

1) Notes 81(2), Third Cross Check 2 Self-Correction

The relativistic momentum is defined by:

$$\underline{p} = m \frac{d\underline{r}}{d\tau} = m \frac{d\underline{r}}{dt} \frac{dt}{d\tau} \quad - (1) \quad \underline{r} = \frac{1}{\gamma} \int \underline{p} dt \quad - (1a)$$

where: $\frac{dt}{d\tau} = \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad - (2)$

so $\underline{r} = \frac{1}{m} \int \underline{p} d\tau = \frac{1}{\gamma m} \int \underline{p} dt = \frac{1}{\gamma} \int \underline{v} dt \quad - (3)$

If we now consider the interaction momentum:

$$\underline{p} = e \underline{A} \quad - (4)$$

then $\underline{r} = \frac{e}{m} \int \underline{A} dt \quad - (5)$

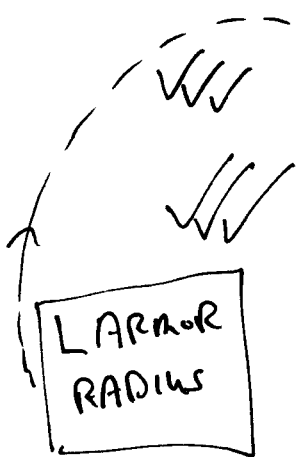
Now assume the plane wave:

$$\underline{A} = A^{(0)} (\underline{i} \cos \phi + \underline{j} \sin \phi) \quad - (6)$$

then: $p_x = e A^{(0)} \cos \phi, \quad p_y = e A^{(0)} \sin \phi$
 $r_x = \frac{e A^{(0)}}{\omega} \sin \phi, \quad r_y = -\frac{e A^{(0)}}{\omega} \cos \phi$

So: $p = (p_x^2 + p_y^2)^{1/2} = e A^{(0)} \quad - (7)$
 $r = (r_x^2 + r_y^2)^{1/2} = \frac{1}{m\omega} e A^{(0)} \quad - (8)$

$$\underline{J} = \underline{r} \cdot \underline{p} = \frac{e^2 A^{(0)2}}{m\omega} = \frac{e^2 c^2 B^{(0)2}}{m\omega^3} \quad - (9)$$



DOUBLE CROSS CHECKED
IN THREE WAYS

More generally:

$$\underline{p} = \gamma m \underline{v} + e \underline{A} \quad - (10)$$

$$\underline{r} = \frac{1}{m} \left(\frac{1}{\gamma} \int m \underline{v} dt + \int e \underline{A} dt \right) - (11)$$

$$\underline{r} = \frac{1}{\gamma} \int \underline{v} dt + \frac{e}{m} \int \underline{A} dt - (12)$$

and

$$\underline{J} = \underline{r} \times \underline{p} \quad - (13)$$

- The relativistic velocity is:

$$\underline{v} = \gamma \underline{v} + \frac{e}{m} \underline{A} \quad - (14)$$

and the angular velocity of the electron in the e/m field is:

$$\Omega = \frac{|\underline{v}|}{|\underline{r}|}, \quad - (15)$$

while its total kinetic energy is:

$$T = \frac{\gamma^2 m^2 v^2 c^2 + e^2 c^2 A^2}{mc^2(1+\gamma) + e\phi} \quad - (16)$$

Hyper-Relativistic Limit

This is the limit:

$$e\phi \gg mc^2(1+\gamma) \quad - (17)$$

so

$$T_{int} = \frac{ec^2 A^2}{\phi} \quad - (18)$$

If we assume that:

$$|A| = A^{(0)}, \quad |\phi| = cA^{(0)} \quad - (19)$$

3) Her:

$$T_{int} = e c A^{(0)} \quad - (20)$$

using de Broglie wave-particle duality

$$e A^{(0)} = \hbar \kappa \quad - (21)$$

Her:

$$T_{int} = \hbar c \kappa = \hbar \omega \quad - (22)$$

which is the quantum of energy of a photon.

- If we define:

$$g = \frac{e}{\hbar} = \frac{\kappa}{A^{(0)}} \quad - (23)$$

Her:

$$\underline{B}^{(3)*} = -i g \underline{A}^{(1)} \times \underline{A}^{(2)} \quad - (24)$$

In this limit, the definition of g defines the ECE spin field in free space. The $e A^{(0)}$ interaction momentum is equal to the photon momentum $\hbar \kappa$. A photon of e/\hbar radiation has transferred all its momentum and energy to the electron. This is therefore an elastic collision between photon and electron. In the opposite limit:

$$\underline{m}^{(3)} = \frac{e}{2m} \underline{J}^{(3)} = -i \frac{e^3}{2m^2 \omega} \underline{A}^{(1)} \times \underline{A}^{(2)}$$

$$\underline{B}^{(3)*} = -i \left(\frac{\mu_0 e^3}{\sqrt{V} 2m^2 \omega} \right) \underline{A}^{(1)} \times \underline{A}^{(2)} \quad - (25)$$

4) Calculation of Angular Momentum and Energy

Assuming that:

$$\underline{v} = v^{(0)} (\underline{i} \cos \phi + \underline{j} \sin \phi) \quad - (26)$$

The relativistic angular momentum can be calculated as follows from eq. (13). We have:

$$r_x = \frac{1}{\omega} \left(\frac{1}{\gamma} v_x + \frac{eA^{(0)}}{m} \right) \sin \phi \quad - (27)$$

$$r_y = -\frac{1}{\omega} \left(\frac{1}{\gamma} v_y + \frac{eA^{(0)}}{m} \right) \cos \phi \quad - (28)$$

$$p_x = (\gamma m v_x + eA^{(0)}) \cos \phi \quad - (29)$$

$$p_y = (\gamma m v_y + eA^{(0)}) \sin \phi \quad - (30)$$

$$J_z = r_x p_y - r_y p_x, \quad - (31)$$

$$J_z = \frac{m v^{(0)2}}{\omega} + \frac{e v^{(0)} A^{(0)}}{\gamma \omega} + \frac{e^2 A^{(0)2}}{m \omega} \quad - (32)$$

So the rotational kinetic energy is:

$$T = \frac{1}{2} \left(m v^{(0)2} + \frac{e v^{(0)} A^{(0)}}{\gamma} + \frac{e^2 A^{(0)2}}{m} \right) \quad - (33)$$

$$= \frac{1}{2} \omega J_z$$

Comparison with Hamiltona - Jacobi Equation

The non-relativistic limit of same second order interaction term is recovered:

$$5) \quad T = \frac{1}{2} \frac{e^2 A^{(0)2}}{m} = \frac{e^2 c^2}{2m\omega^2} B^{(0)2} \quad - (34)$$

CHECKED IN FOUR WAYS

The first order term is:

$$T_1 = \frac{1}{2} e v^{(0)} \left(1 - \frac{u^2}{c^2} \right)^{1/2} A^{(0)} \quad - (35)$$

In the limit:

$$v^{(0)} \rightarrow c, \quad u \rightarrow 0 \quad - (36)$$

$$T_1 \rightarrow \frac{1}{2} e c A^{(0)} = \frac{e c^2}{2\omega} B^{(0)} \quad - (37)$$

which is the ultra-relativistic limit of the Hamiltona - Jacobi method. In this limit the velocity of the orbiting electron approaches the speed of light, at which no frame can move faster, so $u \rightarrow 0$. So this method gives the same results as the Hamiltona - Jacobi method. The latter therefore applies to an electron orbiting according to eq. (26).

Finally the angular velocity of the electron in the e/r field is given by:

5)

$$\Omega = \frac{d\theta}{dt} = \frac{v}{r_L} \quad - (38)$$

where: $r_L = (r_x^2 + r_y^2)^{1/2} \quad - (39)$

$$v = (v_x^2 + v_y^2)^{1/2} \quad - (40)$$

The Larmor radius is:

$$r_L = \left(\left(\frac{v^{(0)}}{\omega} \right)^2 + \left(\frac{eA^{(0)}}{m\omega} \right)^2 \right)^{1/2} \quad - (41)$$

and the net orbital velocity is:

$$v = \left((\gamma v^{(0)})^2 + \left(\frac{eA^{(0)}}{m} \right)^2 \right)^{1/2} \quad - (42)$$

In the non-relativistic limit:

$$\gamma \rightarrow 1 \quad - (43)$$

and:

$$\Omega \rightarrow \left(\frac{\left(\frac{v^{(0)}}{\omega} \right)^2 + \left(\frac{eA^{(0)}}{m\omega} \right)^2}{\left(\left(\frac{v^{(0)}}{\omega} \right)^2 + \left(\frac{eA^{(0)}}{m\omega} \right)^2 \right)^{1/2}} \right)^{1/2}$$

$$= \omega \quad - (43)$$

which is the result of paper 78.