1)

Notes 59(5): Ampère Maxwell Resonance Equation

The complete Ampère Maxwell resonance law is given by eq. (99) of paper 52:

$$\frac{1}{c^2}\frac{\partial^2 \underline{A}^a}{\partial t^2}+\frac{1}{c}\left(\underline{\nabla}A^{0a}-A^{0b}\underline{\omega}^a{}_b+\omega^{0a}{}_b\underline{A}^b\right) \quad -(1)$$
$$+\underline{\nabla}\times\left(\underline{\nabla}\times\underline{A}^a-\underline{\omega}^a{}_b\times\underline{A}^b\right)=\frac{\mu_0}{c}\underline{\tilde{J}}^a$$

and can be approximated in various ways to produce various types of resonances. One approximation given in paper 52 led to eqn. (136):

$$\frac{\partial^2 \underline{A}^a}{\partial t^2}+\omega_0^2\underline{A}^a=c\mu_0\underline{\tilde{J}}^a \quad -(2)$$

Now let:

$$\underline{\tilde{J}}^a=J^a(0)\cos\omega t, \quad -(3)$$

and consider each component, to find:

$$\frac{\partial^2 A}{\partial t^2}+\omega_0^2 A=\alpha\cos\omega t \quad -(4)$$

where:

$$\alpha=\mu_0 c\, J(0). \quad -(5)$$

Eq. (4) is an undamped driven oscillator. Its resonance frequency is:

$$\omega_R=\omega_0. \quad -(6)$$

At resonance:

$$\boxed{A_p=\frac{\alpha}{(\omega_0^2-\omega_R^2)}\rightarrow\infty} \quad -(7)$$

2) Another type of resonance can be obtained from the magnetostatic Ampère Law (eq. (35) of paper 53):

$$\underline{\nabla} \times \underline{B}^a + \underline{\omega}'^a{}_b \times \underline{B}^b = \mu_0 \underline{J}^{a'} \quad -(8)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(9)$$

These give:

$$\boxed{\begin{aligned}&\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) - \underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{A}^b) \\ &+ \underline{\omega}'^a{}_b \times (\underline{\nabla} \times \underline{A}^a) - \underline{\omega}'^a{}_b \times (\underline{\omega}^a{}_b \times \underline{A}^c) = \mu_0 \underline{J}^{a'} \\ &\qquad -(10)\end{aligned}}$$

Now assume:

$$\underline{J}^{a'} = J^a(0) \cos(\underline{\kappa} \cdot \underline{r}) \quad -(11)$$

and eq. (10) is a driven, damped oscillator with resonant solutions.

Therefore the complete Ampère-Maxwell law (1) will have several resonant frequencies in general. The same is true for the magneto-static Ampère Law (10), and also the electro-static Coulomb Law:

$$\boxed{\begin{aligned}&\underline{\nabla} \cdot \underline{\nabla} A^{a0} + \underline{\omega}^a{}_b \cdot \underline{\nabla} A^{0b} + (\underline{\nabla} \cdot \underline{\omega}^a{}_b) A^{0b} \\ &\qquad = -\mu_0 \tilde{J}^{0a} \quad -(12)\end{aligned}}$$

3) Both eqns. (10) and (12) are damped, driven oscillator equations. The origin of resonance is the presence of the spin connection in eqns. (10) and (12). In the off-resonance condition or when the spin connection is very small, eqns. (10) and (11) reduce to the standard model results:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^{a}) = \mu_0 \underline{J}^{a} \quad - (13)$$

$$\underline{\nabla} \cdot \underline{\nabla} A^{0a} = - \mu_0 \bar{J}^{0a} \quad - (14)$$

Experiments show that eqns. (13) and (14) are excellent approximations in the usual situations of electrostatics and magnetostatics. However this is not always the case. For example Johnson vortices in magnets are described by eqn. (10). The resonance point is the centre of the vortex.