133(1): Field Theory in Terms of Spin Connections.

The major advance being made now is that field theory is being shown to be due entirely to the spin connection of spacetime. On the u(1) level, paper 132 shows that if:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(1)$$

and:

$$\partial_\mu A_\nu = -\partial_\nu A_\mu \quad -(2)$$

then:

$$F_{\mu\nu} = ?\,0. \quad -(3)$$

Therefore:

$$\partial_\mu A_\nu = \partial_\nu A_\mu \quad -(4)$$

From eqns. (2) and (4):

$$\boxed{\begin{aligned}\partial_\mu A_\nu &= 0,\\ \partial_\nu A_\mu &= 0.\end{aligned}} \quad -(5)$$

Therefore u(1) gauge symmetry electrodynamics is incorrect fundamentally. Fields cannot be constructed from potentials. In consequence there is no gauge theory of any kind.

On the ECE level:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + A^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right) \quad -(6)$$

2.

Therefore define:

$$G^{a}_{\mu\nu} = F^{a}_{\mu\nu} - A^{(0)}\left(\omega^{a}_{\mu\nu} - \omega^{a}_{\nu\mu}\right) \quad -(7)$$

so that:

$$G^{a}_{\mu\nu} = \partial_\mu A^{a}_{\nu} - \partial_\nu A^{a}_{\mu} \quad -(8)$$

For each a:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(9)$$

where

$$\partial_\mu A_\nu = -\partial_\nu A_\mu, \quad -(10)$$

so

$$\boxed{G^{a}_{\mu\nu} = 0} \quad -(11)$$

and

$$\partial_\mu A^{a}_{\nu} = \partial_\nu A^{a}_{\mu} = 0. \quad -(12)$$

The major advance is made that all fields are defined by spin connections of general relativity.

The gravitational field is:

$$g^{a}_{\mu\nu} = \partial_\mu \Phi^{a}_{\nu} - \partial_\nu \Phi^{a}_{\mu} + \Phi^{(0)}\left(\omega^{a}_{\mu\nu} - \omega^{a}_{\nu\mu}\right) \quad -(13)$$

Therefore:

3.

$$F^a_{\mu\nu} = A^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right), \quad (14)$$
$$g^a_{\mu\nu} = \Phi^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right).$$

where:

$$\omega^a_{\mu\nu} = -\omega^a_{\nu\mu} \quad (15)$$

In vector notation:

$$\underline{E}^a = E^{(0)}\underline{\omega}^a_E$$
$$\underline{B}^a = B^{(0)}\underline{\omega}^a_B \quad (16)$$
$$\underline{g}^a = g^{(0)}\underline{\omega}^a_g$$
$$\underline{h}^a = h^{(0)}\underline{\omega}^a_h$$

Electrodynamical Field Equations

These are:

$$D_\mu F^{a\mu\nu} = A^{(0)} R^{a\ \mu\nu}_{\mu} \quad (17)$$
$$D_\mu \tilde{F}^{a\mu\nu} = A^{(0)} \tilde{R}^{a\ \mu\nu}_{\mu} \quad (18)$$

Gravitational Field Equations

$$D_\mu g^{a\mu\nu} = \Phi^{(0)} R^{a\ \mu\nu}_{\mu} \quad (19)$$
$$D_\mu \tilde{g}^{a\mu\nu} = \Phi^{(0)} \tilde{R}^{a\ \mu\nu}_{\mu} \quad (20)$$

4) The field equations are defined entirely in terms of eqs. (14), which involve only spin connections.

Collapse of U(1) Electrodynamics

This is easily illustrated through the field equations of U(1) electrodynamics in the differential form notation:

$$F = d \wedge A \quad -(21)$$

$$d \wedge F = 0 \quad -(22)$$

$$d \wedge \tilde{F} = \tilde{J} \quad -(23)$$

From eqs. (1) to (5):

$$\left.\begin{aligned} F = d \wedge A &= 0 \\ d \wedge F &= 0 \\ d \wedge \tilde{F} &= 0 \\ \tilde{F} &= 0 \end{aligned}\right\} \quad -(24)$$

So there are no potentials and no fields. All unified field theories based on a U(1) sector symmetry are incorrect. The gravitational sector of such theories is based on the wholly incorrect Einstein field equation.

) 133(2): Antisymmetry applied to the Riemann Tensor in Gravitational Theory.

The symmetries of the Riemann tensor are now known to be:

$$R^{\rho}{}_{\sigma\mu\nu} = -R^{\rho}{}_{\sigma\nu\mu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \quad -(1)$$

where:

$$\partial_\mu \Gamma^{\rho}_{\nu\sigma} = -\partial_\nu \Gamma^{\rho}_{\mu\sigma}. \quad -(2)$$

For each ρ and σ in eq. (2):

$$\partial_\mu \Gamma_\nu = -\partial_\nu \Gamma_\mu. \quad -(3)$$

Define:

$$S^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma}, \quad -(4)$$

so for each ρ and σ:

$$S_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu, \quad -(5)$$

$$S_{0i} = \partial_0 \Gamma_i - \partial_i \Gamma_0, \quad -(6)$$

$$S_{ij} = \partial_i \Gamma_j - \partial_j \Gamma_i. \quad -(7)$$

In vector notation:

$$\underline{S_1} = -\underline{\nabla}\Gamma_0 - \frac{\partial \underline{\Gamma}}{\partial t} \quad -(8)$$

$$\underline{S_2} = \underline{\nabla} \times \underline{\Gamma} \quad -(9)$$

where:

$$\underline{\Gamma} = \Gamma_1 \underline{i} + \Gamma_2 \underline{j} + \Gamma_3 \underline{k} \quad -(10)$$

2) By antisymmetry:

$$\underline{\nabla} \Gamma_0 = \frac{1}{c}\frac{\partial \underline{\Gamma}}{\partial t} \quad -(11)$$

Therefore

$$\underline{\nabla} \times \underline{\nabla} \Gamma_0 = \frac{1}{c}\underline{\nabla} \times \frac{\partial \underline{\Gamma}}{\partial t} = \underline{0} \quad -(12)$$

and

$$\frac{\partial}{\partial t}\left(\underline{\nabla} \times \underline{\Gamma}\right) = \boxed{\frac{\partial \underline{S}_2}{\partial t} = \underline{0}} \quad -(13)$$

From eq. (12):

$$\boxed{\underline{\nabla} \times \underline{S}_1 = \underline{0}} \quad -(14)$$

Therefore:

$$\boxed{\underline{\nabla} \times \underline{S}_1 + \frac{1}{c}\frac{\partial \underline{S}_2}{\partial t} = \underline{0}} \quad -(15)$$

In gravitational theory on the ECE level:

$$\underline{g} = \Phi^{(0)} \underline{S}_1 \quad -(16)$$

$$\underline{h} = \frac{\Phi^{(0)}}{c} \underline{S}_2 \quad -(17)$$

so

$$\underline{\nabla} \times \underline{g} + \frac{\partial \underline{h}}{\partial t} = \underline{0} \quad -(18)$$

$$\underline{\nabla} \times \underline{g} = \underline{0} \quad -(19)$$

$$\frac{\partial \underline{h}}{\partial t} = \underline{0} \quad -(20)$$

i) Since

$$\underline{h} \parallel \frac{\partial \underline{h}}{\partial t} \quad -(21)$$

antisymmetry in the Riemann tensor means:

$$\underline{h} = \underline{0} \quad -(22)$$

from the S tensor part of the Riemann curvature. This means:

$$\boxed{\partial_i \Gamma^\rho{}_{j\sigma} = \partial_j \Gamma^\rho{}_{i\sigma} = 0} \quad -(23)$$

This is a new general property of the Riemann tensor that greatly reduces the number of independent elements of the tensor, giving the result:

$$\boxed{R^\rho{}_{\sigma ij} = \Gamma^\rho{}_{i\lambda} \Gamma^\lambda{}_{j\sigma} - \Gamma^\rho{}_{j\lambda} \Gamma^\lambda{}_{i\sigma}} \quad -(24)$$

$$i, j = 1, 2, 3$$

This case describes the "magnetic" part of the curvature tensor. The electric part is:

$$R^\rho{}_{\sigma 0 i} = \partial_0 \Gamma^\rho{}_{i\sigma} - \partial_i \Gamma^\rho{}_{0\sigma} + \Gamma^\rho{}_{0\lambda} \Gamma^\lambda{}_{i\sigma} - \Gamma^\rho{}_{i\lambda} \Gamma^\lambda{}_{0\sigma} \quad -(25)$$

$$= 2 \left(\partial_0 \Gamma^\rho{}_{i\sigma} + \Gamma^\rho{}_{0\lambda} \Gamma^\lambda{}_{i\sigma} \right)$$

By antisymmetry:

$$R^\rho{}_{\sigma ij} = 2 \Gamma^\rho{}_{i\lambda} \Gamma^\lambda{}_{j\sigma} \quad -(26)$$

4) In the special case:

$$\partial_0 \Gamma^\rho_{i\sigma} = 0 \quad -(27)$$

then

$$R^\rho{}_{\sigma\mu\nu} = 2\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} \quad -(28)$$

This procedure greatly simplifies the computation of the Riemann tensor. The constraint (27) can be seen as that of a static connection. By antisymmetry eq. (27) implies:

$$\partial_i \Gamma^\rho_{0\sigma} = 0 \quad -(29)$$

Conclusion

the Riemann tensor is:

$$R^\rho{}_{\sigma ij} = 2\Gamma^\rho_{i\lambda}\Gamma^\lambda_{j\sigma}$$

$$i, j = 1, 2, 3$$

$$R^\rho{}_{\sigma 0 i} = 2\left(\partial_0 \Gamma^\rho_{i\sigma} + \Gamma^\rho_{0\lambda}\Gamma^\lambda_{i\sigma}\right)$$

$$i = 1, 2, 3$$

$$-(30)$$

1. 135(3). Antisymmetry laws without the constraint $\underline{A} = \underline{0}$.

On the u(1) level:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}, \quad -(1)$$

$$\underline{B} = \underline{\nabla} \times \underline{A}. \quad -(2)$$

They are now constrained by:

$$\underline{\nabla}\phi = \frac{\partial \underline{A}}{\partial t} \quad -(3)$$

$$\frac{\partial A_i}{\partial x_j} = -\frac{\partial A_j}{\partial x_i} \quad -(4)$$

It follows that:

$$\underline{\nabla} \times \underline{\nabla}\phi = \underline{\nabla} \times \frac{\partial \underline{A}}{\partial t} = \underline{0}, \quad -(5)$$

so:

$$\underline{B} = \underline{\nabla} \times \underline{A} = \underline{0}. \quad -(6)$$

The antisymmetry constraints (3) and (4) cause the magnetic field to vanish. So u(1) electrodynamics is restricted by antisymmetry to electric fields only. In this note a potential is derived which is in accordance with the antisymmetry rule. The only situation in which the electric field does not vanish is defined by this potential:

$$\oint \underline{A} \cdot d\underline{l} = \int_S (\underline{\nabla} \times \underline{A}) \cdot \underline{n} \, dA \quad -(7)$$

$$= 0,$$

i.e. by

$$\underline{\nabla} \times \underline{A} = \underline{0}, \quad -(8)$$

and by:

$$\frac{\partial A_X}{\partial Y} = -\frac{\partial A_Y}{\partial X} \quad \text{etc.} \quad -(9)$$

Therefore:

$$\underline{A} = A_X \underline{i} + A_Y \underline{j} + A_Z \underline{k} \quad -(10)$$

It also obeys the antisymmetry equation:

$$\frac{\partial \phi}{\partial Z} = \frac{\partial A_Z}{\partial t} \quad \text{etc.} \quad -(11)$$

<u>Travelling Waves</u>

Attempt the solution:

$$\phi = \frac{\phi^{(0)}}{\sqrt{2}} \exp\left(i(\omega t - \kappa Z)\right) \quad -(12)$$

Then:

$$\frac{\partial \phi}{\partial Z} = \frac{\partial A_Z}{\partial t} = -i\kappa\phi \quad -(13)$$

$$A_Z = -i\kappa \int \phi \, dt \quad -(14)$$

In general:

$$\nabla \phi = -i\kappa\phi\left(\underline{i} + \underline{j} + \underline{k}\right) \quad -(15)$$

$$\frac{\partial \underline{A}}{\partial t} = -i\frac{\omega}{c}\phi\left(\underline{i} + \underline{j} + \underline{k}\right) \quad -(16)$$

$$\underline{E} = 2i\kappa\phi\left(\underline{i} + \underline{j} + \underline{k}\right) e^{i(\omega t - \kappa Z)} \quad -(17)$$

$$Re(\underline{E}) = -2\kappa\phi \sin(\omega t - \kappa Z)\left(\underline{i} + \underline{j} + \underline{k}\right) \quad -(18)$$

In free space on the U(1) level:

$$\nabla \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(19)$$

$$\nabla \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(20)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(21)$$

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(22)$$

A solution of the type (18) does not obey:

$$\underline{\nabla} \times \underline{E} = \underline{0}, \quad \frac{\partial \underline{E}}{\partial t} = \underline{0}, \quad \underline{\nabla} \cdot \underline{E} = 0 \quad -(23)$$

so cannot be a free space solution. If for example:

$$\underline{E} = E_Z \underline{k} \quad -(24)$$

$$E_Z = 2ik\phi e^{i(\omega t - kz)} \quad -(25)$$

the

$$\underline{\nabla} \cdot \underline{E} = \frac{\partial E_Z}{\partial Z} = 2k^2 e^{i(\omega t - kz)} \phi$$

$$= \rho / \epsilon_0 \quad -(26)$$

Therefore:

$$\boxed{\rho = 2\epsilon_0 k^2 \phi e^{i(\omega t - kz)}} \quad -(27)$$

Similarly:

$$-\frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(28)$$

From eq. (25):

$$\frac{\partial E_Z}{\partial t} = -2k\omega\phi e^{i(\omega t - kz)} \quad -(29)$$

So:

$$\boxed{J_Z = \frac{2k\omega\phi}{\mu_0 c^2} e^{i(\omega t - kz)}} \quad -(30)$$

4) If this wave propagates at a velocity v:

$$\kappa = \frac{\omega}{v} \quad -(31)$$

so:

$$\rho = 2\epsilon_0 \frac{\omega^2}{v^2} \phi e^{i(\omega t - \kappa z)} \quad -(32)$$

$$J_Z = 2\epsilon_0 \frac{\omega^2}{v} \phi e^{i(\omega t - \kappa z)} \quad -(33)$$

On average:

$$\langle \rho \rangle = 0 \quad -(34)$$

$$\langle J_Z \rangle = 0. \quad -(35)$$

and:

$$\langle E \rangle = \underline{0}. \quad -(36)$$

The root mean squares are:

$$\langle \rho^2 \rangle^{1/2} = \epsilon_0 \left(\frac{\omega}{v}\right)^2 \phi \quad -(37)$$

$$\langle J_Z^2 \rangle^{1/2} = \epsilon_0 \frac{\omega^2}{v} \phi \quad -(38)$$

$$\langle E_Z^2 \rangle^{1/2} = \kappa \phi \quad -(39)$$

$$= \frac{\omega}{v} \phi \quad -(40)$$

Coulomb Law

In this case:

$$\phi = -\frac{e}{4\pi\epsilon_0 Z} \quad -(41)$$

$$\frac{\partial \phi}{\partial Z} = \frac{\partial A_Z}{\partial t} = \frac{e}{4\pi\epsilon_0 Z^2} \quad -(42)$$

3), So:

$$A_Z = \int \frac{e}{4\pi\epsilon_0 Z^2} dt \quad -(43)$$

i.e

$$A_Z(t) = \frac{e t}{4\pi\epsilon_0 Z^2} + A_Z(0) \quad -(44)$$

$$\boxed{A_Z(t) - A_Z(0) = \frac{e t}{4\pi\epsilon_0 Z^2}} \quad -(45)$$

Define:

$$v = \frac{Z}{t} \quad -(46)$$

then:

$$\boxed{\phi = -v A_Z} \quad -(47)$$

$$E_Z = \frac{-2e}{4\pi\epsilon_0 Z^2} \quad -(48)$$

ECE Level if $\underline{A} \neq \underline{0}$

The result is:

$$\boxed{\begin{aligned} \underline{E}^a &= \underline{E}_A^a + \phi^{(0)} \underline{\omega}_E \quad -(49) \\ \underline{B}^a &= \frac{\phi^{(0)}}{c} \underline{\omega}_B \quad -(50) \end{aligned}}$$

where $\underline{E}_A^a$ is due to a non-zero $\underline{A}$.

133(4): Notes by Douglas Lindstrom of 27th May 2009.

I)
$$\underline{\nabla} \times \underline{A} = 0 \Rightarrow$$
$$\frac{\partial A_i}{\partial x_j} = \frac{\partial A_j}{\partial x_i} \quad -(1)$$

but by antisymmetry:
$$\frac{\partial A_i}{\partial x_j} = -\frac{\partial A_j}{\partial x_i} \quad -(2)$$

so:
$$\frac{\partial A_i}{\partial x_j} = \frac{\partial A_j}{\partial x_i} = 0 \quad -(3)$$

i.e.
$$A_i = A_i(x_i, t), \; i = 1, 2, 3 \quad -(4)$$ ✓✓

II) & III):
$$\underline{B} = \underline{\nabla} \times \underline{A} + \frac{\phi^{(0)}}{c} \underline{\omega}_B \quad -(5)$$
$$\underline{E} = \underline{E}_A + \underline{\omega}_E \phi^{(0)} \quad -(6)$$

IV) Now use the u(1) (Maxwell Heaviside) equations. These are obsolete dogma, as is well known by now, so this part of Doug's notes investigate what they give with eqns (5) and (6). We have:
$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(7)$$
$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(8)$$
$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(9)$$
$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(10)$$

v) It is found that:

$$\underline{\nabla} \cdot \left(\phi^{(0)} \underline{\omega}_B \right) = 0 \quad -(11)$$

$$\underline{\nabla} \times \left(\underline{E}_A + \underline{\omega}_E \phi^{(0)} \right) + \frac{\partial}{\partial t} \left(\underline{\nabla} \times \underline{A} + \frac{\phi^{(0)}}{c} \underline{\omega}_B \right) = \underline{0} \quad -(12)$$

$$\underline{\nabla} \cdot \left(\underline{E}_A + \underline{\omega}_E \phi^{(0)} \right) = \rho / \epsilon_0 \quad -(13)$$

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A} + \frac{\phi^{(0)}}{c} \underline{\omega}_B \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \left(\underline{E}_A + \phi^{(0)} \underline{\omega}_E \right) = \mu_0 \underline{J} \quad -(14)$$

Now use:

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A} \right) = -\nabla^2 \underline{A} + \underline{\nabla} \left(\underline{\nabla} \cdot \underline{A} \right) \quad -(15)$$

so:

$$\nabla^2 \underline{A} = \underline{\nabla} \left(\underline{\nabla} \cdot \underline{A} \right) \quad -(16)$$

In coordinates:

$$\frac{\partial^2 A_i(x_i)}{\partial x_i^2} = \frac{\partial^2 A_i}{\partial x_i^2} \quad -(17) \checkmark$$

double checks eqs. (14) and (16).

vi) Use (11) and (12) with

$$\underline{\nabla} \times \underline{A} = \underline{0} \quad -(18)$$

then:

$$\underline{\nabla} \cdot \left(\phi^{(0)} \underline{\omega}_B \right) = 0 \quad -(19)$$

$$\underline{\nabla} \times \underline{E}_A + \phi^{(0)} \underline{\nabla} \times \underline{\omega}_E + \frac{\phi^{(0)}}{c} \frac{\partial \underline{\omega}_B}{\partial t} = \underline{0} \quad -(20)$$

3) Note (MWE)

This procedure is valid if the homogeneous charge current density is assumed to be zero. The ECE level equations are then:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad - (21)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{0} \quad - (22)$$

valid for each a and written in the general 4-D spacetime. In Minkowski spacetime there are no spin connections. In general, the homogeneous ECE equations are:

$$D_\mu \tilde{F}^{a\mu\nu} = A^{(0)} \tilde{R}^{a\ \mu\nu}_{\ \mu} \quad - (23)$$

i.e.

$$\partial_\mu \tilde{F}^{a\mu\nu} = A^{(0)} \left(\tilde{R}^{a\ \mu\nu}_{\ \mu} - \omega^a_{\ \mu b} \tilde{T}^{b\mu\nu} \right) \quad - (24)$$

These reduce to eqns. (21) and (22) if:

$$\boxed{\tilde{R}^{a\ \mu\nu}_{\ \mu} = \omega^a_{\ \mu b} \tilde{T}^{b\mu\nu}} \quad - (25)$$

Experimentally, eqn. (25) is well justified, because of the absence of a magnetic monopole and the accuracy of the Faraday induction law (22).

4) The inhomogeneous ECE laws are:

$$D_\mu F^{a\mu\nu} = A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu} \quad -(21)$$

from the Cartan Evans dual identity:

$$\boxed{D_\mu T^{a\mu\nu} := R^{a}{}_{\mu}{}^{\mu\nu}} \quad -(22)$$

using:

$$F^{a\mu\nu} = A^{(0)} T^{a\mu\nu} \quad -(23)$$

Therefore:

$$\partial_\mu F^{a\mu\nu} = A^{(0)} \left(R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} T^{b\mu\nu} \right)$$

$$:= J^{a\nu} / \epsilon_0 \quad -(24)$$

where

$$J^{a\nu} = \epsilon_0 A^{(0)} \left(R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} T^{b\mu\nu} \right) \quad -(25)$$

So

$$\boxed{\partial_\mu F^{a\mu\nu} = \frac{J^{a\nu}}{\epsilon_0}} \quad -(26)$$

In vector notation:

$$\underline{\nabla} \cdot \underline{E}^{a} = \rho / \epsilon_0 \quad -(27)$$

$$\underline{\nabla} \times \underline{B}^{a} - \frac{1}{c^2} \frac{\partial \underline{E}^{a}}{\partial t} = \mu_0 \underline{J}^{a} \quad -(28)$$

in the general spacetime where the spin connection is not zero.

5) <u>For each a:</u>

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(29)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(30)$$

Therefore <u>for each a</u>:

$$\underline{\nabla} \cdot \underline{E}_A + \underline{\nabla} \cdot \left(\phi^{(0)} \underline{\omega}_E \right) = \frac{\rho}{\epsilon_0} \quad -(31)$$

$$\frac{\phi^{(0)}}{c} \left(\underline{\nabla} \times \underline{\omega}_B \right) - \frac{1}{c^2} \left(\frac{\partial \underline{E}_A}{\partial t} + \phi^{(0)} \frac{\partial \underline{\omega}_E}{\partial t} \right) = \mu_0 \underline{J} \quad -(32)$$

Use:

$$\underline{E}_A = - \frac{\partial \underline{A}}{\partial t} - \underline{\nabla} \phi \quad -(33)$$

$$= - 2 \frac{\partial \underline{A}}{\partial t} = - 2 \underline{\nabla} \phi \quad -(34)$$

<u>This is OK</u> (MWE).

Thus:

$$\underline{\nabla} \times \underline{E}_A = \underline{0} \quad -(35)$$

and eq. (32) is

$$\frac{\phi^{(0)}}{c} \underline{\nabla} \times \underline{\omega}_B - \frac{2}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} - \frac{\phi^{(0)}}{c^2} \frac{\partial \underline{\omega}_E}{\partial t} = \mu_0 \underline{J} \quad -(36)$$

133(5): Fundamental New Results in Cartan Geometry

The tetrad postulate is:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda = 0 \quad -(1)$$

This may be simplified to:

$$\boxed{D_\mu q^a_\nu - \partial_\mu q^a_\nu = \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}} \quad -(2)$$

Therefore:

$$\boxed{\partial_\mu q^a_\nu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}} \quad -(3)$$

and:

$$\Box q^a_\nu = R q^a_\nu \quad -(4)$$

where:

$$R := q^\nu_a \partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right)$$

The first Cartan structure equation is:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu\nu} - \omega^a_{\nu\mu} \quad -(5)$$

and simplifies to:

$$\boxed{T^a_{\mu\nu} = \Gamma^a_{\mu\nu} - \Gamma^a_{\nu\mu}} \quad -(6)$$

using eq. (3). From eq. (6):

$$T^\lambda_{\mu\nu} = q^\lambda_a T^a_{\mu\nu}. \quad -(7)$$

2. Define: $$\Omega^{a}_{\mu\nu} = \omega^{a}_{\mu\nu} - \omega^{a}_{\nu\mu} \quad -(8)$$

$$\tau^{a}_{\mu\nu} = T^{a}_{\mu\nu} - \Omega^{a}_{\mu\nu}. \quad -(9)$$

Therefore: $$\boxed{\tau^{a}_{\mu\nu} = \partial_\mu q^{a}_{\nu} - \partial_\nu q^{a}_{\mu}} \quad -(10)$$

For each a:

$$\tau_{\mu\nu} = \partial_\mu q_\nu - \partial_\nu q_\mu \quad -(11)$$

Define: $$q_\mu = \left(q_0, -\underline{q}\right) \quad -(12)$$

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right), \quad -(13).$$

then: $$\underline{\tau}_1 = -\underline{\nabla} q_0 - \frac{1}{c}\frac{\partial \underline{q}}{\partial t} \quad -(14)$$

$$\underline{\tau}_2 = \underline{\nabla} \times \underline{q} \quad -(15)$$

Antisymmetry

$$\partial_\mu q_\nu = -\partial_\nu q_\mu \quad -(16)$$

so: $$\underline{\nabla} q_0 = \frac{1}{c}\frac{\partial \underline{q}}{\partial t} \quad -(17)$$

$$\frac{\partial q_i}{\partial x_j} = -\frac{\partial q_j}{\partial x_i} \quad -(18)$$

From eqns. (14) and (17):

3)

$$\underline{\nabla} \times \frac{\partial \underline{v}}{\partial t} = \underline{0} \quad -(19)$$

$$\boxed{\underline{\nabla} \times \tau_1 = \underline{0}} \quad -(20)$$

1) Therefore $\underline{\tau_1}$ in Cartan geometry is irrotational.

2) If:

$$\underline{v} \;//\; \frac{\partial \underline{v}}{\partial t} \quad -(21)$$

then:

$$\boxed{\underline{\tau_2} = \underline{0}} \quad -(22)$$

3) Eq. (22) implies:

$$\frac{\partial v_i}{\partial x_j} = \frac{\partial v_j}{\partial x_i} \quad -(23)$$

and from eqs. (18) and (23):

$$\boxed{\frac{\partial v_i}{\partial x_j} = \frac{\partial v_j}{\partial x_i} = 0} \quad -(24)$$

4) Therefore:

$$\boxed{\begin{aligned} T^a_{\ ij} &= \omega^a_{\ ij} - \omega^a_{\ ji} \\ &= \Gamma^a_{\ ij} - \Gamma^a_{\ ji} \end{aligned}} \quad -(25)$$

These are all consequences of antisymmetry.

4)

5) From eq. (4):

$$\boxed{\begin{array}{l} \text{If} \quad i, j = 1, 2, 3 \\ \text{then} \quad R = 0 \end{array}} \quad -(26)$$

6) The vectors $\underline{\tau}_1^a$ and $\underline{\tau}_2^a$ are:

$$\underline{\tau}_1^a = \tau_{01}^a \, \underline{i} + \tau_{02}^a \, \underline{j} + \tau_{03}^a \, \underline{k}, \quad -(27)$$

$$\underline{\tau}_2^a = \tau_{23}^a \, \underline{i} + \tau_{31}^a \, \underline{j} + \tau_{12}^a \, \underline{k} \quad -(28)$$

7) Therefore:

$$T_{0i}^a = \partial_0 q_i^a - \partial_i q_0^a + \Omega_{0i}^a \quad -(29)$$

$$\underline{\nabla} \times \underline{\tau}_1 = \underline{0}$$

$$T_{ij}^a = \Omega_{ij}^a$$

$$\underline{\tau}_2 = \underline{0}$$

THE CARTAN TORSION

Electrodynamics

$$F_{0i}^a = \partial_0 A_i^a - \partial_i A_0^a + A^{(0)} \Omega_{0i}^a \quad -(30)$$

$$F_{ij}^a = \frac{A^{(0)}}{c} \Omega_{ij}^a \quad -(31)$$

5) <u>Gravitation</u>

$$g^a_{0i} = \partial_0 \Phi^a_i - \partial_i \Phi^a_0 + \Phi^{(0)} \Omega^a_{0i} \quad -(32)$$

$$h^a_{ij} = \frac{\Phi^{(0)}}{c} \Omega^a_{ij} \quad -(33)$$

<u>Vector Notation</u>

$$\underline{E}^a = -\underline{\nabla} \phi^a - \frac{\partial \underline{A}^a}{\partial t} + A^{(0)} \underline{\omega}^a_E \quad -(34)$$

$$\underline{B}^a = \frac{A^{(0)}}{c} \underline{\omega}^a_B \quad -(35)$$

with:

$$\underline{\nabla} \phi^a = \frac{\partial \underline{A}^a}{\partial t} \quad -(36)$$

$$\underline{\nabla} \times \underline{A}^a = \underline{0} \quad -(37)$$

$$\underline{g}^a = -\underline{\nabla} \underline{\Phi}^a - \frac{1}{c} \frac{\partial \underline{\Phi}^a}{\partial t} + \Phi^{(0)} \underline{\omega}^a_g \quad -(38)$$

$$\underline{h}^a = \frac{\Phi^{(0)}}{c} \underline{\omega}^a_h \quad -(39)$$

$$\underline{\nabla} \times \underline{\Phi}^a = \underline{0} \quad -(40)$$

1. 133(7): Various Expressions of the Engineering Model.

The field potential relations are based on the first Cartan structure equation, which has recently been simplified by application of antisymmetry. These advances are summarized here. The original first Cartan structure equation is:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu . \quad -(1)$$

By antisymmetry, the spin torsion reduces to:

$$T^a_{ij} = \omega^a_{ib} q^b_j - \omega^a_{jb} q^b_i , \quad -(2)$$

$$i, j = 1, 2, 3 .$$

This further simplifies as follows:

$$\omega^a_{ij} = -\omega^a_{ji} = \omega^a_{ib} q^b_j , \quad -(3)$$

so:

$$T^a_{ij} = \omega^a_{ij} - \omega^a_{ji} \quad -(4)$$

The spin, or magnetic, part of the electromagnetic field is:

$$F^a_{ij} = \omega^a_{ib} A^b_j - \omega^a_{jb} A^b_i \quad -(5)$$

In vector notation:

$$\underline{B}^a = -\underline{\omega}^a_b \times \underline{A}^b \quad -(6)$$

Eq. (5) can be simplified to:

$$F^a_{ij} = A^{(0)} \left(\omega^a_{ij} - \omega^a_{ji} \right) \quad -(7)$$

or in vector notation:

$$\underline{B}^a = A^{(0)} \underline{\omega}^a_B \quad -(8)$$

Summary

$$\boxed{\underline{B}^a = A^{(0)} \underline{\omega}^a_B = -\underline{\omega}^a{}_b \times \underline{A}^b} \quad -(9)$$

Here:
$$a = (0), (1), (2), (3), \quad -(10)$$

and $\underline{\omega}^a_B$ is the magnetic spin connection.

Similarly, the gravitomagnetic field is:

$$\boxed{\underline{h}^a = \frac{\Phi^{(0)}}{c} \underline{\omega}^a_h = -\underline{\omega}^a{}_b \times \frac{\underline{\Phi}^b}{c}} \quad -(11)$$

where $\underline{\omega}^a_h$ is the gravitomagnetic spin connection.

By antisymmetry, the orbital torsion reduces to:

$$T^a{}_{0i} = \partial_0 q^a{}_i - \partial_i q^a{}_0 + \omega^a{}_{0b} q^b{}_i - \omega^a{}_{ib} q^b{}_0 \quad -(12)$$

$$i = 1, 2, 3$$

This further simplifies as follows:

$$\omega^a{}_{0i} = -\omega^a{}_{i0} = \omega^a{}_{0b} q^b{}_i \quad -(13)$$

so:

$$T^a{}_{0i} = \partial_0 q^a{}_i - \partial_i q^a{}_0 + \omega^a{}_{0i} - \omega^a{}_{i0} \quad -(14)$$

The orbital part of the electromagnetic field is

$$E^a_{0i} = \partial_0 \phi^a_i - \partial_i \phi^a_0 + \omega^a_{0b}\phi^b_i - \omega^a_{ib}\phi^b_0 \quad -(15)$$

or in vector notation:

$$\underline{E}^a = -\underline{\nabla}\phi^a_0 - \frac{\partial \underline{A}^a}{\partial t} - \omega^a_{0b}\underline{\phi}^b + \underline{\omega}^a_b \phi^b_0 \quad -(16)$$

with:

$$\underline{\nabla}\phi^a_0 = \frac{\partial \underline{A}^a}{\partial t} \quad -(17)$$

Eq. (16) simplifies to:

$$\underline{E}^a = -\underline{\nabla}\phi^a_0 - \frac{\partial \underline{A}^a}{\partial t} + \phi^{(0)}\underline{\omega}^a_E \quad -(18)$$

with:

$$\underline{\nabla}\times\underline{A}^a = \underline{0} \quad -(19)$$

and

$$\frac{\partial A_i}{\partial x_j} = \frac{\partial A_j}{\partial x_i} \quad -(20)$$

So:

$$\phi^{(0)}\underline{\omega}^a_E = -\omega^a_{0b}\underline{\phi}^b + \underline{\omega}^a_b\phi^b_0 \quad -(21)$$

Here $\underline{\omega}^a_E$ is the electric spin connection.

If

$$\underline{A}^a = \underline{0} \quad -(22)$$

4) Then

$$\underline{E}^a = \phi^{(0)} \underline{\omega}^a_E \qquad -(23)$$

Similarly, the gravitational field is:

$$\underline{g}^a = -\underline{\nabla}\Phi^a - \frac{1}{c}\frac{\partial \underline{\Phi}^a}{\partial t} + \Phi^{(0)}\underline{\omega}^a_g \qquad -(24)$$

where ω^a_g is the gravitational spin connection, defined by:

$$\Phi^{(0)}\underline{\omega}^a_g = -\omega^a_{\ 0b}\underline{\Phi}^b + \underline{\omega}^a_{\ b}\Phi^b. \qquad -(25)$$

Here:

$$\nabla \times \underline{\Phi}^a = \underline{0} \qquad -(26)$$

$$\frac{\partial \Phi_i}{\partial x_j} = -\frac{\partial \Phi_j}{\partial x_i} \qquad -(27)$$

If $\underline{\Phi}^a = \underline{0}$ $\qquad -(28)$

$$\underline{g}^a = \Phi^{(0)}\underline{\omega}^a_g \qquad -(29)$$

133(6): Implications of Antisymmetry for the Scalar Curvature of the ECE Lemma.

This scalar curvature is defined by:

$$R := q^{\mu}_{a}\, \partial^{\mu} \left(\Gamma^{a}_{\mu\nu} - \omega^{a}_{\mu\nu} \right) \quad -(1)$$

The spin part of the Cartan torsion is defined by:

$$T^{a}_{ij} = \omega^{a}_{ij} - \omega^{a}_{ji} = \Gamma^{a}_{ij} - \Gamma^{a}_{ji} \quad -(2)$$

i.e.

$$\Gamma^{a}_{ij} = \omega^{a}_{ij}. \quad -(3)$$

Therefore

$$\boxed{R(\text{spin}) = 0} \quad -(4)$$

The orbital part of the Cartan torsion is defined by:

$$T^{a}_{oi} = \partial_o q^{a}_{i} - \partial_i q^{a}_{o} + \omega^{a}_{oi} - \omega^{a}_{io} \quad -(5)$$

$$= \Gamma^{a}_{oi} - \Gamma^{a}_{io}$$

so

$$\boxed{R(\text{orbital}) \neq 0} \quad -(6)$$

Only the orbital part of the torsion contributes to the scalar curvature. Therefore curvature is orbital torsion.

If $\mu = 0$:

$$\Box q^a_0 = R_0 q^a_0 \quad - (7)$$

where:

$$R_0 = q^0_a \left(\partial^1 \left(\Gamma^a_{10} - \omega^a_{10} \right) + \partial^2 \left(\Gamma^a_{20} - \omega^a_{20} \right) + \partial^3 \left(\Gamma^a_{30} - \omega^a_{30} \right) \right) \quad - (8)$$

If $\mu = 1$

$$\Box q^a_1 = R_1 q^a_1 \quad - (9)$$

where:

$$R_1 = q^1_a \left(\partial^0 \left(\Gamma^a_{01} - \omega^a_{01} \right) + \partial^2 \left(\Gamma^a_{21} - \omega^a_{21} \right) + \partial^3 \left(\Gamma^a_{31} - \omega^a_{31} \right) \right) \quad - (10)$$

and similarly for $\mu = 2$, $\mu = 3$.

However:

$$\Gamma^a_{21} = \omega^a_{21} \quad - (11)$$

$$\Gamma^a_{31} = \omega^a_{31} \quad - (12)$$

so:

$$R_1 = q^1_a \partial^0 \left(\Gamma^a_{01} - \omega^a_{01} \right) \quad - (13)$$

$$R_2 = q^2_a \partial^0 \left(\Gamma^a_{02} - \omega^a_{02} \right) \quad - (14)$$

$$R_3 = q^3_a \partial^0 \left(\Gamma^a_{03} - \omega^a_{03} \right) \quad - (15)$$

3)

Finally:

$$R = R_0 + R_1 + R_2 + R_3 \quad -(16)$$

$$R = q^0_a \left(\partial^1 \left(\Gamma^a_{10} - \omega^a_{10} \right) + \partial^2 \left(\Gamma^a_{20} - \omega^a_{20} \right) + \partial^3 \left(\Gamma^a_{30} - \omega^a_{30} \right) \right) + q^1_a \partial^0 \left(\Gamma^a_{01} - \omega^a_{01} \right) + q^2_a \partial^0 \left(\Gamma^a_{02} - \omega^a_{02} \right) + q^3_a \partial^0 \left(\Gamma^a_{03} - \omega^a_{03} \right)$$

Electrodynamics

$$\Box A^a_\mu = R A^a_\mu \quad -(17)$$

Gravitation

$$\Box \Phi^a_\mu = R \Phi^a_\mu \quad -(18)$$

Quantum Mechanics

$$\Box q^a_\mu = R q^a_\mu \quad -(19)$$

1. 133(8): Effect of Antisymmetry on Curvature

The torsion and curvature are defined respectively by the first and second Cartan structure equations:

$$T = D \wedge q = d \wedge q + \omega \wedge q \quad -(1)$$

and

$$R = D \wedge \omega = d \wedge \omega + \omega \wedge \omega \quad -(2)$$

The antisymmetry law shows that:

$$T(\text{spin}) = \omega \wedge q \quad -(3)$$

$$R(\text{spin}) = \omega \wedge \omega \quad -(4)$$

In tensor notation:

$$T^a{}_{ij}(\text{spin}) = \omega^a{}_{ib}\, q^b{}_j - \omega^a{}_{jb}\, q^b{}_i \quad -(5)$$

$$= \omega^a{}_{ij} - \omega^a{}_{ji}, \qquad i, j = 1, 2, 3$$

and

$$\boxed{R^a{}_{bij}(\text{spin}) = \omega^a{}_{ic}\, \omega^c{}_{jb} - \omega^a{}_{jc}\, \omega^c{}_{ib}, \qquad i, j = 1, 2, 3} \quad -(6)$$

In vector notation:

$$\underline{T}^a(\text{spin}) = \underline{\omega}^a{}_b \times \underline{q}^b \quad -(7)$$

$$\underline{R}^a{}_b(\text{spin}) = \underline{\omega}^a{}_c \times \underline{\omega}_b \quad -(8)$$

with:

$$\underline{\nabla} \times \underline{q}^a = \underline{0} \quad -(9)$$

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \underline{0} \quad -(10)$$

2) These are major new mathematical results which show that the tetrad vector q^a and spin connection vector $\underline{\omega}^a{}_b$ are <u>irrotational</u>.

In ECE theory:

$$F = D \wedge A \quad -(11)$$

and

$$F(\text{spin}) = \omega \wedge A \quad -(12)$$

where $F(\text{spin})$ is the magnetic field.

The antisymmetry law shows that:

$$R^a{}_{b0i}(\text{orbital}) = \partial_0 \omega^a{}_{jb} - \partial_i \omega^a{}_{0b} + \omega^a{}_{0c}\,\omega^c{}_{ib} - \omega^a{}_{ic}\,\omega^c{}_{0b} \quad -(13)$$

$$i = 1, 2, 3$$

In vector notation:

$$\underline{R}^a{}_b(\text{orbital}) = -\frac{1}{c}\frac{\partial \underline{\omega}^a{}_b}{\partial t} - \underline{\nabla}\,\omega^a{}_{0b} - \omega^a{}_{0c}\,\underline{\omega}^c{}_b + \underline{\omega}^a{}_c\,\omega^c{}_{0b} \quad -(14)$$

where:

$$\omega^a{}_{\mu b} = \left(\omega^a{}_{0b}, -\underline{\omega}^a{}_b\right) \quad -(15)$$

with:

$$\frac{1}{c}\frac{\partial \underline{\omega}^a{}_b}{\partial t} = \underline{\nabla}\,\omega^a{}_{0b} \quad -(16)$$

$$\frac{\partial \omega^a{}_{ib}}{\partial x_j} = -\frac{\partial \omega^a{}_{jb}}{\partial x_i} \quad -(17)$$

3)

The curvature vanishes if the spin connection is zero, and the torsion vanishes if the tetrad is zero. The curvature and torsion are related by the Cartan Bianchi identity:

$$D \wedge T := q \wedge R \quad -(18)$$

and the Cartan Evans identity:

$$D \wedge \tilde{T} := q \wedge \tilde{R} \quad -(19)$$

These are:

$$d \wedge T := q \wedge R - \omega \wedge T \quad -(20)$$

$$d \wedge \tilde{T} := q \wedge \tilde{R} - \omega \wedge \tilde{T} \quad -(21)$$

In tensor notation, eqns. (20) and (21) are respectively:

$$D_\mu \tilde{T}^{a\mu\nu} = \tilde{R}^{a}{}_{\mu}{}^{\mu\nu} \quad -(22)$$

and

$$D_\mu T^{a\mu\nu} = R^{a}{}_{\mu}{}^{\mu\nu} \quad -(23)$$

i.e.

$$\partial_\mu \tilde{T}^{a\mu\nu} = \tilde{R}^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} \tilde{T}^{b\mu\nu} \quad -(24)$$

$$\partial_\mu T^{a\mu\nu} = R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} T^{b\mu\nu} \quad -(25)$$

On experimental grounds, in electrodynamics:

$$\partial_\mu \tilde{F}^{a\mu\nu} = 0 \quad -(26)$$

$$\partial_\mu F^{a\mu\nu} = \frac{J^{a\nu}}{\epsilon_0} \quad -(27)$$

4) The absence of a magnetic monopole and the Faraday law of induction therefore mean:

$$\boxed{\tilde{R}^{a\ \mu\nu}_{\ \mu} = \omega^a_{\ \mu b}\tilde{T}^{b\mu\nu}} \quad -(28)$$

This is a new fundamental law of [illegible] gravitation and electromagnetism. It implies that there is no gravito-magnetic monopole, and that there is a gravitomagnetic law of induction.

In vector notation eq. (26) is:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(29)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{0} \quad -(30)$$

and eq. (27) is:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(31)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(32)$$

The magnetic field is:

$$\underline{B}^a = -\underline{\omega}^a_{\ b} \times \underline{A}^b = 2\,\underline{\omega}^a_{\ B} \quad -(33)$$

and the electric field is:

5)

$$\underline{E}^a = -\underline{\nabla}\phi_0^a - \frac{\partial \underline{A}^a}{\partial t} + 2\phi^{(0)}\underline{\omega}^a{}_E \quad -(34)$$

where

$$2\underline{\omega}^a{}_E = -\omega_0{}^a{}_b\underline{\phi}^b + \underline{\omega}^a{}_b\phi_0^b \quad -(35)$$

These equations are constrained by:

$$\underline{\nabla} \times \underline{A}^a = \underline{0}, \quad -(36)$$

$$\underline{\nabla}\phi_0^a = \frac{\partial \underline{A}^a}{\partial t}, \quad -(37)$$

$$\frac{\partial A_i}{\partial x_j} = -\frac{\partial A_j}{\partial x_i} \quad -(38)$$

The current density is constrained by the foregoing equations of antisymmetry applied to torsion and curvature.

133(9): Some Points Raised in Discussion

1) The transformation law of the connection is given by Carroll in eq. (3.6)

$$\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial x^{\nu'}}{\partial x^{\nu}}\Gamma^{\nu}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial^2 x^{\nu'}}{\partial x^{\mu}\partial x^{\lambda}} \quad -(1)$$

This is not tensorial. Considering the second term:

$$\Gamma^{\nu'}_{\mu'\lambda'} - \Gamma^{\nu'}_{\lambda'\mu'} = -\left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial^2 x^{\nu'}}{\partial x^{\mu}\partial x^{\lambda}} - \frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial^2 x^{\nu'}}{\partial x^{\lambda}\partial x^{\mu}}\right) \quad -(2)$$

$$+ \dots$$

We have:

$$\left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\right)\left(\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\right) = \left(\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\right)\left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\right) \quad -(3)$$

$$\frac{\partial^2 x^{\nu'}}{\partial x^{\mu}\partial x^{\lambda}} = \frac{\partial^2 x^{\nu'}}{\partial x^{\lambda}\partial x^{\mu}} \quad -(4)$$

So:

$$T^{\nu}_{\mu\lambda} = \Gamma^{\nu}_{\mu\lambda} - \Gamma^{\nu}_{\lambda\mu} \quad -(5)$$

transforms as a tensor.

2) We have:

2)

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \underline{0} \quad -(6)$$

$$\underline{\nabla} \times \underline{A}^a = 0 \quad (7)$$

$$\frac{\partial \omega^a{}_{ib}}{\partial x_j} = -\frac{\partial \omega^a{}_{jb}}{\partial x_i} \quad -(8)$$

$$\frac{\partial A^a_i}{\partial x_j} = -\frac{\partial A^a_j}{\partial x_i} \quad -(9)$$

For example:

$$\underline{\omega}^a{}_b = \omega^a{}_{xb}\,\underline{i} + \omega^a{}_{yb}\,\underline{j} + \omega^a{}_{zb}\,\underline{k} \quad -(10)$$

$$\underline{A}^a = A^a_x\,\underline{i} + A^a_y\,\underline{j} + A^a_z\,\underline{k} \quad -(11)$$

So:

$$\underline{\omega}^a{}_b \times \underline{A}^b = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega^a{}_{xb} & \omega^a{}_{yb} & \omega^a{}_{zb} \\ A^a_x & A^a_y & A^a_z \end{vmatrix} \quad -(12)$$

$$= \left(\omega^a{}_{yb} A^a_z - \omega^a{}_{zb} A^a_y\right)\underline{i} - \left(\omega^a{}_{xb} A^a_z - \omega^a{}_{zb} A^a_x\right)\underline{j} + \left(\omega^a{}_{xb} A^a_y - \omega^a{}_{yb} A^a_x\right)\underline{k} \quad -(13)$$

The curls of (10) and (11) are:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial X & \partial/\partial Y & \partial/\partial Z \\ \omega^a{}_{xb} & \omega^a{}_{yb} & \omega^a{}_{zb} \end{vmatrix}$$

$$= \left(\frac{\partial \omega^a{}_{zb}}{\partial Y} - \frac{\partial \omega^a{}_{yb}}{\partial Z} \right) \underline{i} - \left(\frac{\partial \omega^a{}_{zb}}{\partial X} - \frac{\partial \omega^a{}_{xb}}{\partial Z} \right) \underline{j} + \left(\frac{\partial \omega^a{}_{yb}}{\partial X} - \frac{\partial \omega^a{}_{xb}}{\partial Y} \right) \underline{k} \quad — (14)$$

$$= \underline{0}$$

and similarly for $\underline{A}^a$. So:

$$\frac{\partial \omega^a{}_{zb}}{\partial Y} = \frac{\partial \omega^a{}_{yb}}{\partial Z} \quad — (15)$$

$$\frac{\partial \omega^a{}_{zb}}{\partial X} = \frac{\partial \omega^a{}_{xb}}{\partial Z} \quad — (16)$$

$$\frac{\partial \omega^a{}_{yb}}{\partial X} = \frac{\partial \omega^a{}_{xb}}{\partial Y} \quad — (17)$$

From eqns. (8) and (15) − (17):

$$\frac{\partial \omega^a{}_{ib}}{\partial x_j} = 0 \quad — (18)$$

$$\frac{\partial A^a{}_i}{\partial x_j} = 0 \quad — (19)$$

4) However,

$$\boxed{\underline{\omega}^a{}_b \times \underline{A}^b \neq \underline{0}} \quad -(20)$$

because it is the cross product of two irrotational vectors.

3) We have:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b = 0 \quad -(21)$$

so:

$$\underline{\nabla} \cdot \left(\underline{\omega}^a{}_b \times \underline{A}^b \right) = 0. \quad -(22)$$

Also:

$$A^{(0)} \underline{\nabla} \times \underline{\omega}^a{}_b - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(23)$$

and

$$\underline{\nabla} \times \underline{E}^a + A^{(0)} \frac{\partial \underline{\omega}^a{}_b}{\partial t} = \underline{0} \quad -(24)$$

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(25)$$

In eq. (22):

$$\frac{\partial}{\partial x} \left(\omega^a{}_{yb} A^b_z - \omega^a{}_{zb} A^b_y \right) + \cdots = 0 \quad -(26)$$

and this is consistent with eqns (18) & (19).

133(10): The Absence of a Magnetic Monopole from the Antisymmetry Law.

We use:

$$\partial \omega^a{}_b = -\underline{\omega}^a{}_b \times \underline{q}^b, \quad - (1)$$

therefore:

$$2\underline{B}^a = -\underline{\omega}^a{}_b \times \underline{A}^b \quad - (2)$$

The antisymmetry constraints are:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \underline{0} \quad - (3)$$

$$\underline{\nabla} \times \underline{A}^a = \underline{0} \quad - (4)$$

and:

$$\frac{\partial \omega^a{}_{ib}}{\partial x_i} = 0, \quad \frac{\partial A^a_i}{\partial x_j} = 0 \quad - (5)$$

Therefore:

$$\underline{\nabla} \cdot \underline{B}^a = -\frac{1}{2} \underline{\nabla} \cdot \left(\underline{\omega}^a{}_b \times \underline{A}^b \right) \quad - (6)$$

$$= -\frac{1}{2} \left(\frac{\partial}{\partial x} \left(\omega^a{}_{yb} A^b_z - \omega^a{}_{zb} A^b_y \right) + \right)$$

$$= 0$$

because of the constraints (3) - (5) from antisymmetry.

2

Therefore, by antisymmetry:

$$\boxed{\nabla \cdot \mathbf{B}^a = 0} \quad (7)$$

Similarly, by antisymmetry:

$$\nabla \cdot \mathbf{j}^a = 0 \quad (8)$$

and there is no spartomagnetic monopole

From the structure of the homogeneous field equation:

$$\partial_\mu \tilde{F}^{a\mu\nu} = 0 \quad (9)$$

it also follows that

$$\nabla \cdot \mathbf{B}^a = 0 \quad (10)$$

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = 0 \quad (11)$$

Therefore eqns. (7) and (10) have been derived self consistently and are self checking.

133(11): Less Restrictive Symmetry Constraint.

Using the results:

$$\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu} \quad -(1)$$

and

$$\Gamma^a_{\mu\nu} = \partial_\mu q^a_\nu + \omega^a_{\mu\nu} \quad -(2)$$

then:

$$\left(\partial_\mu q^a_\nu + \omega^a_{\mu\nu}\right) = -\left(\partial_\nu q^a_\mu + \omega^a_{\nu\mu}\right) \quad -(3)$$

This is the correct general result. If we we the particular solution:

$$\partial_\mu q^a_\nu = -\partial_\nu q^a_\mu \quad -(4)$$

$$\omega^a_{\mu\nu} = -\omega^a_{\nu\mu} \quad -(5)$$

then it can be seen that the fields are restricted to static fields. This is because:

$$\underline{E}^a = -\underline{\nabla}\phi^a_0 - \frac{\partial \underline{A}^a}{\partial t} - c\omega^a_{0b}\underline{A}^b + cA_0^{b(0)}\underline{\omega}^a_b \quad -(6)$$

and

$$\underline{\nabla}\times\underline{E}^a = \underline{0} \quad -(7)$$

because:

$$\underline{\nabla}\phi^a_0 = \frac{\partial \underline{A}^a}{\partial t} \quad -(8)$$

$$cA^{(0)}\underline{\omega}^a_b = -c\omega^a_{0b}\underline{A}^b \quad -(9)$$

2) and: $\nabla \times \boldsymbol{\omega}^a{}_b = \nabla \times \mathbf{A}^b = \mathbf{0}$. —(10)

The field equations reduce to the static:

$$\left.\begin{aligned} \nabla \cdot \mathbf{B}^a &= 0 \\ \nabla \cdot \mathbf{E}^a &= \rho / \epsilon_0 \\ \frac{\partial \mathbf{B}^a}{\partial t} &= 0 = \nabla \times \mathbf{E}^a \\ \nabla \times \mathbf{B}^a &= \mu_0 \mathbf{J}^a \end{aligned}\right] \quad -(11)$$

For electrostatics and dynamics in general, eq. (3) must be used. Thus:

$$\boxed{\partial_\mu A^a_\nu + \omega^a{}_{\mu b} A^b_\nu = -\left(\partial_\nu A^a_\mu + \omega^a{}_{\nu b} A^b_\mu\right)} \quad -(12)$$

For example:

$$c \nabla A^a_0 - c A^b_0 \boldsymbol{\omega}^a{}_b = \frac{\partial \mathbf{A}^a}{\partial t} + c\, \omega^a{}_{0b} \mathbf{A}^b \quad -(13)$$

and

$$\partial_i A^a_j + \omega^a{}_{ib} A^b_j = -\left(\partial_j A^a_i + \omega^a{}_{jb} A^b_i\right) \quad -(14)$$

$$i, j = 1, 2, 3 .$$

) 133(12): Antisymmetries generated by the Commutator.

The fundamental equation is:

$$[D_\mu, D_\nu]V^\rho = \partial_\mu\partial_\nu V^\rho + (\partial_\mu\Gamma^\rho_{\nu\sigma})V^\sigma + \Gamma^\rho_{\nu\sigma}\partial_\mu V^\sigma$$
$$-\Gamma^\lambda_{\mu\nu}\partial_\lambda V^\rho - \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\lambda\sigma}V^\sigma + \Gamma^\rho_{\mu\sigma}\partial_\nu V^\sigma + \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\nu\lambda}V^\lambda$$
$$-(\mu \leftrightarrow \nu) \quad -(1)$$
$$= (\partial_\mu\Gamma^\rho_{\nu\sigma} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma})V^\sigma$$
$$-(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})D_\lambda V^\rho \quad -(2)$$

Each of the six terms on the right hand side of eq. (2) is antisymmetric so:

$$\partial_\mu\Gamma^\rho_{\nu\sigma} = -\partial_\nu\Gamma^\rho_{\mu\sigma} \quad -(3)$$
$$\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} = -\Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \quad -(4)$$
$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \quad -(5)$$

The reason is that:

$$[D_\mu, D_\nu]V^\rho = -[D_\nu, D_\mu]V^\rho \quad -(6)$$

Standard gravitational theory is trivially incorrect for at least two reasons:

1) $$\Gamma^\lambda_{\mu\nu} = ? \; \Gamma^\lambda_{\nu\mu} \quad -(7)$$

2) it asserts that only the sum of the first four terms on the right hand side is antisymmetric, i.e.

2)

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu} \qquad -(8)$$

where:

$$R^\rho{}_{\sigma\mu\nu} := \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \qquad -(9)$$

This is called the curvature tensor or Riemann tensor.

The correct result is eqs. (3) to (5). This is easily checked from eq. (1), where:

$$\partial_\mu \partial_\nu V^\rho = -\partial_\nu \partial_\mu V^\rho \qquad -(10)$$

but, from space coordinate orthogonality:

$$\partial_\mu \partial_\nu V^\rho = \partial_\nu \partial_\mu V^\rho \qquad -(11)$$

so:

$$[\partial_\mu, \partial_\nu] V^\rho = 0 \qquad -(12)$$

Q.E.D.

In Cartan geometry, the tetrad postulate is:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda = 0 \qquad -(13)$$

where:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu, \quad \Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda \qquad -(14)$$

so:

$$\Gamma^a_{\mu\nu} = q^a_\lambda \Gamma^\lambda_{\mu\nu} \qquad -(15)$$

$$= -\Gamma^a_{\nu\mu}$$

$$\boxed{\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu} = \partial_\mu q^a_\nu + \omega^a_{\mu\nu}} \qquad -(16).$$

1) 133(13): Simple Demonstration of the Incorrectness of U(1) E/m Theory

In the U(1) gauge theory, methods are used which are borrowed from Riemann geometry. In U(1) electrodynamics the covariant derivative is:

$$D_\mu = \partial_\mu - i g A_\mu \quad -(1)$$

where g is a proportionality that is scalar valued. Here A_μ is the four potential. The commutator of covariant derivatives acts on the gauge field ϕ. Thus:

$$[D_\mu, D_\nu]\phi = [\partial_\mu - igA_\mu, \partial_\nu - igA_\nu]\phi \quad -(2)$$

$$= [\partial_\mu, \partial_\nu]\phi - ig[A_\mu, \partial_\nu]\phi - ig[\partial_\mu, A_\nu]\phi - g^2[A_\mu, A_\nu]\phi$$

by multiplying out the right hand side of eq. (2). Now use antisymmetry:

$$[D_\mu, D_\nu]\phi = -[D_\nu, D_\mu]\phi \quad -(3)$$

$$[\partial_\mu, \partial_\nu]\phi = -[\partial_\nu, \partial_\mu]\phi \quad -(4)$$

$$[A_\mu, \partial_\nu]\phi = -[\partial_\nu, A_\mu]\phi \quad -(5)$$

$$[\partial_\mu, A_\nu]\phi = -[A_\nu, \partial_\mu]\phi \quad -(6)$$

$$[A_\mu, A_\nu]\phi = -[A_\nu, A_\mu]\phi \quad -(7)$$

It is known from coordinate orthogonality that:

$$[\partial_\mu, \partial_\nu]\phi = 0. \quad -(8)$$

So we obtain:

2)

$$[D_\mu, D_\nu]\phi = -ig[\partial_\mu, A_\nu]\phi + ig[\partial_\nu, A_\mu]\phi - g^2[A_\mu, A_\nu]\phi \quad -(9)$$

$$= -ig\left([\partial_\mu, A_\nu]\phi - [\partial_\nu, A_\mu]\phi - ig[A_\mu, A_\nu]\phi\right) \quad -(10)$$

By definition:

$$[\partial_\mu, A_\nu]\phi = \partial_\mu(A_\nu\phi) - A_\nu(\partial_\mu\phi). \quad -(11)$$

Use the Leibnitz Theorem:

$$\partial_\mu(A_\nu\phi) = (\partial_\mu A_\nu)\phi + A_\nu(\partial_\mu\phi) \quad -(12)$$

Therefore:

$$[\partial_\mu, A_\nu]\phi = (\partial_\mu A_\nu)\phi. \quad -(13)$$

Similarly:

$$[\partial_\nu, A_\mu]\phi = (\partial_\nu A_\mu)\phi \quad -(14)$$

From eqs. (5) and (6):

$$(\partial_\mu A_\nu)\phi = -(\partial_\nu A_\mu)\phi \quad -(15)$$

$$(\partial_\nu A_\mu)\phi = -(\partial_\mu A_\nu)\phi \quad -(16)$$

and

$$\boxed{\partial_\mu A_\nu = -\partial_\nu A_\mu} \quad -(17)$$

From eq. (7):

$$\boxed{[A_\mu, A_\nu] = -[A_\nu, A_\mu]} \quad -(18)$$

Using these results:

$$[D_\mu, D_\nu]\phi = -ig\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]\right)\phi \quad -(19)$$

The Fundamental Errors of U(1) Electrodynamics

1) It asserts incorrectly that only the following quantity is antisymmetric:

$$F_{\mu\nu} = -F_{\nu\mu} \quad -(20)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \quad -(21)$$

There is no reason for this, the correct result is eq. (17). $F_{\mu\nu}$ is the U(1) electromagnetic field tensor.

2) It asserts:

$$[A_\mu, A_\nu] \overset{?}{=} 0. \quad -(22)$$

Again this is incorrect experimentally. The inverse Faraday effect shows that this commutator is non-zero. This has been known for almost sixty years.

As shown in recent work, eq. (17) means:

$$\underline{\nabla}\,\phi = \frac{\partial \underline{A}}{\partial t} \quad -(23)$$

so

$$\underline{\nabla} \times \underline{\nabla}\,\phi = \underline{\nabla} \times \left(\frac{\partial \underline{A}}{\partial t}\right) = \underline{0} \quad -(24)$$

Therefore:

$$\frac{\partial}{\partial t}\left(\underline{\nabla} \times \underline{A}\right) = \underline{0} \quad -(25)$$

In u(1) electrodynamics:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(26)$$

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \underline{\nabla}\phi \quad -(27)$$

so:

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(28)$$

$$\frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(29)$$

Therefore eq. (17) implies that if $\underline{A}$ is non-zero, $\underline{E}$ and $\underline{B}$ are static. In u(1) electrodynamics there can be no radiation, an incorrect result. The usual u(1) assumption for a static electric field is:

$$\underline{A} = \underline{0} \quad -(30)$$

so the static electric field is:

$$\underline{E} = -\underline{\nabla}\phi . \quad -(31)$$

If this assumption is used, then eq. (17) implies:

$$\underline{E} = \underline{0} \quad -(32)$$

$$\underline{B} = \underline{0} \quad -(33)$$

and so u(1) is completely incorrect, as are all attempts at a unified field theory based on u(1).

1) 133(14): Final Version of the Engineering Model

$$\underline{E}^a = -\underline{\nabla}\phi^a - \frac{\partial \underline{A}^a}{\partial t} - c\omega^a_{0b}\underline{A}^b + cA^b_0\underline{\omega}^a_b \quad -(1)$$

$$\underline{B}^a = \underline{\nabla}\times\underline{A}^a - \underline{\omega}^a_b\times\underline{A}^b \quad -(2)$$

Antisymmetry Constraint

Define:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b}q^b_\nu \quad -(3)$$

then:

$$\boxed{\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu}} \quad -(4)$$

where:

$$\Gamma^a_{\mu\nu} = \partial_\mu q^a_\nu + \omega^a_{\mu\nu} \quad -(5)$$

$$= \partial_\mu q^a_\nu + \omega^a_{\mu b}q^b_\nu \quad -(6)$$

Therefore:

$$\boxed{\begin{aligned}&\partial_\mu A^a_\nu + \omega^a_{\mu b}A^b_\nu \\ &= -\left(\partial_\nu A^a_\mu + \omega^a_{\nu b}A^b_\mu\right)\end{aligned}} \quad -(7)$$

Electric Field

$$\boxed{\partial_0 A^a_i + \omega^a_{0b}A^b_i = -\left(\partial_i A^a_0 + \omega^a_{ib}A^b_0\right)}$$

$$i = 1, 2, 3 \quad -(8)$$

2)

Magnetic Field

$$\partial_i A^a_j + \omega^a_{\ ib} A^b_j = -\left(\partial_i A^a_j + \omega^a_{\ ib} A^b_j\right)$$

$i, j = 1, 2, 3.$ — (9)

Field Equations

If the magnetic monopole is zero:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(10)$$

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{0} \quad -(11)$$

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(12)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(13)$$

If there is polarization and magnetization, eqs. (12) and (13) become:

$$\underline{\nabla} \cdot \underline{D}^a = \rho^a \quad -(14)$$

$$\underline{\nabla} \times \underline{H}^a - \frac{\partial \underline{D}^a}{\partial t} = \underline{J}^a \quad -(15)$$

$$\underline{D}^a = \epsilon_0 \underline{E}^a + \underline{P}^a \; ; \quad \underline{B}^a = \mu_0 \left(\underline{H}^a + \underline{M}^a\right) \quad -(16)$$

Here:

$a = (0), (1), (2), (3)$

1. 133(15) : Linearized Constraints.

The electromagnetic field is:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + A^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right) \quad -(1)$$

with the antisymmetry constraint:

$$\partial_\mu A^a_\nu + A^{(0)}\omega^a_{\mu\nu} = -\left(\partial_\nu A^a_\mu + A^{(0)}\omega^a_{\nu\mu}\right) \quad -(2)$$

For each a:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + A^{(0)}\left(\omega_{\mu\nu} - \omega_{\nu\mu}\right) \quad -(3)$$

In vector notation:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} + \underline{E}\,(\text{connection}) \quad -(4)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} + \underline{B}\,(\text{connection}) \quad -(5)$$

where:

$$\underline{E}\,(\text{connection}) = cA^{(0)}\underline{\omega}_E \quad -(6)$$

$$\underline{B}\,(\text{connection}) = A^{(0)}\underline{\omega}_B \quad -(7)$$

Here:

$$\underline{\omega}_E = \omega_{XE}\,\underline{i} + \omega_{YE}\,\underline{j} + \omega_{ZE}\,\underline{k}, \quad -(8)$$

$$\underline{\omega}_B = \omega_{XB}\,\underline{i} + \omega_{YB}\,\underline{j} + \omega_{ZB}\,\underline{k}, \quad -(9)$$

with

$$\left.\begin{aligned} \omega_{XE} &= -\left(\omega_{01} - \omega_{10}\right) \\ \omega_{YE} &= -\left(\omega_{02} - \omega_{20}\right) \\ \omega_{ZE} &= -\left(\omega_{03} - \omega_{30}\right) \end{aligned}\right\} \quad -(10)$$

2.

$$\left.\begin{aligned} \omega_{xB} &= -(\omega_{23} - \omega_{32}) \\ \omega_{yB} &= -(\omega_{31} - \omega_{13}) \\ \omega_{zB} &= -(\omega_{12} - \omega_{21}) \end{aligned}\right\} \quad -(11)$$

Electric Antisymmetry Constraints -

$$\left.\begin{aligned} \partial_0 A_1 + \partial_1 A_0 &= \underline{A}^{(0)}(\omega_{01} + \omega_{10}) \\ \partial_0 A_2 + \partial_2 A_0 &= \underline{A}^{(0)}(\omega_{02} + \omega_{20}) \\ \partial_0 A_3 + \partial_3 A_0 &= \underline{A}^{(0)}(\omega_{03} + \omega_{30}) \end{aligned}\right\} \quad -(12)$$

Magnetic Antisymmetry Constraints -

$$\left.\begin{aligned} \partial_1 A_2 + \partial_2 A_1 &= \underline{A}^{(0)}(\omega_{12} + \omega_{21}) \\ \partial_3 A_1 + \partial_1 A_3 &= \underline{A}^{(0)}(\omega_{31} + \omega_{13}) \\ \partial_2 A_3 + \partial_3 A_2 &= \underline{A}^{(0)}(\omega_{23} + \omega_{32}) \end{aligned}\right\} \quad -(13)$$

Here

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right) \quad -(14)$$

$$A_\mu = (A_0, -\underline{A}) \quad -(15)$$

Therefore eq. (12) is:

$$-\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \underline{\nabla} A_0 = -A^{(0)} \underline{\Omega}_E \quad -(16)$$

$$\underline{\nabla} \times \underline{A} = -A^{(0)} \underline{\Omega}_B \quad -(17)$$

where:

$$\underline{\Omega}_E = -(\omega_{01} + \omega_{10})\underline{i} - (\omega_{02} + \omega_{20})\underline{j} - (\omega_{03} + \omega_{30})\underline{k} \quad -(18)$$

3.

$$\underline{\Omega}_B = -(\omega_{23} + \omega_{32})\underline{i} - (\omega_{31} + \omega_{13})\underline{j} - (\omega_{12} + \omega_{21})\underline{k} \quad -(19)$$

Summary for each a.

$$\underline{E} - \underline{E}\,(\text{connection}) = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} \quad -(20)$$

$$\underline{B} - \underline{B}\,(\text{connection}) = \underline{\nabla} \times \underline{A} \quad -(21)$$

$$\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} = -\phi^{(0)}\underline{\Omega}_E \quad -(22)$$

$$\underline{\nabla} \times \underline{A} = -\frac{\phi^{(0)}}{c}\underline{\Omega}_B \quad -(23)$$

where

$$\phi^{(0)} = c A^{(0)} \quad -(24)$$

Therefore, for each polarization a:

$$\underline{E}\,(ECE) := \underline{E} - \underline{E}\,(\text{connection}) = -2\underline{\nabla}\phi \mp \phi^{(0)}\underline{\Omega}_E$$

$$\underline{B}\,(ECE) := \underline{B} - \underline{B}\,(\text{connection}) = -\frac{\phi^{(0)}}{c}\underline{\Omega}_B$$

$$-(25)$$

133(16): Spin Connection Resonance with Antisymmetry

The electromagnetic field is:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + A^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right) \quad -(1)$$

where:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b}\, q^b_\nu . \quad -(2)$$

For each a:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \omega_{\mu b} A^b_\nu - \omega_{\nu b} A^b_\mu \quad -(3)$$

If $a = b$ (field has only one polarization):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \omega_\mu A_\nu - \omega_\nu A_\mu \quad -(4)$$

The electric field is given by:

$$F_{0i} = \partial_0 A_i - \partial_i A_0 + \omega_0 A_i - \omega_i A_0 \quad -(5)$$

so:

$$\underline{E} = -\underline{\nabla}\,\phi_0 - \frac{\partial \underline{A}}{\partial t} - \omega_0 \underline{\phi} + \underline{\omega}\,\phi_0 \quad -(6)$$

where:

$$A_\mu = (A_0, -\underline{A}) = \left(\frac{\phi_0}{c}, -\underline{A}\right) \quad -(7)$$

$$\omega_\mu = (\omega_0, -\underline{\omega}) \quad -(8)$$

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right) \quad -(9)$$

$$A_0 = c\,\phi_0, \quad \underline{A} = c\,\underline{\phi} \quad -(9a)$$

2)

Antisymmetry Constraint

$$\partial_\mu A_\nu + \omega_\mu A_\nu = -\left(\partial_\nu A_\mu + \omega_\nu A_\mu\right) \quad -(10)$$

i.e.

$$\boxed{\partial_\mu A_\nu + \partial_\nu A_\mu = -\left(\omega_\mu A_\nu + \omega_\nu A_\mu\right)} \quad -(11)$$

For the electric field:

$$\partial_0 A_i + \partial_i A_0 = -\left(\omega_0 A_i + \omega_i A_0\right) \quad -(12)$$

i.e.

$$-\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \underline{\nabla} A_0 = -\left(-\omega_0 \underline{A} - \underline{\omega} A_0\right) \quad -(13)$$

$$= \omega_0 \underline{A} + \underline{\omega} A_0$$

$$\underline{\nabla} A_0 - \underline{\omega} A_0 = \frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \omega_0 \underline{A} \quad -(14)$$

Finally use:

$$\left.\begin{aligned} c\phi_0 &= c A_0 \\ c\underline{\phi} &= c\underline{A} \end{aligned}\right\} \quad -(15)$$

$$\boxed{\underline{\nabla}\phi_0 - \underline{\omega}\phi_0 = \frac{\partial \underline{\phi}}{\partial t} + \omega_0 \underline{\phi}} \quad -(16)$$

Spin Connection Resonance in the Coulomb Law

For each a the Coulomb law is:

3)

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(17)$$

where:

$$\boxed{\begin{aligned}\underline{E} &= 2\left(\underline{\nabla}\phi_0 - \underline{\omega}\,\phi_0\right) \\ &= 2\left(\frac{\partial \underline{\phi}}{\partial t} + \omega_0 \underline{\phi}\right)\end{aligned}} \quad -(18)$$

In this case there is only one, longitudinal, polarization. Therefore:

$$\underline{\nabla} \cdot \left(\underline{\nabla}\phi_0 - \underline{\omega}\,\phi_0\right) = \frac{1}{2}\frac{\rho}{\epsilon_0} \quad -(19)$$

$$\nabla^2 \phi_0 - \underline{\nabla} \cdot \left(\underline{\omega}\,\phi_0\right) = \frac{1}{2}\frac{\rho}{\epsilon_0}$$

$$\boxed{\begin{aligned}\nabla^2 \phi_0 - \left(\underline{\nabla} \cdot \underline{\omega}\right)\phi_0 - \underline{\omega} \cdot \underline{\nabla}\phi_0 \\ = \frac{1}{2}\frac{\rho}{\epsilon_0}\end{aligned}} \quad -(20)$$

This produces Euler - Bernoulli resonance if $\underline{\nabla} \cdot \underline{\omega}$ is negative valued, and if ρ is oscillatory, as in previous work.

11

133(17): Hodge Duality Constraints

4(1) Homogeneous Equations

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(1)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(2)$$

where:

$$\underline{B} = \underline{\nabla} \times \underline{A} \; ; \; \underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} \quad -(3)$$

Now write this as:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad -(3)$$

Let:

$$\underline{B} = -i\underline{E}/c, \quad \underline{E} = ic\underline{B} \quad -(4)$$

then we obtain:

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(5)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(6)$$

i.e.

$$\partial_\mu F^{\mu\nu} = 0 \quad -(7)$$

Eqs. (4) imply:

$$\boxed{\underline{\nabla} \times \underline{A} = \frac{i}{c}\left(\underline{\nabla}\phi + \frac{\partial \underline{A}}{\partial t}\right)} \quad -(8)$$

This is a constraint equation. It applies to the vacuum fields $\underline{E}$ and $\underline{B}$, i.e.

2) to the case:

$$\rho = 0, \quad \underline{J} = 0 \quad -(9)$$

or

$$J^{\mu} = 0 \quad -(10)$$

$$-(11)$$

<u>ECE Homogeneous Equations</u>.

$$\underline{E}^a = -\underline{\nabla}\phi^a - \frac{\partial \underline{A}^a}{\partial t} - c\omega^a_{0b}\underline{A}^b + c\underline{\omega}^a_{\ b}A^b_0$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a_{\ b} \times \underline{A}^b \quad -(12)$$

These are constrained by antisymmetry, and also by the ECE level of eq. (8). For the electric field, antisymmetry means:

$$\underline{\nabla}\phi^a + \underline{\omega}^a_{\ b}\phi^b_0 = \frac{\partial \underline{A}^a}{\partial t} + c\omega^a_{0b}\underline{A}^b \quad -(13)$$

and eq. (8) generalizes to:

$$\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a_{\ b} \times \underline{A}^b = \frac{i}{c}\left(\underline{\nabla}\phi^a + \frac{\partial \underline{A}^a}{\partial t} + c\omega^a_{0b}\underline{A}^b - c\underline{\omega}^a_{\ b}A^b_0\right)$$

$$-(14)$$

This simplifies <u>for one polarization</u> to:

$$\underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}$$
$$= \frac{\partial i}{c} \left(\underline{\nabla} \phi - \underline{\omega} \phi \right)$$
$$= \frac{\partial i}{c} \left(\frac{\partial \underline{A}}{\partial t} + c\omega_0 \underline{A} \right)$$

— (15)

The inhomogeneous equations at the u(1) level are:

$$\underline{\nabla} \cdot \underline{D} = \rho \quad - (16)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \quad - (17)$$

i.e.

$$\partial_\mu G^{\mu\nu} = J^\nu \quad - (18)$$

These may be transformed into:

$$\partial_\mu \tilde{G}^{\mu\nu} = \tilde{J}^\nu, \quad - (19)$$

with:

$$\underline{H} = -ic\underline{D}, \quad \underline{D} = i\underline{H}/c$$

— (20)

where:

$$\left.\begin{array}{l} \underline{D} = \epsilon_0 \underline{E} + \underline{P} \\ \underline{B} = \mu_0 (\underline{H} + \underline{M}) \end{array}\right\} \quad - (21)$$