1) On The Spin Connection in The Inhomogeneous Field Equation

As argued in paper 137 the homogeneous field equation uses the Γ connection and the inhomogeneous field equation uses the $\Lambda = \tilde{\Gamma}$ connection. The former equation is built up from:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \quad -(1)$$

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \quad -(2)$$

which implies the tetrad postulate:

$$\begin{aligned} D_\mu q^a_\nu &= \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda \\ &= \partial_\mu q^a_\nu + \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \\ &= 0 \end{aligned} \quad -(3)$$

So

$$\boxed{\omega^a_{\mu\nu} = \partial_\mu q^a_\nu + \Gamma^a_{\mu\nu}} \quad -(4)$$

If the connection $\Gamma^a_{\mu\nu}$ is changed to:

$$\Lambda^a_{\mu\nu} = \tilde{\Gamma}^a_{\mu\nu} \quad -(5)$$

then the spin connection in eq. (4) is also changed to:

$$\boxed{\Omega^a_{\mu\nu} = \partial_\mu q^a_\nu + \tilde{\Gamma}^a_{\mu\nu}} \quad -(6)$$

The spin connection in the inhomogeneous field

2) equation is different from that in the homogeneous field equation. Therefore the HFE is:

$$d \wedge F^a = j^a = A^{(0)} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right) \quad -(7)$$

and the IFE is:

$$d \wedge \tilde{F}^a = J^a = A^{(0)} \left(\tilde{R}^a{}_b \wedge q^b - \Omega^a{}_b \wedge \tilde{T}^b \right) \quad -(8)$$

Here

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(9)$$

Magnetic Monopole and Magneticity

These phenomena are described by:

$$\boxed{d \wedge F^a = j^a} \quad -(10)$$

i.e. by

$$\underline{\nabla} \cdot \underline{B}^a = c j^0 \quad -(11)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{j} \quad -(12)$$

In the standard model:

$$j^0 = 0 \quad -(13)$$

$$\underline{j} = \underline{0} \quad -(14)$$

In ECE, the existence of a magnetic monopole implies the existence of a magnetic current $\underline{j}$. These are described by:

$$R^a{}_b \wedge q^b \neq \omega^a{}_b \wedge T^b \quad -(15)$$

i.e. $$\omega^a{}_b \neq \kappa \epsilon^a{}_{bc} q^c \quad -(16)$$

As shown in previous work the existence of the magnetic current j implies that the circular polarization of light is changed when it grazes a massive object.

<u>The Inhomogeneous J^a</u>

This is defined by the interaction of the electromagnetic field with matter, i.e. by the Coulomb law and the Ampère Maxwell law:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(17)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(18)$$

for each polarization a.

This means that:

$$\tilde{R}^a{}_b \wedge q^b \neq \Omega^a{}_b \wedge \tilde{T}^b \quad -(19)$$

38(2): Spin Connection of the Inhomogeneous Field Equation.

Write the tetrad postulate as:

$$\Gamma^a_{\mu\nu} = \partial_\mu q^a_\nu + \omega^a_{\mu\nu} \qquad (1)$$

then:

$$\Lambda^a_{\mu\nu} = \tilde{\Gamma}^a_{\mu\nu} = \left(\partial_\mu q^a_\nu\right)_{HD} + \tilde{\omega}^a_{\mu\nu}. \qquad (2)$$

where the $\|g\|^{1/2}$ factor cancels out

Define:

$$\partial_\mu q^a_\nu + \Omega^a_{\mu\nu} = \left(\partial_\mu q^a_\nu\right)_{HD} + \tilde{\omega}^a_{\mu\nu} \qquad (3)$$

then:

$$\boxed{\Omega^a_{\mu\nu} = \left(\partial_\mu q^a_\nu\right)_{HD} - \partial_\mu q^a_\nu + \tilde{\omega}^a_{\mu\nu}} \qquad (4)$$

and

$$\boxed{\Lambda^a_{\mu\nu} = \partial_\mu q^a_\nu + \Omega^a_{\mu\nu}} \qquad (5)$$

The Cartan Evans dual identity is therefore

$$D_\mu T^{a\mu\nu} := R^{a\ \ \mu\nu}_{\ \mu} \qquad (6)$$

i.e.

2)

$$\partial_\mu T^{a\mu\nu} = J^{a\nu} \quad -(7)$$

where:

$$\boxed{J^{a\nu} = R^{a\ \mu\nu}_{\ \mu} - \Omega^{a}_{\ \mu b} T^{b\mu\nu}}$$

$$-(8)$$

The <u>inhomogeneous</u> field equation of e/R is:

$$\boxed{\partial_\mu F^{a\mu\nu} = A^{(0)} J^{a\nu}} \quad -(9)$$

The spin connection in the inhomogeneous field equation is:

$$\Omega^{a}_{\ \mu b} = \Omega_{\mu\nu}^{\ \ a}\, \tilde{q}^{\nu}_{b} \quad -(10)$$

The <u>homogeneous</u> field equation of e/R is:

$$\partial_\mu \tilde{F}^{a\mu\nu} = A^{(0)} j^{a\nu} \quad -(11)$$

where

$$j^{a\nu} = A^{(0)}\left(\tilde{R}^{a\ \mu\nu}_{\ \mu} - \omega^{a}_{\ \mu b}\tilde{T}^{b\mu\nu}\right)$$

$$-(12)$$

The existence of the magnetic charge

3) current density $j^{a\mu}$ is defined by the geometry of eq. (12).

Vector Notation

In the absence of polarization and magnetization eq. (9) gives:

$$\underline{\nabla}\cdot\underline{E}^a = \rho^a/\epsilon_0 \qquad -(13)$$

which is the Coulomb law for each polarization a. Eq. (9) also gives:

$$\underline{\nabla}\times\underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \qquad -(14)$$

which is the Ampere Maxwell law for each a.

Eq. (11) gives:

$$\boxed{\underline{\nabla}\cdot\underline{B}^a = \mu_0\rho_m^a} \qquad -(15)$$

which is the Gauss law with magnetic monopole ρ_m^a for each a. Eq. (11) also gives:

$$\boxed{\underline{\nabla}\times\underline{E}^a + \frac{\partial\underline{B}^a}{\partial t} = c\mu_0\underline{J}_m^a} \qquad -(16)$$

4) which is the Faraday law with magnetic current j_m^a for each a.

The prediction of eq. (16) in ECE was made several years ago. It has been shown that eq. (16) describes the experimentally observed change of polarization of light grazing a heavy mass.

Magnetic monopoles (charges) are described by eq. (15), and magnetic currents by eq. (16).

138(3): The Flaw in "The First Bianchi Identity".

The "First Bianchi Identity" of the standard model is, in shorthand notation:

$$R \wedge q = 0. \quad -(1)$$

In the notation of differential geometry this is:

$$R^a{}_b \wedge q^b = 0. \quad -(2)$$

In tensor notation this is:

$$R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} = 0 \quad -(3)$$

Eq. (3) comes from the wedge product of a two-form $(R^a{}_b)$ and a one-form (q^b) (see GCUFT and Carroll). "The First Bianchi Identity" in its usual format is:

$$R^\kappa{}_{\mu\nu\rho} + R^\kappa{}_{\rho\mu\nu} + R^\kappa{}_{\nu\rho\mu} = 0 \quad -(4)$$

As should be well known by now, eq. (4) is not an identity at all. The true identity was given by Cartan and is:

$$D \wedge T := R \wedge q \quad -(5)$$

In tensor notation eq. (5) can be written as:

$$\boxed{D_\mu T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\rho\mu} := R^\kappa{}_{\mu\nu\rho} + R^\kappa{}_{\rho\mu\nu} + R^\kappa{}_{\nu\rho\mu} \neq 0} \quad -(6)$$

2) As proven in ~~eq.~~ paper 102 eq. (6) is the cyclic sum on the right hand side identically equal to the same cyclic sum of the definition of each of the curvature tensors. Cartan derived an exact identity. It is not even clear that Bianchi derived eq. (4), and eq. (4) is not an identity.

The Riemannian torsion in eq. (6) is:

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} \quad -(7)$$

where $\Gamma^{\lambda}_{\mu\nu}$ is the connection of the Riemann manifold. Eq. (6) follows from the fundamental commutator equation:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^{\lambda}_{\mu\nu} D_\lambda V^\rho . \quad -(8)$$

Eq. (7) also follows from eq. (8). Written out more fully, eq. (8) is:

$$[D_\mu, D_\nu] V^\rho = -\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right) D_\lambda V^\rho + R^\rho{}_{\sigma\mu\nu} V^\sigma \quad -(9)$$

Therefore:

$$\boxed{[D_\mu, D_\nu] V^\rho = -\Gamma^{\lambda}_{\mu\nu} D_\lambda V^\rho + \ldots} \quad -(10)$$

By definition:

3) $[D_\mu, D_\nu] := -[D_\nu, D_\mu]$ — (11)

so from eq. (10):

$$\boxed{\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}}. \quad \text{— (12)}$$

Note carefully that if:

$$\mu = \nu \quad \text{— (13)}$$

then:

$$[D_\mu, D_\nu] = 0 \quad \text{— (14)}$$

and:

$$\boxed{\Gamma^\lambda_{\mu\nu} = 0} \quad \text{— (15)}$$

Also, if eq. (13) is used

$$R^\rho{}_{\sigma\mu\nu} = 0 \quad \text{— (16)}$$

$$T^\lambda_{\mu\nu} = 0 \quad \text{— (17)}$$

Therefore if eq. (13) is used the eq. (6) reduces to

$$\boxed{0 = 0} \quad \text{— (18)}$$

The Standard Model Error.

The error in the standard model of cosmology is catastrophic, and is eq. (13).

4) The error (13) works its way through the entire mathematics of general relativity, the major equations of which fail catastrophically. Notably:

1) The standard equation linking the connection to the metric is incorrect because it assumes eq. (13):

$$\Gamma^{\sigma}_{\mu\nu} \overset{?}{=} \frac{1}{2} g^{\sigma\rho}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right) \quad -(19)$$

So we see that textbooks that use this equation are unreliable.

2) The connection of the standard cosmology is

$$\Gamma^{\lambda}_{\mu\nu} \overset{?}{=} \Gamma^{\lambda}_{\nu\mu} \quad -(20)$$

$$\overset{?}{\neq} 0$$

This is incorrect from eq. (10). Eq. (19) is true if and only if eq. (20) is true.

3) The standard cosmology uses:

$$T^{\lambda}_{\mu\nu} \overset{?}{=} 0 \quad -(21)$$

and at the same time uses:

$$R^{\rho}{}_{\sigma\mu\nu} \neq 0 \quad -(22)$$

This is incorrect from eq. (18). If the torsion is zero, so is the curvature.

1) 138(4)
Errors in the Second Bianchi Identity

The second Bianchi identity of the standard model is again incorrect because of the omission of torsion. In shorthand notation the second Bianchi identity is:

$$D \wedge R = 0 \quad (1)$$

and in the notation of differential geometry it is:

$$D \wedge R^a{}_b = 0. \quad (2)$$

Eq. (2) may be expanded out as:

$$d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b = 0 \quad (3)$$

In tensor notation it is:

$$D_\lambda R^\rho{}_{\sigma\mu\nu} + D_\sigma R^\lambda{}_{\rho\mu\nu} + D_\rho R^\sigma{}_{\lambda\mu\nu} = 0 \quad (4)$$

(Carroll, eq. (3.87) of notes).

However, eq. (4) is true if and only if

$$\Gamma^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu} \quad (5)$$

in which case: $R^\rho{}_{\sigma\mu\nu} = 0. \quad (6)$

So eq. (4) means:

$$\boxed{0 = 0} \quad (7)$$

As Carroll mentions on page 81 of his notes, "Notice that for a general connection there would be additional terms involving the torsion tensor."

In paper 88 the true second Bianchi

2) identity was given:

$$D \wedge (D \wedge T) := D \wedge (R \wedge q) \quad - (8)$$

which is derived by taking $D \wedge$ on both sides of the Cartan identity:

$$D \wedge T := R \wedge q. \quad - (9)$$

Note carefully that Carroll does not realize that:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu} \quad - (10)$$

and does not realize that there is no symmetric part to the connection. In consequence, chapters 4 onwards by Carroll are erroneous. However, the pure geometry in his chapters one to three is correct.

Albert Einstein used the erroneous eq. (4) in the format:

$$D^{\mu} G_{\mu\nu} = 0 \quad - (11)$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad - (12)$$

where:

$$R_{\mu\nu} = R_{\nu\mu}, \quad - (13)$$

$$g_{\mu\nu} = g_{\nu\mu} \quad - (14)$$

The erroneous Einstein field equation is based on making eq. (11) proportional to the covariant Noether Theorem:

$$D^\mu T_{\mu\nu} = 0. \quad -(15)$$

So:

$$D^\mu G_{\mu\nu} = k D^\mu T_{\mu\nu} \quad -(16)$$

where k is Einstein's constant. The field equation is the particular solution:

$$G_{\mu\nu} = k T_{\mu\nu}. \quad -(17)$$

However, eq. (17) assumes the erroneous eq. (5), which means that

$$G_{\mu\nu} \overset{?}{=} 0 \quad -(18)$$

Therefore the Einstein field equation produces the erroneous result

$$T_{\mu\nu} \overset{?}{=} 0 \quad -(19)$$

The correct field equations are the ECE field equations based on:

$$\boxed{\begin{aligned} D \wedge T &= R \wedge q \quad -(20) \\ D \wedge \tilde{T} &= \tilde{R} \wedge q \quad -(21) \end{aligned}}$$

38(5): Proof of the Second Bianchi Identity from the First Bianchi Identity

Standard Model

The first Bianchi identity is:

$$R^{\kappa}{}_{\rho\mu\nu} + R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\nu\rho\mu} = 0 \quad -(1)$$

because it is assumed that:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu} \quad -(2)$$

Thus:

$$D_\sigma R^{\kappa}{}_{\rho\mu\nu} + D_\sigma R^{\kappa}{}_{\mu\nu\rho} + D_\sigma R^{\kappa}{}_{\nu\rho\mu} = 0 \quad -(3)$$

Similarly:

$$R^{\kappa}{}_{\rho\sigma\mu} + R^{\kappa}{}_{\mu\rho\sigma} + R^{\kappa}{}_{\sigma\mu\rho} = 0 \quad -(4)$$

and

$$D_\nu R^{\kappa}{}_{\rho\sigma\mu} + D_\nu R^{\kappa}{}_{\mu\rho\sigma} + D_\nu R^{\kappa}{}_{\sigma\mu\rho} = 0 \quad -(5)$$

Thirdly:

$$R^{\kappa}{}_{\rho\nu\sigma} + R^{\kappa}{}_{\sigma\rho\nu} + R^{\kappa}{}_{\nu\sigma\rho} = 0 \quad -(6)$$

and

$$D_\mu R^{\kappa}{}_{\rho\nu\sigma} + D_\mu R^{\kappa}{}_{\sigma\rho\nu} + D_\mu R^{\kappa}{}_{\nu\sigma\rho} = 0 \quad -(7)$$

Add (3), (5) and (7):

$$D_\sigma R^{\kappa}{}_{\rho\mu\nu} + D_\nu R^{\kappa}{}_{\rho\sigma\mu} + D_\mu R^{\kappa}{}_{\rho\nu\sigma}$$
$$+ D_\sigma\left(R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\nu\rho\mu}\right) + D_\nu\left(R^{\kappa}{}_{\mu\rho\sigma} + R^{\kappa}{}_{\sigma\mu\rho}\right)$$
$$+ D_\mu\left(R^{\kappa}{}_{\sigma\rho\nu} + R^{\kappa}{}_{\nu\sigma\rho}\right) = 0 \quad -(8)$$

Finally add to both sides of eq. (8):

$$D_\sigma R^{\kappa}{}_{\rho\mu\nu} + D_\nu R^{\kappa}{}_{\rho\sigma\mu} + D_\mu R^{\kappa}{}_{\rho\nu\sigma}$$

*) to find:

$$D_\sigma R^{\kappa}{}_{\rho\mu\nu} + D_\nu R^{\kappa}{}_{\rho\sigma\mu} + D_\mu R^{\kappa}{}_{\rho\nu\sigma} = 0 \qquad (9)$$

which is the second Bianchi identity, QED.

Eq. (9) was actually discovered by Ricci, and is true if and only if eqs. (1) and (2) are assumed.

The Correct Identity

This was first given by Cartan and is:

$$D_\mu T^{a}{}_{\nu\rho} + D_\rho T^{a}{}_{\mu\nu} + D_\nu T^{a}{}_{\rho\mu} := R^{a}{}_{\mu\nu\rho} + R^{a}{}_{\rho\mu\nu} + R^{a}{}_{\nu\rho\mu} \qquad (10)$$

so the correct version of eq. (9) is:

$$\boxed{\begin{aligned} &D_\sigma R^{a}{}_{\rho\mu\nu} + D_\nu R^{a}{}_{\rho\sigma\mu} + D_\mu R^{a}{}_{\rho\nu\sigma} \\ &:= D_\sigma D_\rho T^{a}{}_{\mu\nu} + D_\nu D_\rho T^{a}{}_{\sigma\mu} + D_\mu D_\rho T^{a}{}_{\nu\sigma} \\ &\neq 0 \end{aligned}} \qquad (11)$$

In eqs. (10) and (11):

$$\boxed{\Gamma^{\lambda}{}_{\mu\nu} = -\Gamma^{\lambda}{}_{\nu\mu}} \qquad (12)$$

138(6): Proof of the Second Cartan Identity

The first Cartan identity is:

$$S^a_{\mu\nu\rho} + S^a_{\rho\mu\nu} + S^a_{\nu\rho\mu} := 0 \quad -(1)$$

where

$$S^a_{\mu\nu\rho} = R^a_{\mu\nu\rho} - D_\mu T^a_{\nu\rho} \quad -(2)$$

and so on.

Thus:

$$D_\sigma \left(S^a_{\mu\nu\rho} + S^a_{\rho\mu\nu} + S^a_{\nu\rho\mu} \right) = 0 \quad -(3)$$

$$D_\nu \left(S^a_{\rho\sigma\mu} + S^a_{\mu\rho\sigma} + S^a_{\sigma\mu\rho} \right) = 0 \quad -(4)$$

$$D_\mu \left(S^a_{\rho\sigma\nu} + S^a_{\sigma\rho\nu} + S^a_{\nu\sigma\rho} \right) = 0 \quad -(5).$$

Add eqns. (3) to (5):

$$D_\sigma S^a_{\rho\mu\nu} + D_\nu S^a_{\rho\sigma\mu} + D_\mu S^a_{\rho\sigma\nu}$$
$$+ D_\sigma \left(S^a_{\mu\nu\rho} + S^a_{\nu\rho\mu} \right) + D_\nu \left(S^a_{\mu\rho\sigma} + S^a_{\sigma\mu\rho} \right)$$
$$+ D_\mu \left(S^a_{\sigma\rho\nu} + S^a_{\nu\sigma\rho} \right) = 0 \quad -(6)$$

Add to both sides of eq. (6) the sum

$$D_\sigma S^a_{\rho\mu\nu} + D_\nu S^a_{\rho\sigma\mu} + D_\mu S^a_{\rho\nu\sigma} \quad -(7)$$

to obtain:

$$2 \left(D_\sigma S^a_{\rho\mu\nu} + D_\nu S^a_{\rho\sigma\mu} + D_\mu S^a_{\rho\nu\sigma} \right)$$
$$+ D_\sigma \left(S^a_{\mu\nu\rho} + S^a_{\nu\rho\mu} + S^a_{\rho\mu\nu} \right)$$
$$+ D_\nu \left(S^a_{\mu\rho\sigma} + S^a_{\sigma\mu\rho} + S^a_{\rho\sigma\mu} \right)$$

2)

$$+ D_\mu \left(S^a_{\sigma\rho\nu} + S^a_{\nu\sigma\rho} + S^a_{\rho\nu\sigma} \right)$$

$$= D_\sigma S^a_{\rho\mu\nu} + D_\nu S^a_{\rho\sigma\mu} + D_\mu S^a_{\rho\nu\sigma} \quad -(8)$$

Finally use eqs (3) to (5) in eq. (8) to find that:

$$\boxed{D_\sigma S^a_{\rho\mu\nu} + D_\nu S^a_{\rho\sigma\mu} + D_\mu S^a_{\rho\nu\sigma} = 0} \quad -(9).$$

Writing out eq. (9) in full:

$$\boxed{\begin{aligned} &D_\sigma D_\rho T^a_{\mu\nu} + D_\nu D_\rho T^a_{\sigma\mu} + D_\mu D_\rho T^a_{\nu\sigma} \\ &:= D_\sigma R^a_{\rho\mu\nu} + D_\nu R^a_{\rho\sigma\mu} + D_\mu R^a_{\rho\nu\sigma} \end{aligned}} \quad -(10)$$

Since eq. (1) is an exact identity, eq. (10) is also an exact identity.

In differential form notation, eq. (10) is:

$$D \wedge (D_\rho T^a) := D \wedge R^a_\rho \quad -(11)$$

The ρ index is the same on either side of eq. (11), so:

3)

$$D \wedge (D T^a) := D \wedge R^a \quad -(12)$$

The misnamed and incorrect "second Bianchi identity" of obsolete physics is:

$$D_\sigma R^\kappa{}_{\rho\mu\nu} + D_\nu R^\kappa{}_{\rho\sigma\mu} + D_\mu R^\kappa{}_{\rho\nu\sigma} \stackrel{?}{=} 0 \quad -(13)$$

Eq. (13) is true if and only if:

$$\Gamma^\lambda_{\mu\nu} \stackrel{?}{=} \Gamma^\lambda_{\nu\mu} \neq 0. \quad -(14)$$

The correct eqn. (13) is:

$$\boxed{D_\sigma D_\rho T^\kappa{}_{\mu\nu} + D_\nu D_\rho T^\kappa{}_{\sigma\mu} + D_\mu D_\rho T^\kappa{}_{\nu\sigma} := D_\sigma R^\kappa{}_{\rho\mu\nu} + D_\nu R^\kappa{}_{\rho\sigma\mu} + D_\mu R^\kappa{}_{\rho\nu\sigma} \neq 0} \quad -(15)$$

The correct connection symmetry is:

$$\boxed{\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}} \quad -(16)$$

Eq. (15) is the identity that should have been used in the Einstein field equation.

4) Also, Einstein used the incorrect connection (14).

By using "the second Bianchi identity", eq. (13) he introduced an over complicated and meaningless procedure. The method he used was to write eq. (13) as:

$$D^\mu G_{\mu\nu} = 0 \quad -(17)$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad -(18)$$

Here

$$R_{\mu\nu} = R_{\nu\mu} = R^\lambda{}_{\mu\lambda\nu} \quad -(19)$$

is the Ricci tensor and

$$R = g^{\mu\nu} R_{\mu\nu}. \quad -(20)$$

The quantity $G_{\mu\nu}$ is known as the "Einstein tensor" but it is meaningless because eqs (13) and (14) are incorrect. Eq. (17) is obtained from eq. (13) by the following steps. First lower indices, e.g.:

$$R_{\kappa\rho\mu\nu} = g_{\kappa\alpha} R^\alpha{}_{\rho\mu\nu} \quad -(21)$$

where

$$g_{\kappa\alpha} = g_{\alpha\kappa}. \quad -(22)$$

Secondly use metric compatibility, e.g.:

5) $$D_\sigma g_{\mu\lambda} = 0 \quad -(23)$$

to find:

$$D_\lambda R_{\rho\sigma\mu\nu} + D_\rho R_{\sigma\lambda\mu\nu} + D_\sigma R_{\lambda\rho\mu\nu} = 0 \quad -(24)$$

Thirdly use:

$$g^{\nu\sigma} g^{\mu\lambda}\left(D_\lambda R_{\rho\sigma\mu\nu} + D_\rho R_{\sigma\lambda\mu\nu} + D_\sigma R_{\lambda\rho\mu\nu}\right)$$

$$= D^\mu R_{\rho\mu} - D_\rho R + D^\nu R_{\rho\nu}$$

$$= 0 \quad -(25)$$

i.e. $$D^\mu R_{\rho\mu} = \frac{1}{2} D_\rho R \quad -(26)$$

Finally: eq. (26) use:

$$D_\rho R = g_{\rho\mu} D^\mu R \quad -(27)$$

so $$D^\mu\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) = 0 \quad -(28)$$

Q.E.D.

In eq. (25) the following definitions are used:

6)

$$D^{\mu} R_{\rho\mu} := g^{\nu\sigma} g^{\mu\lambda} D_{\lambda} R_{\rho\sigma\mu\nu} \quad -(29)$$

$$D^{\sigma} R_{\rho\sigma} := g^{\nu\sigma} g^{\mu\lambda} D_{\sigma} R_{\lambda\rho\mu\nu} \quad -(30)$$

$$D_{\rho} R := -g^{\nu\sigma} g^{\mu\lambda} D_{\rho} R_{\sigma\lambda\mu\nu} \quad -(31)$$

As S. P. Carroll states on p. 81, (chapter 3) of his downloadable notes, these definitions (29) to (31) are unique if and only if the ... is correct eq. (14) is used.

Conclusion

The Einstein field theory is meaningless geometrically

ECE Cosmology

This is much simpler and mathematically correct. It is based on the Cartan identity:

$$D \wedge T^a := R^a_{\ b} \wedge q^b \quad -(32)$$

and the Cartan Evans identity:

$$D \wedge \tilde{T}^a := \tilde{R}^a_{\ b} \wedge q^b \quad -(33)$$

1) 138(7). The Jacobi Identity in Riemann Geometry.

Another fundamental error of the obsolete physics is the incorrect claim that the Jacobi identity gives the incorrect "second Bianchi identity". The Jacobi identity is

$$[[A,B],C] + [[C,A],B] + [[B,C],A] := 0 \quad -(1)$$

where

$$[A,B] = -[B,A] = AB - BA. \quad -(2)$$

Eq. (1) is true and is easily proven as follows:

$$[(AB-BA),C] + [(CA-AC),B] + [(BC-CB),A]$$
$$= (AB-BA)C - C(AB-BA) + (CA-AC)B - B(CA-AC) + (BC-CB)A - A(BC-CB) \quad -(3)$$
$$= 0$$

Q.E.D.

Therefore applying (1) to covariant derivatives in differential Riemann geometry:

$$\left([D_\rho,[D_\mu,D_\nu]] + [D_\nu,[D_\rho,D_\mu]] + [D_\mu,[D_\nu,D_\rho]]\right)V^\sigma = 0 \quad -(4)$$

It is incorrectly claimed that eq. (4) gives the incorrect "second Bianchi identity".

2) Working out eq. (4) gives:

$$[D_\rho,[D_\mu,D_\nu]]V^\sigma = D_\rho\left(R^\sigma{}_{\alpha\mu\nu}V^\alpha - T^\lambda{}_{\mu\nu}D_\lambda V^\sigma\right) - [D_\mu,D_\nu]D_\rho V^\sigma \quad -(5)$$

$$[D_\nu,[D_\rho,D_\mu]]V^\sigma = D_\nu\left(R^\sigma{}_{\alpha\rho\mu}V^\alpha - T^\lambda{}_{\rho\mu}D_\lambda V^\sigma\right) - [D_\rho,D_\mu]D_\nu V^\sigma \quad -(6)$$

$$[D_\mu,[D_\nu,D_\rho]]V^\sigma = D_\mu\left(R^\sigma{}_{\alpha\nu\rho}V^\alpha - T^\lambda{}_{\nu\rho}D_\lambda V^\sigma\right) - [D_\nu,D_\rho]D_\mu V^\sigma \quad -(7)$$

Now use:

$$[D_\rho,D_\sigma]X^{\mu_1\ldots\mu_k}{}_{\nu_1\ldots\nu_l} = R^{\mu_1}{}_{\lambda\rho\sigma}X^{\lambda\mu_2\ldots\mu_k}{}_{\nu_1\ldots\nu_l} + \ldots$$
$$- R^\lambda{}_{\nu_1\rho\sigma}X^{\mu_1\ldots\mu_k}{}_{\lambda\nu_2\ldots\nu_l} - \ldots$$
$$- T^\lambda{}_{\rho\sigma}D_\lambda X^{\mu_1\ldots\mu_k}{}_{\nu_1\ldots\nu_l} \quad -(8)$$

This is the rule for the action of the commutator of covariant derivatives on the arbitrary tensor X of any rank. In eqs. (5) to (7) the quantities $D_\rho V^\sigma$, $D_\nu V^\sigma$ and $D_\mu V^\sigma$, acted upon by the commutators, are <u>second rank tensors</u>. Thus:

$$[D_\mu,D_\nu]D_\rho V^\sigma = R^\sigma{}_{\lambda\mu\nu}D_\rho V^\lambda - R^\lambda{}_{\rho\mu\nu}D_\lambda V^\sigma - T^\lambda{}_{\mu\nu}D_\lambda V^\sigma \quad -(9)$$

$$[D_\rho,D_\mu]D_\nu V^\sigma = R^\sigma{}_{\lambda\rho\mu}D_\nu V^\lambda - R^\lambda{}_{\nu\rho\mu}D_\lambda V^\sigma - T^\lambda{}_{\rho\mu}D_\lambda V^\sigma \quad -(10)$$

$$[D_\nu,D_\rho]D_\mu V^\sigma = R^\sigma{}_{\lambda\nu\rho}D_\mu V^\lambda - R^\lambda{}_{\mu\nu\rho}D_\lambda V^\sigma - T^\lambda{}_{\nu\rho}D_\lambda V^\sigma \quad -(11)$$

3) So eq. (4) is:

$$\boxed{\begin{aligned}&\left(D_\rho R^\sigma{}_{\alpha\mu\nu} + D_\nu R^\sigma{}_{\alpha\rho\mu} + D_\mu R^\sigma{}_{\alpha\nu\rho}\right) V^\alpha \\ &+ \left(T^\lambda{}_{\mu\nu} + T^\lambda{}_{\rho\mu} + T^\lambda{}_{\nu\rho}\right) D_\lambda V^\sigma \\ &- \left(R^\sigma{}_{\lambda\mu\nu} D_\rho V^\sigma + R^\sigma{}_{\lambda\rho\mu} D_\nu V^\sigma + R^\sigma{}_{\lambda\nu\rho} D_\mu V^\sigma\right) \\ &= 0 \qquad -(12)\end{aligned}}$$

where we have used the Cartan identity:

$$D_\rho T^\lambda{}_{\mu\nu} + D_\nu T^\lambda{}_{\rho\mu} + D_\mu T^\lambda{}_{\nu\rho} = R^\lambda{}_{\rho\mu\nu} + R^\lambda{}_{\nu\rho\mu} + R^\lambda{}_{\mu\nu\rho} \qquad -(13)$$

It is seen that eq. (12) does not give the incorrect "second Bianchi identity":

$$D_\rho R^\sigma{}_{\alpha\mu\nu} + D_\nu R^\sigma{}_{\alpha\rho\mu} + D_\mu R^\sigma{}_{\alpha\nu\rho} \stackrel{?}{=} 0 \qquad -(14),$$

Q.E.D.

1) 138(8): Contractions in the Incorrect "Second Bianchi Identity".

The incorrect "second Bianchi identity" is:

$$D_\lambda R_{\rho\sigma\mu\nu} + D_\rho R_{\sigma\lambda\mu\nu} + D_\sigma R_{\lambda\rho\mu\nu} =?\ 0 \quad -(1)$$

This is the incorrect basis of the incorrect Einstein field equation. The procedure adopted is to contract eq. (1) as follows:

$$g^{\nu\sigma} g^{\mu\lambda} \left(D_\lambda R_{\rho\sigma\mu\nu} + D_\rho R_{\sigma\lambda\mu\nu} + D_\sigma R_{\lambda\rho\mu\nu} \right) =?\ 0 \quad -(2)$$

By metric compatibility

$$g^{\mu\lambda} D_\lambda \left(g^{\nu\sigma} R_{\rho\sigma\mu\nu} \right) + D_\rho \left(g^{\nu\sigma} g^{\mu\lambda} R_{\sigma\lambda\mu\nu} \right) + g^{\nu\sigma} D_\sigma \left(g^{\mu\lambda} R_{\lambda\rho\mu\nu} \right) =?\ 0. \quad -(3)$$

Here, the metric is symmetric:

$$g^{\mu\lambda} = g^{\lambda\mu} \quad -(4)$$

etc. It is implicitly assumed that the connection is also symmetric

$$\Gamma^\lambda_{\mu\nu} =?\ \Gamma^\lambda_{\nu\mu} \neq ?\ 0. \quad -(5)$$

If eq. (5) is assumed:

$$R_{\rho\sigma\nu\mu} =?\ -R_{\sigma\rho\nu\mu} \quad -(6)$$

and:

$$R_{\mu\nu\rho\sigma} =?\ R_{\rho\sigma\mu\nu}. \quad -(7)$$

These are incorrect symmetries. The only

2) correct symmetry is:

$$R_{\rho\sigma\mu\nu} = -R_{\rho\sigma\nu\mu}. \quad -(8)$$

The incorrect symmetry (7) is used to define the Ricci tensor:

$$R_{\mu\rho} =? R_{\rho\mu} = g^{\sigma\nu} R_{\mu\sigma\rho\nu} = g^{\sigma\nu} R_{\mu\sigma\rho\nu}$$

$$(9)$$

The basic error in eq. (5) works through into eqs. (7) and (9). Finally the following contraction is made:

$$g^{\nu\sigma} g^{\mu\lambda} R_{\sigma\lambda\mu\nu} =? -g^{\nu\sigma} g^{\mu\lambda} R_{\lambda\sigma\mu\nu}$$

$$-(10)$$

This contraction again depends on the use of the incorrect eq. (5). Eq. (10) is written as:

$$R = -g^{\mu\lambda} R_{\lambda\mu} = -g^{\nu\sigma} g^{\mu\lambda} R_{\lambda\sigma\mu\nu} \quad -(11)$$

So eq. (3) becomes the incorrect:

$$D^{\mu} R_{\rho\mu} - D_{\rho} R + D^{\sigma} R_{\rho\sigma} =? 0 \quad -(12)$$

with the incorrect:

$$R_{\rho\mu} =? R_{\mu\rho}. \quad -(13)$$

Eq. (12) is written as:

$$D^{\mu} G_{\mu\nu} =? 0 \quad -(14)$$

3) in which the Einstein field tensor is incorrectly defined:

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} =? 0 \quad -(15)$$

Einstein further compounded this error by the claim that

$$D^\mu G_{\mu\nu} =? k D^\mu T_{\mu\nu} \quad -(16)$$

where

$$T_{\mu\nu} = T_{\nu\mu} \quad -(17)$$

is the canonical energy-momentum density. Finally it was claimed that:

$$G_{\mu\nu} =? k T_{\mu\nu}, \quad -(18)$$

a meaningless equation.

The correct field equations are based directly and simply on the Cartan and Evans identities:

$$D \wedge T := R \wedge q \quad -(19)$$

and

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(20)$$

in which

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}. \quad -(21)$$

1) 138(9) : Invariance of the Vector Field under Coordinate Transformation.

This is denoted in general as in eq. (1.26) of Carroll (downloadable notes):

$$V = V^{\mu} \hat{e}_{(\mu)} = V^{\nu'} \hat{e}_{(\nu')} \quad -(1)$$

For example, considering a Lorentz boost in the x axis:

$$V^{\mu} = \begin{bmatrix} ct \\ x \end{bmatrix} \quad -(2)$$

The y and z axis remain the same, so we need only consider (2). The vector field is

$$V = ct\, \underline{e}_0 + x\, \underline{i} \quad -(3)$$

in vector notation. So:

$$V = V' = (ct)'\, \underline{e}_0' + x' \underline{i}'. \quad -(4)$$

The x axis Lorentz boost is:

$$\Lambda = \begin{bmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{bmatrix} \quad -(5)$$

The inverse Lorentz boost is Λ^{-1} defined by $\Lambda \Lambda^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad -(6)$

2) So:
$$\Lambda^{-1} = \begin{bmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{bmatrix} \quad -(7)$$

The <u>components</u> V^μ transform as:

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} \quad -(8)$$

i.e.
$$ct' = ct\cosh\phi - x\sinh\phi \quad -(9)$$
$$x' = -ct\sinh\phi + x\cosh\phi \quad -(10)$$

The <u>unit vectors</u> $\hat{e}_{(\mu)}$ transform as:

$$\begin{bmatrix} \underline{e}_0' \\ \underline{i}' \end{bmatrix} = \begin{bmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{bmatrix} \begin{bmatrix} \underline{e}_0 \\ \underline{i} \end{bmatrix} \quad -(11)$$

i.e.
$$\underline{e}_0' = \underline{e}_0\cosh\phi + \underline{i}\sinh\phi \quad -(12)$$
$$\underline{i}' = \underline{e}_0\sinh\phi + \underline{i}\cosh\phi \quad -(13)$$

Both components and unit vectors <u>transform</u> <u>covariantly</u> according to eqns. (8) and (11).

<u>Check</u>

We have:

3)

$$\underline{V} = ct\,\underline{e}_0 + x\,\underline{i} \qquad -(14)$$

$$\underline{V}' = ct'\,\underline{e}_0' + x'\,\underline{i}' \qquad -(15)$$

Eq. (15) is:

$$\underline{V}' = (ct\cosh\phi - x\sinh\phi)(\cosh\phi\,\underline{e}_0 + \sinh\phi\,\underline{i})$$

$$+ (x\cosh\phi - ct\sinh\phi)(\sinh\phi\,\underline{e}_0 + \cosh\phi\,\underline{i})$$

$$= ct(\cosh^2\phi - \sinh^2\phi)\,\underline{e}_0 + x(\cosh^2\phi - \sinh^2\phi)\,\underline{i}$$

$$- x\sinh\phi\cosh\phi\,\underline{e}_{(0)} + x\sinh\phi\cosh\phi\,\underline{e}_{(0)}$$

$$- ct\sinh\phi\cosh\phi\,\underline{i} + ct\sinh\phi\cosh\phi\,\underline{i}.$$

$$= ct\,\underline{e}_0 + x\,\underline{i} \qquad -(16)$$

$$= \underline{V}$$

Q.E.D.

Application to B Cyclic Theorem

The B Cyclic Theorem is:

$$\underline{B}^{(1)} \times \underline{B}^{(2)} = i\,\underline{B}^{(3)*} \qquad -(17)$$

et cyclicum $\qquad -(18)$

where

$$\underline{B}^{(1)} = B^{(0)}\,\underline{e}^{(1)}\,e^{i\phi}$$

$$\underline{B}^{(2)} = B^{(0)}\,\underline{e}^{(2)}\,e^{-i\phi} \qquad -(19)$$

4) $$\underline{B}^{(3)*} = \underline{B}^{(3)} = B^{(0)}\underline{e}^{(3)} \quad -(20)$$

So eq. (17) is:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\underline{e}^{(3)*} \quad -(21)$$

et cyclicum

The basis vectors $\underline{e}^{(1)}$, $\underline{e}^{(2)}$ and $\underline{e}^{(3)}$ are Lorentz covariant by definition. So the B Cyclic Theorem (17) is Lorentz covariant, Q.E.D.

The complex circular basis vectors are:

$$\left.\begin{aligned} \underline{e}^{(1)} &= \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \\ \underline{e}^{(2)} &= \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \\ \underline{e}^{(3)} &= \underline{k} \\ \underline{e}^{(0)} &= \underline{e}_0 \end{aligned}\right\} \quad -(22)$$

They are complex combinations of the Cartesian unit vectors.

1. Notes 138(10): Antisymmetry of the Connection, Further Details

The fundamental theorem of Riemann geometry is:

$$[D_\mu, D_\nu] V^\rho = \left(\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}\right) V^\sigma \quad -(1)$$

$$- \left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}\right) D_\lambda V^\rho . \quad -(2)$$

If $\mu = \nu$

$$[D_\mu, D_\nu] V^\rho = 0. \quad -(3)$$

Therefore

$$\Gamma^\lambda_{00} = \Gamma^\lambda_{11} = \Gamma^\lambda_{22} = \Gamma^\lambda_{33} = 0, \quad -(4)$$

$$T^\lambda_{00} = T^\lambda_{11} = T^\lambda_{22} = T^\lambda_{33} = 0. \quad -(5)$$

Also,

$$\left.\begin{aligned}
\partial_0 \Gamma^\rho_{0\sigma} - \partial_0 \Gamma^\rho_{0\sigma} + \Gamma^\rho_{0\lambda}\Gamma^\lambda_{0\sigma} - \Gamma^\rho_{0\lambda}\Gamma^\lambda_{0\sigma} &= 0 \\
\partial_1 \Gamma^\rho_{1\sigma} - \partial_1 \Gamma^\rho_{1\sigma} + \Gamma^\rho_{1\lambda}\Gamma^\lambda_{1\sigma} - \Gamma^\rho_{1\lambda}\Gamma^\lambda_{1\sigma} &= 0 \\
\partial_2 \Gamma^\rho_{2\sigma} - \partial_2 \Gamma^\rho_{2\sigma} + \Gamma^\rho_{2\lambda}\Gamma^\lambda_{2\sigma} - \Gamma^\rho_{2\lambda}\Gamma^\lambda_{2\sigma} &= 0 \\
\partial_3 \Gamma^\rho_{3\sigma} - \partial_3 \Gamma^\rho_{3\sigma} + \Gamma^\rho_{3\lambda}\Gamma^\lambda_{3\sigma} - \Gamma^\rho_{3\lambda}\Gamma^\lambda_{3\sigma} &= 0
\end{aligned}\right] \quad -(6)$$

The only non-zero connections are:

$$\left.\begin{aligned}
&\Gamma^\lambda_{01} = -\Gamma^\lambda_{10}, \; \Gamma^\lambda_{02} = -\Gamma^\lambda_{20}, \; \Gamma^\lambda_{03} = -\Gamma^\lambda_{30} \\
&\Gamma^\lambda_{12} = -\Gamma^\lambda_{21}, \; \Gamma^\lambda_{13} = -\Gamma^\lambda_{31} \\
&\Gamma^\lambda_{23} = -\Gamma^\lambda_{32}.
\end{aligned}\right] \quad -(7)$$

Therefore in the Riemann tensor:

$$R^\rho{}_{\sigma\mu\nu} := \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \quad -(8)$$

$$\mu \neq \nu \quad -(9)$$

and

$$R^\rho{}_{\sigma\mu\nu} = - R^\rho{}_{\sigma\nu\mu}. \quad -(10)$$

2. The other symmetries are:

$$T^{\lambda}_{\mu\nu} = -T^{\lambda}_{\nu\mu}, \quad -(11)$$

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}, \quad -(12)$$

$$\partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} = -\left(\partial_\nu \Gamma^{\rho}_{\mu\sigma} - \partial_\mu \Gamma^{\rho}_{\nu\sigma}\right), \quad -(13)$$

$$\Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} = -\left(\Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma}\right), \quad -(14)$$

$$\Gamma^{\rho}_{\nu\sigma} = -\Gamma^{\rho}_{\sigma\nu} \quad -(15)$$

$$\Gamma^{\rho}_{\mu\sigma} = -\Gamma^{\rho}_{\sigma\mu} \quad -(16)$$

$$\Gamma^{\rho}_{\mu\lambda} = -\Gamma^{\rho}_{\lambda\mu} \quad -(17)$$

$$\Gamma^{\lambda}_{\nu\sigma} = -\Gamma^{\lambda}_{\sigma\nu} \quad -(18)$$

$$\Gamma^{\rho}_{\nu\lambda} = -\Gamma^{\rho}_{\lambda\nu} \quad -(19)$$

$$\Gamma^{\lambda}_{\mu\sigma} = -\Gamma^{\lambda}_{\sigma\mu} \quad -(20)$$

The Error in the Twentieth Century Cosmology

This was to assume that the connection could be symmetric and non-zero. This is a glaring error because it assumes that there is a non-zero symmetric commutator. This assumption was used to write the incorrect equation:

$$[D_\mu, D_\nu] V^\rho = ? \left(\partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}\right) V^\sigma \neq 0 \quad -(21)$$

In this equation there is no indication of the symmetry of the connection, whereas the correct eq. (1) fixes the antisymmetry (12) through:

5.

$$[D_\mu, D_\nu] V^\rho = -\Gamma^\lambda_{\mu\nu} + \dots \qquad -(22)$$

The commutator $[D_\mu, D_\nu]$ and the connection $\Gamma_{\mu\nu}$ must both be antisymmetric.

In the incorrect eqn. (21), there is nothing to indicate this, and the error was compounded by assuming that:

$$\Gamma^\lambda_{\mu\nu} =? \frac{1}{2}\left(\Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\mu\nu}\right) + \frac{1}{2}\left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\mu\nu}\right) \qquad -(23)$$

in which:

$$\Gamma^\lambda_{\mu\nu}(S) =? \Gamma^\lambda_{\nu\mu}(S) \qquad -(24)$$

and

$$\Gamma^\lambda_{\mu\nu}(A) = -\Gamma^\lambda_{\nu\mu}(A) \qquad -(25)$$

The correct eq. (22) shows that eq. (25) is the correct antisymmetry.

Scientific History

The basic error is so glaring:

$$\boxed{[D_\mu, D_\nu] V^\rho =? [D_\nu, D_\mu] V^\rho \neq? 0} \qquad -(26)$$

that some research is needed into why it was made, and why it was repeated for ninety years.

38(11) : Parallel Transport and Geodesics

! The theory of parallel transport depends on the connection and different connections will give different answers. The parallel transport equation is, for a symmetric connection:

$$\frac{dV^\mu}{d\lambda} + \Gamma^\mu_{\sigma\rho} \frac{dx^\sigma}{d\lambda} V^\rho = 0, \quad -(1)$$

where the connection appears as $\Gamma^\mu_{\sigma\rho}$. Solving this for a vector V^μ amounts to finding a matrix $P^\mu{}_\rho(\lambda, \lambda_0)$ which relates the vector at its initial value $V^\mu(\lambda_0)$ to its value later in the path.

$$V^\mu(\lambda) = P^\mu{}_\rho(\lambda, \lambda_0) V^\rho(\lambda_0). \quad -(2)$$

Define the matrix:

$$A^\mu{}_\rho(\lambda) = -\Gamma^\mu_{\sigma\rho} \frac{dx^\sigma}{d\lambda} \quad -(3)$$

Then:

$$\frac{d}{d\lambda} P^\mu{}_\rho(\lambda, \lambda_0) = A^\mu{}_\sigma(\lambda) P^\sigma{}_\rho(\lambda, \lambda_0) - (4)$$

Schroedinger's equation for a time ordered operator has the same form as eq. (5). Its solution can be expressed as a path ordered exponential similar to Dyson's solution:

$$P^\mu{}_\nu(\lambda, \lambda_0) = \hat{P} \exp\left(-\int_{\lambda_0}^{\lambda} \Gamma^\mu_{\sigma\nu} \frac{dx^\sigma}{d\eta} d\eta\right)$$

$$-(5)$$

If the path is a loop, starting and ending at the same point, then $P^\mu{}_\nu(\lambda, \lambda_0)$ is a Lorentz transform

... on the tangent space at the point. The transformation is the holonomy of the loop. Knowing the holonomy of every possible loop is equivalent to knowing the metric. If the connection is not symmetric, all of geodesic theory is changed.

The tangent vector to a path $x^\mu(\lambda)$ is The tangent vector is $dx^\mu / d\lambda$. Parallel transport of the tangent vector is

$$\frac{D}{d\lambda}\left(\frac{dx^\mu}{d\lambda}\right) = 0 \quad — (6)$$

i.e.

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0 \quad — (7)$$

The proper time is calculated using the definition of a time-like path (Carroll notes, eq. (3.48)):

$$\tau = \int\left(-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right)^{1/2} d\lambda \quad — (8)$$

The calculus of variations gives the shortest path:

$$\frac{d^2x^\rho}{d\tau^2} + \frac{1}{2}g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0 \quad — (9)$$

Eq (9) reduces to eq. (1) if and only

3) if the connection is symmetric. In other words if and only if:

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right) \quad — (10)$$

In general, eq. (10) is not true, and in general, eqs. (1) and (9) are not the same. Eq. (6) is the path that parallel transports its own tangent vector. Eq. (7) is the shortest distance between two points. When the connection is not symmetric, these concepts do not lead to the same result.

Einstein used eq. (7) to derive the Newtonian limit (Carroll eq. (4.7) ff). So Einstein's theory depends on the assumption of a symmetric connection. It is now known that the connection is antisymmetric, and can never be symmetric. Einstein's method was to consider the Newtonian limit as:

$$\frac{dx^i}{d\tau} << \frac{dt}{d\tau} \quad — (11)$$

so eq. (7) reduces to:

4)
$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2 = 0 \quad -(12)$$

Einstein again assumes the is connect symmetric connection:

$$\Gamma^\mu_{00} = \frac{1}{2} g^{\mu\lambda}\left(\partial_0 g_{\lambda 0} + \partial_0 g_{0\lambda} - \partial_\lambda g_{00}\right)$$

$$= -\frac{1}{2} g^{\mu\lambda} \partial_\lambda g_{00} \quad -(13)$$

and at this point it may be concluded that Einstein's procedure is meaningless.

For the sake of completeness it is described as follows. The metric is expanded as a perturbation of the Minkowski metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| << 1. \quad -(14)$$

Then we:
$$g^{\mu\nu} g_{\nu\sigma} = \delta^\mu_\sigma \quad -(15)$$

so:
$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}. \quad -(16)$$

Thus:
$$\Gamma^\mu_{00} = -\frac{1}{2}\eta^{\mu\lambda}\partial_\lambda h_{00} \quad -(17)$$

From eq. (17) in eq. (12):

$$\frac{d^2 x^\mu}{d\tau^2} = \frac{1}{2}\eta^{\mu\lambda}\partial_\lambda h_{00}\left(\frac{dt}{d\tau}\right)^2 \quad -(18)$$

5) Then use: $\partial_0 h_{00} = 0$ — (19)

so $\frac{d^2 t}{d\tau^2} = 0$ — (20)

i.e. $dt / d\tau$ is constant.

The spacelike components are given by:

$$\frac{d^2 x^i}{d\tau^2} = \frac{1}{2} \left(\frac{dt}{d\tau} \right)^2 \partial_i h_{00} \quad — (21)$$

i.e. $$\frac{d^2 x^i}{dt^2} = \frac{1}{2} \partial_i h_{00} \quad — (22)$$

Finally the incorrect eq. (22) is claimed to be the Newtonian theory by the arbitrary assertion:

$$h_{00} = -2\Phi \quad — (23)$$

where $$\Phi = -\frac{GM}{r} \quad — (24)$$

It is claimed incorrectly that eq. (24) was derived by Schwarzschild in 1916 from a metric solution of the Einstein field equation.

Eq. (10) is derived from the incorrect equation

$$\Gamma^{\lambda}_{\mu\nu} \overset{?}{=} \Gamma^{\lambda}_{\nu\mu} \quad - (25)$$

and the assumption of metric compatibility (Carroll eq. (3.17)):

$$D_{\rho} g_{\mu\nu} = 0 . \quad - (26)$$

From eq. (26):

$$D_{\rho} g_{\mu\nu} = \partial_{\rho} g_{\mu\nu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda} = 0 \quad - (27)$$

$$D_{\mu} g_{\nu\rho} = \partial_{\mu} g_{\nu\rho} - \Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho} - \Gamma^{\lambda}_{\mu\rho} g_{\nu\lambda} = 0 . \quad - (28)$$

$$D_{\nu} g_{\rho\mu} = \partial_{\nu} g_{\rho\mu} - \Gamma^{\lambda}_{\nu\rho} g_{\lambda\mu} - \Gamma^{\lambda}_{\nu\mu} g_{\rho\lambda} = 0 \quad - (29)$$

Sustract eqs. (28) and (29) from eq. (27):

$$\partial_{\rho} g_{\mu\nu} - \partial_{\mu} g_{\nu\rho} - \partial_{\nu} g_{\rho\mu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda} + \Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho} + \Gamma^{\lambda}_{\mu\rho} g_{\nu\lambda} + \Gamma^{\lambda}_{\nu\rho} g_{\lambda\mu} + \Gamma^{\lambda}_{\nu\mu} g_{\rho\lambda} = 0 \quad - (30)$$

It is now assumed incorrectly that:

$$\Gamma^{\lambda}_{\mu\rho} \overset{?}{=} \Gamma^{\lambda}_{\rho\mu} \quad - (31)$$

$$\Gamma^{\lambda}_{\rho\nu} \overset{?}{=} \Gamma^{\lambda}_{\nu\rho} . \quad - (32)$$

7)

The metric is symmetric, so it is assumed that

$$\Gamma^\lambda_{\rho\mu}\, g_{\lambda\nu} \stackrel{?}{=} \Gamma^\lambda_{\mu\rho}\, g_{\nu\lambda} \qquad (33)$$

and

$$\Gamma^\lambda_{\rho\nu}\, g_{\mu\lambda} \stackrel{?}{=} \Gamma^\lambda_{\nu\rho}\, g_{\lambda\mu} \qquad (34)$$

Eq. (30) therefore becomes

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} + \Gamma^\lambda_{\mu\nu}\, g_{\lambda\rho} + \Gamma^\lambda_{\nu\mu}\, g_{\rho\lambda} = 0 \qquad (35)$$

Now it is again incorrectly assumed that

$$\Gamma^\lambda_{\mu\nu} \stackrel{?}{=} \Gamma^\lambda_{\nu\mu} \qquad (36)$$

so:

$$\Gamma^\sigma_{\mu\nu} \stackrel{?}{=} \frac{1}{2} g^{\sigma\rho}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right) \qquad (37)$$

This incorrect formula is found in all the textbooks of the last 92 years on general relativity. ECE theory does not use eq. (37).