1)

Notes for Paper 47, Part 1

It was shown in paper 46 that the homogeneous field equation in the presence of the homogeneous current is

$$\underline{\nabla} \times \underline{D}^a + \frac{1}{c^2} \frac{\partial \underline{H}^a}{\partial t} = 0 \quad -(1)$$

where:

$$\underline{D}^a = \epsilon_0 \underline{E}^a + \underline{P}^a$$
$$= \epsilon_r \epsilon_0 \underline{E}^a \quad -(2)$$
$$\underline{B}^a = \mu_0 (\underline{H}^a + \underline{M}^a)$$
$$= \mu_0 \mu_r \underline{M}^a \quad -(3)$$

Here ϵ_r and μ_r are the relative permittivity and permeability of ECE spacetime:

$$\epsilon_r = \epsilon / \epsilon_0 \quad -(4)$$
$$\mu_r = \mu / \mu_0 \quad -(5)$$

and

$$c^2 = \frac{1}{\mu_0 \epsilon_0} \quad -(6)$$

Therefore

$$\underline{\nabla} \times \underline{E}^a + \frac{\mu_0 \epsilon_0}{\mu \epsilon} \underline{B}^a = \underline{0}$$

or

$$\boxed{\frac{\partial \underline{B}^a}{\partial t} + \frac{\mu \epsilon}{\mu_0 \epsilon_0} \underline{\nabla} \times \underline{E}^a = \underline{0}}$$

2)

In the absence of EMG coupling:

$$\mu \epsilon = \mu_0 \epsilon_0 \quad -(7).$$

Therefore EMG coupling introduces the refractive index of ECE spacetime:

$$n^2 = \frac{\mu \epsilon}{\mu_0 \epsilon_0} \quad -(8)$$

One solution of eq. (7) consists of plane waves with phase velocity v. The phase is now:

$$\phi = \omega t - n^2 \kappa Z \quad -(9)$$

and:

$$\underline{B}^a = \frac{B^{(0)}}{\sqrt{2}} \left(i \underline{i} + \underline{j} \right) e^{i\phi} \quad -(10)$$

$$\underline{E}^a = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) e^{i\phi} \quad -(11)$$

$$E^{(0)} = c B^{(0)}. \quad -(12)$$

$$\boxed{\text{Therefore } \kappa \text{ is shifted to } \frac{\mu \epsilon}{\mu_0 \epsilon_0} \kappa.}$$

The shift therefore depends on the permeability and permittivity of ECE spacetime.
