1) 123(1): Simplification of the Derivation of the ECE Engineering Model.

The derivation of the ECE engineering model can be simplified and therefore strengthened by developing a theory with two index tensors replacing three index tensors. The complete information is still contained in the three index tensors, but for practical purposes the two index tensors are sufficient. In general a two index tensor for example can be integrated over a hypersurface to give a one index tensor. This is described for example in:

Lewis H. Ryder, "Quantum Field Theory", (Cambridge University Press, 2nd. ed. 1996), in his chapter 3.

In the context of Ryder's chapter three, a conserved charge of the Noether theorem is defined by:

$$Q_\sim = \int_\sigma J^\mu_\sim \, d\sigma_\mu \quad -(1)$$

where the integral is over a spacelike hypersurface σ_μ. In the same way as a two-dimensional surface can be defined in a 3-D space, a three dimensional hypersurface can be defined in a 4-D spacetime. Thus, for example:

$$Q_\sim = \int_V J^0_\sim \, d^3x \quad -(2)$$

in which the time-like component is held constant and denoted 0, i.e. we choose:

$$\mu = 0 \quad - (3)$$

and $$d\sigma_0 = d^3x = dV. \quad - (4)$$

Therefore $Q_\sim$ is an integration of $J^0_\sim$ over volume:

$$\boxed{Q_\sim = \int J^0_\sim \, dV}. \quad - (5)$$

Similarly, Ryder defines the three index tensor:

$$J^{\mu\rho\sigma} = -\frac{1}{2}\left(T^{\mu\rho}x^\sigma - T^{\mu\sigma}x^\rho\right) \quad - (6)$$

where:

$$T^{\mu\rho} = T^{\rho\mu} \quad - (7)$$

is the canonical energy momentum) density tensor and x^μ is the coordinate vector:

$$x^\mu = (ct, X, Y, Z). \quad - (8)$$

Thus $J^{\mu\rho\sigma}$ is the canonical angular energy angular momentum density tensor. As argued in previous work $J^{\mu\rho\sigma}$ is proportional to the spacetime torsion tensor $T^{\mu\rho\sigma}$.

Ryder defines the angular momentum tensor

3)

9:

$$J^{\rho\sigma} = -J^{\sigma\rho} \quad -(9)$$

$$= \int J^{0\rho\sigma} dV$$

i.e. is the integral over the volume occupied by the tensor density $J^{0\rho\sigma}$. In analogy it is possible to define the tensor:

$$\boxed{\begin{aligned} T^{\rho\sigma} &= -T^{\sigma\rho} \\ &= \int T^{0\rho\sigma} dV \end{aligned}} \quad -(10)$$

where the anti-symmetric tensor of rank two in eq. (10) is an angular momentum with in the Einstein constant c/k:

$$\boxed{J^{\mu\nu} = \frac{c}{k} T^{\mu\nu}} \quad -(11)$$

Check a Units

The units of $T^{\mu\nu}$ are m^2 because the units of the three index torsion tensor are m^{-1} and it is integrated over volume (m^3). The units of angular momentum are $kgm\, m^2 s^{-1}$. The Einstein constant is:

$$k = \frac{8\pi G}{c^2} = 1.86595 \times 10^{-26} \, N s^2 kg^{-2} \quad -(12)$$

where

$$N = kgm \, m \, s^{-2} \quad -(13)$$

so the units of k are $m \, kgm^{-1}$. Therefore the units either side of eq. (11) are:

$$kgm \, m^2 s^{-1} = m s^{-1} kgm \, m^{-1} m^2 \quad -(14)$$

Adapting the tensor $T^{\mu\nu}$ is very useful for a simplified derivation of the ECE engineering model:

$$T^{\mu\nu} = \frac{k}{c} J^{\mu\nu} \quad -(15)$$

The electromagnetic field tensor is therefore:

$$F^{\mu\nu} = A^{(0)} T^{\mu\nu} = -F^{\nu\mu} \quad -(16)$$

The theory of angular momentum is highly developed, and it is well known that angular momentum operators are rotation generators within T. Therefore by integrating the three index torsion tensor it becomes a rotation generator. The integration is carried out over $T^{\kappa\mu\nu}$ with:

$$\boxed{\kappa = 0} \quad -(17)$$

i.e.:

$$T^{\mu\nu} = \int T^{0\mu\nu} \, dV \quad -(18)$$

and:

$$F^{\mu\nu} = \int F^{0\mu\nu} \, dV. \quad -(19)$$

Similarly:

$$\tilde{T}^{\mu\nu} = \int \tilde{T}^{0\mu\nu} \, dV \quad -(20)$$

and

$$\tilde{F}^{\mu\nu} = \int \tilde{F}^{0\mu\nu} \, dV. \quad -(21)$$

The homogeneous and inhomogeneous field equations of ECE theory are given respectively by:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa\ \mu\nu}_{\ \mu} \quad -(22)$$

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \mu\nu}_{\ \mu} \quad -(23)$$

Therefore integrating a both sides of each equation, with eq. (17):

$$\int D_\mu \tilde{T}^{0\mu\nu} \, dV = \int \tilde{R}^{0\ \mu\nu}_{\ \mu} \, dV \quad -(24)$$

$$\int D_\mu T^{0\mu\nu} \, dV = \int R^{0\ \mu\nu}_{\ \mu} \, dV \quad -(25)$$

In the next note it will be shown how this procedure simplifies the ECE field equations and derivation of the ECE engineering model.

1) 123(2): Simplification of the ECE Field Equations

Start with the homogeneous and inhomogeneous field equations, respectively:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu} \quad -(1)$$

and

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \ \mu}. \quad -(2)$$

These can be expanded as:

$$\partial_\mu \tilde{T}^{\kappa\mu\nu} + \omega^\kappa_{\ \mu\lambda} \tilde{T}^{\lambda\mu\nu} = \tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu} \quad -(3)$$

and

$$\partial_\mu T^{\kappa\mu\nu} + \omega^\kappa_{\ \mu\lambda} T^{\lambda\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \ \mu} \quad -(4)$$

and rewritten as:

$$\partial_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{j}^{\kappa\nu} \quad -(5)$$

and

$$\partial_\mu T^{\kappa\mu\nu} = j^{\kappa\nu} \quad -(6)$$

where:

$$\tilde{j}^{\kappa\nu} = \tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu} - \omega^\kappa_{\ \mu\lambda} \tilde{T}^{\lambda\mu\nu} \quad -(7)$$

and

$$j^{\kappa\nu} = R^{\kappa\ \ \mu\nu}_{\ \ \mu} - \omega^\kappa_{\ \mu\lambda} T^{\lambda\mu\nu}. \quad -(8)$$

Now use:

$$\boxed{T^{\mu\nu} = \int_\sigma T^{\kappa\mu\nu} \sigma_\kappa} \quad -(9)$$

and

$$\boxed{j^{\nu} = \int_\sigma j^{\kappa\nu} \sigma_\kappa} \quad -(10)$$

and similarly for the homogeneous equation. Here σ_κ is the hypersurface:

2)

$$\sigma_\kappa = (\sigma_0, -\underline{\sigma}) \qquad -(11)$$

$$= (V, -\underline{\sigma})$$

where V is the volume. So:

$$\sigma_0 = V. \qquad -(12)$$

Now consider:

$$\kappa = 0 \qquad -(13)$$

in eqns. (9) and (10), so:

$$T^{\mu\nu} = \int T^{0\mu\nu}\, dV \qquad -(14)$$

$$j^\nu = \int j^{0\nu}\, dV. \qquad -(15)$$

Thus:

$$\boxed{\begin{aligned} T^{0\mu\nu} &= T^{\mu\nu} / V \\ j^{0\nu} &= j^\nu / V \end{aligned}} \qquad \begin{aligned} -(16) \\ -(17) \end{aligned}$$

With these definitions, eqs (1) and (2) reduce to:

$$\boxed{\begin{aligned} \partial_\mu \tilde{T}^{\mu\nu} &= \tilde{j}^\nu \\ \partial_\mu T^{\mu\nu} &= j^\nu \end{aligned}} \qquad -(18)$$

where:

$$\boxed{\begin{aligned} j^\nu &= V \left(R^{0}{}_{\mu}{}^{\mu\nu} - \omega^{0}{}_{\mu\lambda} T^{\lambda\mu\nu} \right) \\ \tilde{j}^\nu &= V \left(\tilde{R}^{0}{}_{\mu}{}^{\mu\nu} - \omega^{0}{}_{\mu\lambda} \tilde{T}^{\lambda\mu\nu} \right) \end{aligned}} \qquad -(19)$$

In electrodynamics:

$$\partial_\mu \tilde{F}^{\mu\nu} = A^{(0)} \tilde{j}^{\nu} \quad -(20)$$

$$\partial_\mu F^{\mu\nu} = A^{(0)} j^{\nu} \quad -(21)$$

and if there is no magnetic monopole:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad -(22)$$

$$\partial_\mu F^{\mu\nu} = A^{(0)} j^{\nu} \quad -(23)$$

In vector notation eq. (22) is:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(24)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(25)$$

and eq. (23) is:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(26)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(27)$$

These are the generally covariant equations of classical electrodynamics. Here:

$$\boxed{\rho = c \epsilon_0 A^{(0)} \left(R^{\circ}{}_{\mu}{}^{\mu 0} - \omega^{\circ}{}_{\mu\lambda} T^{\lambda\mu 0} \right)} \quad -(28)$$

in $C m^{-3}$, because $c A^{(0)} = J C^{-1} = \text{volt}$, and $\epsilon_0 = J^{-1} C^2 m^{-1}$

and the current density is:

$$\underline{J} = J_x \underline{i} + J_y \underline{j} + J_z \underline{k} \quad -(29)$$

where:

$$\left.\begin{aligned} J_x &= \epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 1} - \omega^{\circ}_{\mu\lambda} T^{\lambda\mu 1}\right) \\ J_y &= \epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 2} - \omega^{\circ}_{\mu\lambda} T^{\lambda\mu 2}\right) \\ J_z &= \epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 3} - \omega^{\circ}_{\mu\lambda} T^{\lambda\mu 3}\right) \end{aligned}\right\} - (30)$$

The index λ is restricted to 0 in eqs. (28) and (30) because of eqs. (16) and (17), so:

$$\boxed{\begin{aligned} \rho &= c\epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 0} - \omega^{\circ}_{\mu 0} T^{0\mu 0}\right) &&- (31) \\ J_x &= \epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 1} - \omega^{\circ}_{\mu 0} T^{0\mu 1}\right) &&- (32) \\ J_y &= \epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 2} - \omega^{\circ}_{\mu 0} T^{0\mu 2}\right) &&- (33) \\ J_z &= \epsilon_0 A^{(0)}\left(R^{\circ}{}_{\mu}{}^{\mu 3} - \omega^{\circ}_{\mu 0} T^{0\mu 3}\right) &&- (34) \end{aligned}}$$

It is seen that this method removes the need to vary K, and simplifies the derivation of the ECE engineering model. Charge and current densities are defined by curvature, spin connection and torsion as in eqs. (31) to (34)

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1)

123(3): Simplified Derivation of the Field Potential Equation

Start with the Cartan Maurer structure equation:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(1)$$

and define:

$$T_{\mu\nu} = \int T^a_{\mu\nu}\, d\sigma_a \quad -(2)$$

where the hypersurface is σ defined by:

$$\sigma_a = (\sigma_0, -\underline{\sigma}) \quad -(3)$$

in the Minkowski spacetime denoted by a. When:

$$a = 0 \quad -(4)$$

then:

$$T_{\mu\nu} = \int T^0_{\mu\nu}\, dV \quad -(4)$$

and similarly:

$$q_\mu = \int q^0_\mu\, dV. \quad -(5)$$

Thus:

$$\boxed{T^0_{\mu\nu} = T_{\mu\nu} / V, \quad q^0_\mu = q_\mu / V} \quad -(6)$$

Therefore eq. (1) simplifies to: $-(7)$

$$T_{\mu\nu} = \partial_\mu q_\nu - \partial_\nu q_\mu + \omega_{\mu b} q^b_\nu - \omega_{\nu b} q^b_\mu .$$

Finally, define:

$$\boxed{\omega_\mu q_\nu := \omega_{\mu b} q^b_\nu} . \quad -(8)$$

This means that the b index is restricted to 0

) because:

$$q_\nu = V q_\nu^{\circ} \quad - (9)$$

So: $$\omega_\mu = \omega_{\mu o} / V \quad - (10)$$

Therefore:

$$\boxed{\begin{aligned} T_{\mu\nu} &= \partial_\mu q_\nu - \partial_\nu q_\mu + \omega_\mu q_\nu - \omega_\nu q_\mu \\ &= (\partial_\mu + \omega_\mu) q_\nu - (\partial_\nu + \omega_\nu) q_\mu \end{aligned}} \quad - (11)$$

and:

$$\boxed{F_{\mu\nu} = (\partial_\mu + \omega_\mu) A_\nu - (\partial_\nu + \omega_\nu) A_\mu} \quad - (12)$$

where:

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \underline{\nabla} \right) \quad - (13)$$

$$A_\mu = \left(\frac{\phi}{c}, -\underline{A} \right) \quad - (14)$$

$$\omega_\mu = \left(\frac{\omega_0}{c}, -\underline{\omega} \right) \quad - (15)$$

so:

$$\boxed{\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} + \underline{\omega} \phi - \omega_0 \underline{A}} \quad - (16)$$

$$\boxed{\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}} \quad - (17)$$