Some Technical Points

In order to define the covariant derivative D_μ a correction is applied to the flat space covariant derivative:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla} \right). \quad -(1)$$

The contravariant flat space derivative is

$$\partial^\mu = \eta^{\mu\nu}\partial_\nu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla} \right) \quad -(2)$$

and the flat space d'Alembertian is:

$$\Box = \partial^\mu \partial_\mu . \quad -(3)$$

It follows that eqn (2) must also be used to define ∂^μ in curved space. The contravariant derivative in curved space is:

$$D^\mu = g^{\mu\nu} D_\nu \quad -(4)$$

but:

$$D^\mu = \partial^\mu + \ldots \quad -(5)$$

Therefore the d'Alembertian $\Box$ remains the same in curved space, i.e:

$$D^\mu D_\mu = \Box + \ldots \quad -(6)$$

i.e $\Box$ is the flat part of $D^\mu D_\mu$. Similarly

$$g^{\mu\nu} = \eta^{\mu\nu} + \ldots \quad -(7)$$

and $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the flat part of $g^{\mu\nu}$.