Notes 59(8): Relation of Electric Field to Scalar Potential in ECE Theory.

The complete equation is:

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c\underline{\nabla} A^{0a} - c\omega^{0a}{}_b \underline{A}^b + c\underline{\omega}^a{}_b A^{0b} \quad -(1)$$

If the vector potential is neglected:

$$\underline{E}^a = -c\underline{\nabla} A^{0a} + c\underline{\omega}^a{}_b A^{0b}$$
$$= -\underline{\nabla}\phi^a + \underline{\omega}^a{}_b \phi^b. \quad -(2)$$

Meaning of Indices

This seems clear if we consider a complex potential:

$$\phi = \phi^{(1)} = \frac{1}{\sqrt{2}}(1-i)\, e^{i(\omega t - \kappa z)} \quad -(3)$$
$$\phi^* = \phi^{(2)} = \frac{1}{\sqrt{2}}(1+i)\, e^{-i(\omega t - \kappa z)} \quad -(4)$$

so the indices label states such as those in eqs. (3) & (4).

For simplicity of argument choose one element of $\underline{\omega}^a{}_b$:

$$\underline{\omega} = -\underline{\omega}^0{}_0 \quad -(5)$$

and use the minus sign as a convenient convention. Then

$$\boxed{\underline{E} = -(\underline{\nabla} + \underline{\omega})\phi} \quad -(6)$$

so $\underline{E}$ is defined by a covariant derivative $\underline{\nabla} + \underline{\omega}$, where $\underline{\omega}$ is a spin connection vector. In fact, this is always what is meant by an electric field. The latter always obeys the Coulomb Law (sign again chosen by convention):

$$\underline{\nabla} \cdot \underline{E} = -\rho/\epsilon_0, \quad -(7)$$

to give:

$$* \boxed{\underline{\nabla} \cdot ((\underline{\nabla} + \underline{\omega})\phi) = \rho/\epsilon_0} * . \quad -(8)$$