1)

Some Background Notes for Paper 61

The Poisson equation for the radial component of the system (r, θ, ϕ) is:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = -\frac{\rho}{\epsilon_0} . \quad -(1)$$

In ECE theory this becomes:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = -\frac{\rho}{\epsilon_0} + A\left(\frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\phi}{r^2}\right)$$

$$:= \frac{1}{\epsilon_0}\left(-\rho + \rho_{eff}\right) \quad -(2)$$

where:

$$\rho_{eff} = A\epsilon_0\left(\frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\phi}{r^2}\right). \quad -(3)$$

The A term decreases $-\rho$, i.e. the negative potential energy of the Coulomb law is added by a positive potential energy inputted from spacetime:

$$V_{eff} = e\,\phi_{eff} \quad -(4)$$

where ϕ_{eff} is the potential caused by ρ_{eff}.
The net effect is that the electron in H is attracted less strongly to the proton than in the absence of A. When:

$$\rho_{eff} > \rho \quad -(5)$$

2) the repulsion exceeds the attraction and the H atom is ionized. The ionization energy of H is 13.6 eV, and when V_{eff} is greater than this amount, the electron breaks free of the proton.

A particular solution of eq. (2) is given by :

$$A = 2 \qquad -(6)$$

where

$$\frac{\partial^2 \phi}{\partial r^2} = \underbrace{-\frac{\rho}{\epsilon_0}}_{\text{attraction}} + \underbrace{\frac{2\phi}{r^2}}_{\text{repulsion}} \qquad -(7)$$

This is an undamped resonant oscillator if ρ is initially oscillatory. If ϕ is made very large and positive, and if ρ_e is positive, the potential energy in eq. (4) is positive:

If the second term on the right hand side of eq (7) dominates, then :

$$\phi \sim \beta / r \qquad -(8)$$

is a possible solution, where β must have the S.I. units of $e / (4\pi\epsilon_0)$. In this simple approximation, the potential energy of the H atom is simply made more positive :

$$V = \frac{-e^2}{4\pi\epsilon_0 r} + \frac{e\beta}{r} \qquad -(9)$$

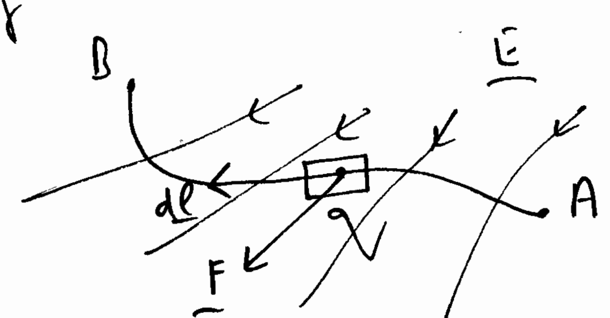
and if β is large enough there will be no negative attraction energy between proton and electron.

Sign Conventions of the Standard Model

The scalar potential in electrostatics is defined by:

$$\underline{E} = -\underline{\nabla}\phi \quad -(10)$$

in the standard model. The work done on a test charge q in moving it from point A to point B in the presence of $\underline{E}$ is sketched as follows:



The force acting on the charge is:

$$\underline{F} = q\underline{E} \quad -(11)$$

The work done is:

$$W = -\int_A^B \underline{F}\cdot d\underline{l} = -q\int_A^B \underline{E}\cdot d\underline{l}$$

$$-(12)$$

The minus sign appears because the work done on the charge is being calculated. This work done is calculated against the action of the electric field.

W.r.t. the definition (10):

4)

$$W = q \int_A^B \underline{\nabla} \phi \cdot d\underline{l}$$

$$= q \int_A^B d\phi$$

$$= q \left(\phi_B - \phi_A \right) \quad -(13)$$

This is the potential energy difference between B and A.

The line integral of the electric field between two points is independent of the path and is the negative of the potential difference:

$$\int_A^B \underline{E} \cdot d\underline{l} = - \left(\phi_B - \phi_A \right) \quad -(14)$$

If the path is closed there is no potential energy difference, so:

$$\oint \underline{E} \cdot d\underline{l} = 0 . \quad -(15)$$

Stokes' theorem then states that:

$$\oint_C \underline{A} \cdot d\underline{l} = \int_S \underline{\nabla} \times \underline{A} \cdot \underline{n} \, da \quad -(16)$$

so:

$$\underline{\nabla} \times \underline{E} = \underline{0} . \quad -(17)$$

If we go through these well known formulae in ECE theory, the electric field

5) is changed to:

$$\underline{E} = -\underline{\nabla}\phi + \phi\,\underline{\omega} \quad - (18)$$

and it becomes less negative by an amount:

$$\boxed{\underline{E}_\omega = \phi\,\underline{\omega}} \quad - (19) \quad (\text{volt m}^{-1}) ✓$$

So the work done in eq. (12) is changed by an amount:

$$W_\omega = -q\int_A^B \underline{E}_\omega \cdot d\underline{l} \quad - (20)$$

$$= -q\int_A^B \phi\,\underline{\omega} \cdot d\underline{l}.$$

So eqn. (13) is changed to:

$$W = q(\phi_B - \phi_A) - W_\omega \quad - (21)$$

This means that less work is needed to move q from A to B against the electric field. The electric field is weaker and the Coulombic attraction or repulsion is weaker. This has assumed that $\underline{\omega}$ is positive.

Repulsion between two like charges produces a positive $\phi_B - \phi_A$, attraction between unlike charges a negative $\phi_B - \phi_A$ for the Coulomb law of the standard model

6) attraction between a proton of charge $+e$ and an electron of charge $-e$ is described by the negative potential difference:

$$V = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(22)$$

A positive $\underline{\omega}$ makes this less negative:

$$\underline{F} = -e\underline{E}, \quad -(23)$$

corresponding to eq. (22), is changed to:

$$\underline{F} = -e\left(\underline{E} - \phi\underline{\omega}\right), \quad -(24)$$

i.e.

$$\boxed{\underline{F} = -e\left(\underline{E} - \underline{E}_\omega\right)} \quad -(24a)$$

The repulsive potential energy difference:

$$\boxed{V_\omega = +q\int_A^B \phi\,\underline{\omega}\cdot d\underline{l}} \quad -(25)$$

is inputted from space-time if $\underline{\omega}$ is positive.

Negative V denotes attraction, positive V denotes repulsion.