1) 128(1): Metric Development of ECE and Derivation of the Engineering Model

The metric is conventionally defined as:

$$x_\mu = g_{\mu\nu} x^\nu \quad -(1)$$

and so:

$$x^\mu = g^\mu{}_\nu x^\nu \quad -(2)$$

where:

$$g^\mu{}_\nu = g^{\mu\alpha} g_{\alpha\nu} \quad -(3)$$

In general: $V^\mu = q^\mu{}_\nu V^\nu$ $-(4)$

where $q^\mu{}_\nu$ is a tetrad. These equations are defined in the general spacetime of any dimension. If the tetrad is defined by:

$$V^a = q^a{}_\mu V^\mu \quad -(5)$$

the first Cartan structure equation is:

$$T^a{}_{\mu\nu} = \partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu \quad -(6)$$

Eq. (2) implies that it is possible to define a torsion tensor through the metric $g^\mu{}_\nu$:

$$\boxed{T^\lambda{}_{\mu\nu} = \partial_\mu g^\lambda{}_\nu - \partial_\nu g^\lambda{}_\mu + \Gamma^\lambda{}_{\mu\kappa} g^\kappa{}_\nu - \Gamma^\lambda{}_{\nu\kappa} g^\kappa{}_\mu} \quad -(7)$$

and so the field density is:

2) $$\boxed{F^{\lambda}_{\mu\nu} = \partial_\mu A^{\lambda}_{\nu} - \partial_\nu A^{\lambda}_{\mu} + \Gamma^{\lambda}_{\mu\kappa} A^{\kappa}_{\nu} - \Gamma^{\lambda}_{\nu\kappa} A^{\kappa}_{\mu}} \quad -(8)$$

This means that the potential density of the electromagnetic field is directly proportional to the metric:

$$A^{\lambda}_{\mu} = A^{(0)} g^{\lambda}_{\mu} \quad -(9)$$

$$\boxed{A^{\lambda}_{\mu} = A^{(0)} g^{\lambda\alpha} g_{\alpha\mu}} \quad -(10)$$

In field ~~theory~~ (Ryder, 1996) the angular energy / momentum is obtained from the angular energy / momentum density by an integration over:

$$\lambda = 0 \quad -(11)$$

$$J_{\mu\nu} = \int J^{0}_{\mu\nu} \, dV \quad -(12)$$

The e/m field density is directly proportional to the angular momentum density as shown in paper 127, so:

$$F_{\mu\nu} = \int F^{0}_{\mu\nu} \, dV \quad -(13)$$

where:

$$F^{0}_{\mu\nu} = \partial_\mu A^{0}_{\nu} - \partial_\nu A^{0}_{\mu} + \Gamma^{0}_{\mu\kappa} A^{\kappa}_{\nu} - \Gamma^{0}_{\nu\kappa} A^{\kappa}_{\mu} \quad -(14)$$

3) Therefore:

$$F_{\mu\nu} = \int \left(\partial_\mu A^{0}_{\nu} - \partial_\nu A^{0}_{\mu} + \Gamma^{0}_{\mu\kappa} A^{\kappa}_{\nu} - \Gamma^{0}_{\nu\kappa} A^{\kappa}_{\mu} \right) dV \quad -(15)$$

expresses the electromagnetic field in terms of the metric. In eq. (15)

$$\partial_\mu A_\nu = \int \partial_\mu A^{0}_{\nu} \, dV \quad -(16)$$

$$\Gamma_{\mu\kappa} A^{\kappa}_{\nu} = \int \Gamma^{0}_{\mu\kappa} A^{\kappa}_{\nu} \, dV \quad -(17)$$

and so on. Therefore:

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \Gamma_{\mu\kappa} A^{\kappa}_{\nu} - \Gamma_{\nu\kappa} A^{\kappa}_{\mu}} \quad -(18)$$

Diagonal Metric

If the metric is diagonal, then:

$$A^{0}_{0} = A^{(0)} g^{00} g_{00} = A^{(0)}, \quad -(19)$$

$$A^{1}_{1} = A^{2}_{2} = A^{3}_{3} = A^{(0)} \quad -(20)$$

similarly:

there are terms such as:

$$A^{0}_{1} = A^{(0)} \left(g^{00} g_{01} + \ldots + g^{03} g_{31} \right) \quad -(21)$$

which exist if there are off diagonal elements of the metric.

The engineering model is obtained from eq. (18)

128(2) : <u>New General Condition for any Metric</u>

Start with the definition of the ~~metric~~ tetrad:

$$V^a = q^a_\mu V^\mu \quad - (1)$$

A particular case of this is:

$$x^\kappa = g^\kappa_\mu x^\mu \quad - (2)$$

where g^κ_μ is the metric. Consider:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda, \quad - (3)$$

and:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b. \quad - (4)$$

Eqs. (3) and (4) imply the tetrad postulate:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda = 0, \quad - (5).$$

i.e.

$$\Gamma^\lambda_{\mu\nu} = q^\lambda_a \left(\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu \right) \quad - (6)$$

using

$$q^a_\lambda q^\alpha_a = \delta^\alpha_\lambda. \quad - (7)$$

The special case of eq. (2) implies ~~that~~ eq. (4) becomes:

$$\partial_\mu g^\kappa_\nu + \Gamma^\kappa_{\mu\lambda} g^\lambda_\nu - \Gamma^\lambda_{\mu\nu} g^\kappa_\lambda = 0 \quad - (8)$$

i.e. <u>a is replaced by κ, b by λ, and ω by Γ</u>. In eq. (8):

$$\Gamma^\kappa_{\mu\lambda} g^\lambda_\nu = \Gamma^\kappa_{\mu\nu} \quad - (9)$$

and

$$\Gamma^\lambda_{\mu\nu} g^\kappa_\lambda = \Gamma^\kappa_{\mu\nu}. \quad - (10)$$

2) Therefore:

$$\boxed{\partial_\mu g^{\kappa}_{\ \nu} = 0} \quad — (11)$$

This is an important new fundamental equation for the metric:

$$g^{\kappa}_{\ \nu} = g^{\kappa\alpha} g_{\alpha\nu}. \quad — (12)$$

Any Riemannian metric obeys eq. (11), and in general any metric in any spacetime of any dimension, in general a spacetime with torsion and curvature.

Diagonal Metric

In this case off-diagonals are zero, so eq. (11) produces:

$$\partial_\mu \left(g^{00} g_{00} \right) = 0 \quad — (13)$$

$$\partial_\mu \left(g^{11} g_{11} \right) = 0 \quad — (14)$$

$$\partial_\mu \left(g^{22} g_{22} \right) = 0 \quad — (15)$$

$$\partial_\mu \left(g^{33} g_{33} \right) = 0 \quad — (16)$$

Eqs. (13) - (16) are true for any μ, because

$$g^{00} g_{00} = g^{11} g_{11} = g^{22} g_{22} = g^{33} g_{33} = 1 \quad — (17)$$

3) In general, in four dimensions:

$$\partial_\mu \left(g^{\kappa\alpha} g_{\alpha\nu} \right) = 0 \quad - (18)$$

where:

$$g^{\kappa\alpha} g_{\alpha\nu} = g^{\kappa 0} g_{0\nu} + g^{\kappa 1} g_{1\nu} + g^{\kappa 2} g_{2\nu} + g^{\kappa 3} g_{3\nu} \quad - (19)$$

and where the metric has diagonal and off-diagonal elements.

The metric of the orbital theorem of paper 111 obeys eq. (11) because it is a diagonal metric.

Computer Test

It is possible now to test metrics with off-diagonal elements by using computer algebra with eq. (18).

1) Tetrad Postulate in the Base Manifold (Note 128(3)).

The derivation of Cartan geometry in tetrad postulate is based on the use of a base manifold labelled μ and a tangent spacetime labelled a. The derivation of the postulate is given in proof three and is reviewed here before specializing to the case of one manifold. The covariant derivative of Riemann geometry is:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \quad -(1)$$

and is defined in the base manifold. The tetrad is defined by:

$$V^a = q^a_\mu V^\mu \quad -(2)$$

In the tangent spacetime the covariant derivative is defined by:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \quad -(3)$$

The basis elements for this tangent space (a Minkowski space) are $\hat{q}_a$, and the basis elements of the base manifold are ∂_μ. The advantage of using the tangent space is that $e(a)$ can be Pauli matrices, for example, so the Dirac equation and spinors can be considered. Therefore, $\omega^a_{\mu b}$ is known as the spin connection.

The complete vector field is the same for eqns. (1) & (3):

$$DV = D_\mu V^\nu dx^\mu \otimes \partial_\nu = D_\mu V^a dx^\mu \otimes \hat{q}_a \quad -(4)$$

The vector components and basis elements are related by equations similar to eq. (2):

$$\hat{q}_a = q^\sigma_a \partial_\sigma , \quad -(5)$$

$$V^a = q^a_\nu V^\nu . \quad -(6)$$

Therefore in eq. (4):

$$D V = \left(\partial_\mu \left(q^a_\nu V^\nu \right) + \omega^a_{\mu b} q^b_\lambda V^\lambda \right) dx^\mu \otimes \left(q^\sigma_a \partial_\sigma \right) \quad -(7)$$

$$= q^\sigma_a \left(\partial_\mu \left(q^a_\nu V^\nu \right) + \omega^a_{\mu b} q^b_\lambda V^\lambda \right) dx^\mu \otimes \partial_\sigma \quad -(8)$$

The Cartan convention is:

$$q^\sigma_a q^a_\nu = \delta^\sigma_\nu \quad -(9)$$

where:

$$\delta^\sigma_\nu = 1 \text{ if } \sigma = \nu, \quad -(10)$$

$$\delta^\sigma_\nu = 0 \text{ if } \sigma \neq \nu. \quad -(11)$$

Eq. (8) is:

$$D V = q^\sigma_a q^a_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_\sigma + \dots \quad -(12)$$

$$= \delta^\sigma_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_\sigma + \dots \quad -(13)$$

$$= \delta^0_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_0 + \delta^1_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_1 + \delta^2_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_2 + \delta^3_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_3 + \dots \quad -(14)$$

$$= \delta^0_0 \partial_\mu V^0 dx^\mu \otimes \partial_0 + \delta^1_1 \partial_\mu V^1 dx^\mu \otimes \partial_1 + \delta^2_2 \partial_\mu V^2 dx^\mu \otimes \partial_2 + \delta^3_3 \partial_\mu V^3 dx^\mu \otimes \partial_3 + \dots \quad -(15)$$

$$= \partial_\mu V^0 dx^\mu \otimes \partial_0 + \partial_\mu V^1 dx^\mu \otimes \partial_1 + \partial_\mu V^2 dx^\mu \otimes \partial_2 + \partial_\mu V^3 dx^\mu \otimes \partial_3 + \dots \quad -(16)$$

$$= \partial_\mu V^\nu dx^\mu \otimes \partial_\nu + \dots \quad -(17)$$

So:

$$D V = \left(\partial_\mu V^\nu + q^\nu_a \left(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda \right) V^\lambda \right) dx^\mu \otimes \partial_\nu \quad -(18)$$

5.

From eqs. (1), (4) and (18):

$$\boxed{\Gamma^{\nu}_{\mu\lambda} = q^{\nu}_{a}\left(\partial_\mu q^{a}_{\lambda} + \omega^{a}_{\mu b} q^{b}_{\lambda}\right)} \quad - (19)$$

The connection can therefore be expanded in terms of the tetrad and the spin connection. As can be seen from eq. (4) this is a basic property of the complete vector field DV, and so is a very fundamental result.

Multiply both sides of eq. (19) by q^{a}_{ν} and use:

$$q^{a}_{\nu} q^{\nu}_{a} = 1 \quad - (20)$$

to obtain __the tetrad postulate__:

$$\boxed{\begin{aligned} D_\mu q^{a}_{\lambda} &= \partial_\mu q^{a}_{\lambda} + \omega^{a}_{\mu b} q^{b}_{\lambda} - \Gamma^{\nu}_{\mu\lambda} q^{a}_{\nu} \\ &= 0 \end{aligned}} \quad - (21)$$

Eq. (21) uses the rule for the covariant derivative of the mixed index rank two tensor q^{a}_{λ}.

__Tetrad Postulate in the Base Manifold.__

Consider the basic definition (2) in the base manifold, when:

$$a = \nu. \quad - (22)$$

Then:

$$V^{\nu} = q^{\nu}_{\mu} V^{\mu} \quad - (23)$$

where V^{ν} and V^{μ} are any vectors. In the special case:

$$V^\mu = x^\mu, \quad V^\nu = x^\nu \quad -(24)$$

then:

$$x^\nu = q^\nu_\mu \, x^\mu. \quad -(25)$$

However, the <u>metric</u> is defined by:

$$x_\mu = g_{\mu\nu} \, x^\nu, \quad -(26)$$

so:

$$\boxed{x^\nu = g^\nu_\mu \, x^\mu}, \quad -(27)$$

where:

$$g^\nu_\mu = g^{\nu\alpha} g_{\alpha\mu} \quad -(28)$$

$$= g^{\nu 0} g_{0\mu} + \ldots + g^{\nu 3} g_{3\mu}. \quad -(29)$$

In the case of eq. (22), eq. (3) becomes:

$$D_\mu V^\nu = \partial_\mu V^\nu + \omega^\nu_{\mu b} V^b. \quad -(30)$$

However, it is known that:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda, \quad -(31)$$

so:

$$\omega^\nu_{\mu b} V^b = \Gamma^\nu_{\mu\lambda} V^\lambda. \quad -(32)$$

Therefore:

$$b = \lambda. \quad -(33)$$

The tetrad postulate (21) becomes:

$$\boxed{D_\mu g^\kappa_\nu = \partial_\mu g^\kappa_\nu + \Gamma^\kappa_{\mu\lambda} g^\lambda_\nu - \Gamma^\lambda_{\mu\nu} g^\kappa_\lambda = 0}$$

$$-(34)$$

5.) It is seen that:

$$q^{\nu}_{\mu} = g^{\nu}_{\mu}. \quad -(35)$$

In Cartan geometry there are results such as:

$$T^{a}_{\mu\nu} = q^{a}_{\lambda} T^{\lambda}_{\mu\nu} \quad -(36)$$

and

$$\omega^{a}_{\mu b} = \omega^{\lambda}_{\mu b} q^{b}_{\lambda}. \quad -(37)$$

Therefore using eqn. (35):

$$\Gamma^{\kappa}_{\mu\nu} = g^{\lambda}_{\nu} \Gamma^{\kappa}_{\mu\lambda}, \quad -(38)$$

$$\Gamma^{\kappa}_{\mu\nu} = g^{\kappa}_{\lambda} \Gamma^{\lambda}_{\mu\nu}. \quad -(39)$$

So we obtain:

$$\boxed{D_\mu g^{\kappa}_{\nu} = \partial_\mu g^{\kappa}_{\nu} = 0} \quad -(40)$$

This is a very fundamental result, valid for any Riemannian spacetime in any dimension.

Computer Tests

Test whether solution of the Einstein field equation obey:

$$\boxed{\partial_\mu (g^{\kappa\alpha} g_{\alpha\nu}) = 0} \quad -(41)$$

1) Note 128(4): Basic Hypotheses.

There are two fundamental hypotheses of ECE theory.

1)
$$A^a_\mu = A^{(0)} q^a_\mu \quad -(1)$$

which implies:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad -(2)$$

and:

2)
$$J^a_{\mu\nu} = \frac{c}{k} T^a_{\mu\nu}. \quad -(3)$$

Here A^a_μ is the electromagnetic potential density and $J^a_{\mu\nu}$ the canonical energy / momentum density. These can be expressed in the base manifold as:

$$A^\kappa_\mu = A^{(0)} q^\kappa_\mu \quad -(4)$$

and

$$J^\kappa_{\mu\nu} = \frac{c}{k} T^\kappa_{\mu\nu}. \quad -(5)$$

In the base manifold:

$$T^\kappa_{\mu\nu} = \partial_\mu q^\kappa_\nu - \partial_\nu q^\kappa_\mu + \Gamma^\kappa_{\mu\lambda} q^\lambda_\nu - \Gamma^\kappa_{\nu\lambda} q^\lambda_\mu \quad -(6)$$

$$= \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu}$$

because:

$$\partial_\mu q^\kappa_\nu = \partial_\nu q^\kappa_\mu = 0 \quad -(7)$$

and

$$\Gamma^\kappa_{\mu\nu} = \Gamma^\kappa_{\mu\lambda} q^\lambda_\nu \quad -(8)$$

$$\Gamma^\kappa_{\nu\mu} = \Gamma^\kappa_{\nu\lambda} q^\lambda_\mu. \quad -(9)$$

Therefore:

$$J^{\kappa}_{\mu\nu} = \frac{c}{k}\left(\Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu}\right) \quad (10)$$

However:

$$J^{\kappa}_{\mu\nu} = x_\mu p^{\kappa}_{\nu} - x_\nu p^{\kappa}_{\mu} \quad (11)$$

which is a generalization of:

$$\underline{J} = \underline{r} \times \underline{p} \quad (12)$$

Here:

$$x_\mu = (ct, -\underline{r}) \quad (13)$$

and

$$\boxed{p^{\kappa}_{\nu} = p^{(0)} q^{\kappa}_{\nu}} \quad (14)$$

so:

$$J^{\kappa}_{\mu\nu} = p^{(0)}\left(x_\mu q^{\kappa}_{\nu} - x_\nu q^{\kappa}_{\mu}\right) \quad (15)$$

and

$$\boxed{\Gamma^{\kappa}_{\mu\nu} = \frac{k}{c} p^{(0)} x_\mu q^{\kappa}_{\nu}} \quad (16)$$

The units of $kp^{(0)}/c$ are $\underline{1}/Ar$, so:

$$\boxed{\Gamma^{\kappa}_{\mu\nu} = \frac{1}{Ar} x_\mu q^{\kappa}_{\nu}} \quad (17)$$

Eq. (14) defines energy momentum density p^{κ}_{ν} in terms of the metric q^{κ}_{ν}.

3) The electromagnetic field density is:

$$F^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\lambda} A^{\lambda}_{\nu} - \Gamma^{\kappa}_{\nu\lambda} A^{\lambda}_{\mu} \qquad (18)$$
$$= A^{(0)} \left(\Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} \right)$$

Using:
$$\Gamma^{\kappa}_{\mu\nu} = -\Gamma^{\kappa}_{\nu\mu} \qquad (19)$$

then
$$\boxed{F^{\kappa}_{\mu\nu} = 2A^{(0)} \Gamma^{\kappa}_{\mu\nu}} \qquad (20)$$

This result is consistent with:

$$F^{\kappa}_{\mu\nu} = q^{\kappa}_{a} F^{a}_{\mu\nu}$$
$$= A^{(0)} q^{\kappa}_{a} \left(\partial_\mu q^{a}_{\nu} - \partial_\nu q^{a}_{\mu} + \omega^{a}_{\mu b} q^{b}_{\nu} - \omega^{a}_{\nu b} q^{b}_{\mu} \right) \qquad (21)$$
$$= A^{(0)} \left(\Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} \right)$$

Therefore the analysis is self-consistent.

128(5): General Interpretation of the Tetrad in the Base Manifold.

In general:

$$V^\mu = q^\mu_{\ \nu} V^\nu \quad — (1)$$

where all quantities are expressed in the base manifold. Here V^μ and V^ν are vectors and $q^\mu_{\ \nu}$ is the tetrad. It is proven here that under all conditions:

$$\partial_\lambda q^\mu_{\ \nu} = 0. \quad — (2)$$

Proof

The torsion is: — (3)

$$T^\kappa_{\ \mu\nu} = \Gamma^\kappa_{\ \mu\nu} - \Gamma^\kappa_{\ \nu\mu}$$
$$= \partial_\mu q^\kappa_{\ \nu} - \partial_\nu q^\kappa_{\ \mu} + \Gamma^\kappa_{\ \mu\lambda} q^\lambda_{\ \nu} - \Gamma^\kappa_{\ \nu\lambda} q^\lambda_{\ \mu}.$$

However:

$$\Gamma^\kappa_{\ \mu\nu} = \Gamma^\kappa_{\ \mu\lambda} q^\lambda_{\ \nu} \quad — (4)$$
$$\Gamma^\kappa_{\ \nu\mu} = \Gamma^\kappa_{\ \nu\lambda} q^\lambda_{\ \mu}, \quad — (5)$$

so:

$$\partial_\mu q^\kappa_{\ \nu} - \partial_\nu q^\kappa_{\ \mu} = 0. \quad — (6)$$

It is known from the tetrad postulate that:

$$\partial_\mu q^\kappa_{\ \nu} = 0, \quad — (7)$$
$$\partial_\nu q^\kappa_{\ \mu} = 0. \quad — (8)$$

In the special case:

$$V^\mu = x^\mu \quad — (9)$$

then:

2)

$$q^{\mu}{}_{\nu} = g^{\mu}{}_{\nu} \quad -(10)$$

where the metric $g^{\mu}{}_{\nu}$ is defined by:

$$x^{\mu} = g^{\mu}{}_{\nu}\, x^{\nu}. \quad -(11)$$

Cartesian Interpretation

An example of the meaning of eqns. (7) and (8) may be given in a Cartesian space, in which for example:

$$x^1 = X, \quad x^2 = Y \quad -(12)$$

In this case there is no connection:

$$\Gamma^{\kappa}{}_{\mu\nu} = 0 \quad -(13)$$

because the space is a flat space. So there is no torsion. Therefore:

$$\partial_{\mu}\, q^1{}_2 = 0 \quad -(14)$$

where:

$$V^1 = q^1{}_2 V^2. \quad -(15)$$

Similarly:

$$\partial_{\mu}\, g^1{}_2 = 0 \quad -(16)$$

where

$$x^1 = g^1{}_2\, x^2 \quad -(17)$$

with

$$g^1{}_2 = g^{1\alpha} g_{\alpha 2}. \quad -(18)$$

However:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(19)$$

so

$$g^1{}_2 = 0 \quad -(20)$$

3) The Existence of Spin Connection Resonance (SCR)

SCR can exist if and only if:

$$V^a = q^a_\mu V^\mu \quad - (21)$$

and:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad - (22)$$

This means that the representation space of V^a must be distinct from that of V^μ. In this case, for example:

$$V^a = (ct, (1), (2), (3)), \quad - (23)$$

$$V^\mu = (ct, X, Y, Z) \quad - (24)$$

where respectively, the complex circular and Cartesian representations are used for the space-like parts of V^a and V^μ. In this case:

$$\boxed{D_\mu q^a_\nu = 0,} \quad - (25)$$

but

$$\boxed{\partial_\mu q^a_\nu \neq 0.} \quad - (26)$$

Therefore the tetrad may be interpreted as a Cartan spinor. Cartan inferred both tetrads and spinors. The latter are of course used in the Dirac equation, whose

4)

basis elements are the Pauli matrices (SU(2)). In strong field theory the basis elements are the Gell-Mann matrices, (SU(n)).

Three dimensional space may be represented by a basis set consisting either of the Cartesian unit vectors, $\underline{i}$, $\underline{j}$ and $\underline{k}$, or of the complex circular unit vectors, $\underline{e}^{(1)}$, $\underline{e}^{(2)}$ and $\underline{e}^{(3)}$. Both are O(3) representation spaces. The electromagnetic potential density may be represented by A_μ^a and the electromagnetic field density by $F_{\mu\nu}^a$. It is this representation that gives rise to SCR. The latter does not arise from eqn. (1) because the latter leads to eqns. (7) and (8).