \quad -(19)$$

so is not invariant under (18). In the shorthand notation with $\hbar = c = 1$ the gauge principle is saved by using:

$$D_\mu = \partial_\mu + ieA_\mu \quad -(20)$$

where A_μ is known as a "gauge field". This is assumed to be the $U(1)$ electromagnetic potential field. It is further required that the gauge field transform as:

$$A_\mu \to A_\mu' = A_\mu + \frac{1}{e}\partial_\mu\alpha \quad -(21)$$

where α is the same as in eq. (18). Then

$$\mathcal{L}_\psi \to \mathcal{L}_\psi' = \bar{\psi}'((i\cancel{\partial} - e\cancel{A}') - m)\psi'$$

$$= \bar{\psi}\exp(i\alpha)\left(i\cancel{\partial} - e\left(\cancel{A} + \frac{1}{e}\cancel{\partial}\alpha\right) - m\right)e^{-i\alpha}\psi$$

$$= \mathcal{L}_\psi - e\bar{\psi}\gamma_\mu\psi A^\mu \quad -(22)$$

7) This is not a particularly clear way to introduce the potential field. Compared with the simple ECE Ansatz:

$$A_\mu^a = A^{(0)} q_\mu^a \quad -(23)$$

the gauge principle is needlessly complicated. It is reviewed here to illustrate this.

In classical electrodynamics the usual $U(1)$ field:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(24)$$

is invariant under:

$$A_\mu \rightarrow A_\mu + \partial_\mu \chi . \quad -(25)$$

If the Lagrangian is then constructed to be:

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad -(26)$$

it is said to be also gauge invariant. However this introduces ~~the~~ a severe problem with photon mass, because the mass Lagrangian:

$$\mathcal{L}_A = -\frac{1}{2} A_\mu A^\mu \quad -(27)$$

(shorthand notation) is not invariant. So the Proca equation cannot be derived from the gauge principle.

8) <u>Gauge Invariance for Non-Abelian Groups.</u>

This subject was initiated by Yang and Mills in 1955 and was suggested by work by Heisenberg in 1932: protons and neutrons can be regarded as degenerate states since their masses are similar and electromagnetic interactions negligible. This means that any arbitrary combination of their wave-function is equivalent. A spinor is then constructed in a similar way to the construction of the Dirac spinor from two Pauli spinors for left and right handed electrons:

$$\psi \equiv \begin{bmatrix} \psi_p \\ \psi_n \end{bmatrix} \quad — (28)$$

The spinor transforms under $SU(2)$ as:

$$\psi' = U\psi \quad - (29)$$

where
$$U^+U = UU^+ = 1, \quad - (30)$$

and
$$\det|U| = 1. \quad — (31)$$

Here:
$$U = \exp\left(-i\frac{\tau^a}{2}\alpha^a\right)$$
$$\sim 1 - i\frac{\tau^a}{2}\alpha^a \quad - (32)$$

where τ^a are the Pauli matrices.

9)

This procedure so far can also be derived from Cartan geometry in ECE theory, following the ECE derivation of the Dirac equation.

In gauge theory, this idea led to the idea of isotopic gauge invariance, introduced by Yang and Mills in 1954. Essentially

$$\alpha^a = \alpha^a(x^\mu) \quad -(33)$$

meaning that α^a must be a function of x^μ. This is known as "local gauge invariance". The idea was generalized by Utiyama in 1956 to any non-Abelian Lie group, defined by the Lie algebra:

$$[t_a, t_b] = i\, C_{abc}\, t_c \quad -(34)$$

where t_a are the group generators and C_{abc} the group structure constant. The gauge transformation of the matter field is then defined as:

$$\phi \rightarrow \phi' = \Omega\phi = \exp\left(-i\, T^a \alpha^a(x)\right)\phi \quad -(35)$$

where T^a is a representation of t^a.

A gauge field is introduced for each

10) generator :

$$D_\mu = \partial_\mu - igT^a A^a_\mu . \quad \text{— (36)}$$

This means that :

$$D_\mu \psi \rightarrow \exp\left(-iT^a \alpha^a (x^\mu)\right)\left(D_\mu \psi\right) \quad \text{— (37)}$$

- This ensures gauge invariance under a local non-Abelian transform as long as the gauge field transformation is:

$$T^a A^a_\mu \rightarrow \Omega \left(T^a A^a_\mu + \frac{i}{g} \partial_\mu \right) \Omega^{-1} \quad \text{— (38)}$$

- where $\quad \Omega = \exp\left(-iT^a \alpha^a (x^\mu)\right), \quad \text{— (39)}$

$$\sim 1 - iT^a \alpha^a (x^\mu) .$$

Thus : $\quad A^{a\prime}_\mu = A^a_\mu - \frac{1}{g} \partial_\mu \alpha^a + C_{abc} \alpha^b A^c_\mu \quad \text{— (40)}$

and $\quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, C_{abc} A^b_\mu A^c_\nu \quad \text{— (41)}$

11)

This is the field tensor, which is introduced in ECE theory as:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad -(42)$$

where $T^a_{\mu\nu}$ is the Cartan torsion.

- In gauge theory:

$$F^{a\,\prime}_{\mu\nu} \rightarrow F^a_{\mu\nu} + C_{abc}\, \alpha^b F^c_{\mu\nu} \quad -(43)$$

In ECE theory:

$$F^{a\,\prime}_{\mu\nu} \rightarrow \underline{\text{coordinate}}\ \underline{\text{transform}}\ \text{of}\ \underline{F^a_{\mu\nu}} \quad -(44)$$

- In gauge theory (shorthand notation):

$$F \propto (\partial A - \partial A) + g\, A A \quad -(45)$$

$$\mathcal{L}_A \propto F^2 = (\partial A - \partial A)^2 + g(\partial A - \partial A) A A + g^2 A A A A \quad -(46)$$

giving self interaction. In ECE theory this same process is introduced by:

$$F = d \wedge A + \omega \wedge A \quad -(47)$$

$$F^2 = (d \wedge A)^2 + (d \wedge A)(\omega \wedge A) + (\omega \wedge A)(\omega \wedge A). \quad -(48)$$

o(3) Electrodynamics

I adapted these gauge principles to develop the spin field:

$$\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(49)$$

and this led to o(3) electrodynamics, and o(3) quantum electrodynamics. The ECE theory of 2003 onwards gives a much more self-consistent theory of electrodynamics than o(3) electrodynamics. So I propose to replace the gauge principle by:

$$\boxed{D_\nu q_\mu^a \rightarrow (D_\nu q_\mu^a)' = 0} \quad -(50)$$

THE INVARIANCE OF THE TETRAD POSTULATE UNDER COORDINATE TRANSFORM