6a(3): Effect of Gravitation on RFR

In the presence of gravitation the key resonance equation in indexless notation is:

$$d \wedge (d \wedge A + \omega \wedge A) = \mu_0 j \quad -(1)$$

It is known that in the off resonance condition, the effect of gravitation is weak, so:

$$\omega \wedge = -g A \quad -(2)$$

and eq. (1) becomes:

$$d \wedge (d \wedge A - i g A \wedge A^*) = \mu_0 j. \quad -(3)$$

For circularly polarized radiation in free space:

$$d \wedge A = -i g A \wedge A^*. \quad -(4)$$

This is true to a good approximation in the presence of gravitation. So:

$$d \wedge (d \wedge A) = \frac{\mu_0}{2} j. \quad -(5)$$

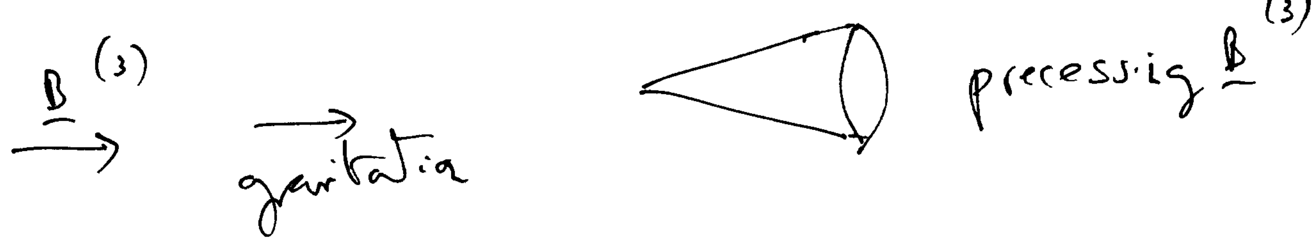
Therefore there are equations such as:

$$\boxed{d \wedge B^{(3)} = \mu_0 \frac{j}{2}}. \quad -(6)$$

The effect of gravitation is to make $B^{(3)}$ space and time dependent. If for example we consider the space part of eq. (6), we obtain:

$$\nabla \times B^{(3)} = \mu_0 \ldots j \quad -(7)$$

2) in vector notation. This is a precessional equation:



We have:

$$\underline{\nabla} \times \underline{B}^{(3)} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial X & \partial/\partial Y & \partial/\partial Z \\ B_X^{(3)} & B_Y^{(3)} & B_Z^{(3)} \end{vmatrix} \quad -(8)$$

So:

$$\frac{\partial B_Z^{(3)}}{\partial Y} - \frac{\partial B_Y^{(3)}}{\partial Z} = \frac{\mu_0}{2} j_X \quad -(9)$$

$$\frac{\partial B_X^{(3)}}{\partial Z} - \frac{\partial B_Z^{(3)}}{\partial X} = \frac{\mu_0}{2} j_Y \quad -(10)$$

$$\frac{\partial B_Y^{(3)}}{\partial X} - \frac{\partial B_X^{(3)}}{\partial Y} = \frac{\mu_0}{2} j_Z \quad -(11)$$

We may now attempt to derive resonance equations from eqs. (9) to (11). The aim is to amplify $B_Z^{(3)}$ at resonance, which would mean that the RFR line would be greatly shifted at resonance. This might lead to a practical way of measuring the effect of gravitation on a spectrum.

3) Differentiate eq. (9):

$$\frac{\partial^2 B_Z^{(3)}}{\partial Y^2} - \frac{\partial}{\partial Y}\left(\frac{\partial B_Y^{(3)}}{\partial Z}\right) = \frac{\mu_0}{2}\frac{\partial j_X}{\partial Y} \quad -(12)$$

This is a resonance equation if:

$$\frac{\partial}{\partial Y}\left(\frac{\partial B_Y^{(3)}}{\partial Z}\right) = -\kappa_0^2 B_Z^{(3)} \quad -(13)$$

i.e.:

$$B_Y^{(3)} = -\kappa_0^2 \iint B_Z^{(3)}\, dZ\, dY \quad -(14)$$

and:

$$\frac{\partial j_X}{\partial Y} = j^{(0)} \cos(\kappa Y). \quad -(15)$$

Thus:

$$\frac{\partial^2 B_Z^{(3)}}{\partial Y^2} + \kappa_0^2 B_Z^{(3)} = \frac{\mu_0}{2} j^{(0)} \cos(\kappa Y) \quad -(16)$$

At resonance, $B_Z^{(3)}$ is greatly amplified, resulting in a possibly measurable RFR shift due to gravity.